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GEOMETRY IN GREEK ART

DEPTH AND MOVEMENT

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GEOMETRY IN GREEK ART

STUDIED IN THE LIGHT
OF EGYPTIAN METHODS

ELSE CHRISTIE KIELLAND

DREYER

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Preface

Brahms about the creating Mozart:

«That is the way Mozart composed. He was once asked what the process was with him while composing and he replied: «The process with me is like a vivid dream.»

«He then went on and described how ideas, clothed in the proper musical setting, streamed down upon him, just as they do with me. Of course, a composer must have mastered the technique of composition, form, theory, harmony, counterpoint, instrumentation – but any musical person can do that if he has the proper application. –»

From Arthur M. Abell
Talks with great Composers
Philosophical Library, New York

The process of creation is analogous with regard to pictorial art and architecture and it is my hope that this way of thinking is evident in the trilogy Depth and Movement.

As a very young painter I realised that the teaching of the academies did not offer the knowledge required to understand how the great masterpieces of art had been created. The ancients I felt, must have possessed a wisdom since lost, and it seemed likely that this wisdom had its basis in geometry. Among certain groups of painters there was a great interest in geometry, and my first main contact with this movement I gained from *L'Esprit Nouveau* (Paris 1920–1928), where names like Le Corbusier dominated. From then on, I was primarily concerned with finding out how geometry had been applied in practice until it eventually became a part of the process of artistic creation. It is not sufficient to be able to demonstrate geometrical proportions in a work of art, for there are hardly limits to these in a work planned geometrically or, as the Greeks said, symmetrically. We must find *the way of thinking* which underlies this process, the method which was subject to laws, the mastery of which provided the creative imagination with inspiration and stimuli.

During the war I started to organise the material I had been collecting since I began my investigations in 1929. In 1947 I published a small book entitled *The Human Figure* (Oslo 1947). The Egyptian part of the book I still accept today, and in *Geometry in Egyptian Art* (London 1955) this material was published in a wider context, and further developed. The Greek investigations on the other hand, I must now reject; I consider them without any bearing on the investigations published in the present work.

The three books published so far, 'Geometry in Egyptian Art' (London 1955), 'Geometry in Greek Art' (Oslo 1983) and 'Stave Churches and Viking Ships' (Oslo 1981), all deal with Depth and Movement. They form an entity, but may be read independently.

I have spent about twenty-five years since the appearance of *Geometry in Egyptian Art* on the study of the Greek material, alternating with periods of creative artistic activity. Now I realise that the great length of time spent on these studies has been beneficial to my work.

Some of my analyses in this book are based on measured drawings which have been prepared by others. This applies first and foremost to the Theseion in Athens, for which I used Herbert Koch's immensely thorough survey. His plates include all the dimensions of this temple, large and small, so that every stage of my analysis can be checked arithmetically. The same is true of two measured drawings by K. Rhomaios (published in Christos Karusos' '*Aristodikos*'). One of these concerns a statue base, the other deals with the great Archaic statue from which his book derives its name.

It may be noticed that the bibliography is rather short, in spite of the fact that there exists a large body of literature on geometry in art, which is of the greatest interest. My own investigations are concerned with one particular aspect of a complicated problem, and I concentrate on this.

I should like to make grateful acknowledgement to all those who by their great kindness and helpfulness facilitated my work at the various museums from which my material is taken. I should also like to thank my good friends, and experts in related fields, for encouraging and stimulating me in my work. My gratitude is also due to the translator, Elizabeth S. Seeberg, for the thoroughness with which she tackled a task which cannot have been easy, and to Ivar Østby who has checked my constructions arithmetically. My thanks also go to Gunnar Aune for his final tracing of my geometrical analyses and for his faithful support and interest during the final phase of the production of my book.

Oslo, February 1984 *Else Christie Kielland*

Introduction

The Importance of Geometry in Greek Philosophy

Most scholars agree that Archaic Greek art was highly dependent on the art of Egypt and of the eastern Mediterranean area. In this book we shall attempt to show that the background of the peculiarly Greek style, of its distinctive way of expression, is to be found in the geometrical methods which all these ancient cultures knew and practised. In *Geometry in Egyptian Art* (Tiranti, London 1955) I investigated these methods, and accounted for their principal features. We therefore continue to call them 'Egyptian', in spite of the fact that extensive study of material from the entire cultural region would seem to show that the basic principles were the same throughout.

Is it really possible to demonstrate these 'Egyptian' geometrical methods also in Greek art? Can we prove that the Greeks not only adopted these methods, but developed them in their own way, using them so extensively that one is justified in saying that their art is based on them? Do they provide part of the explanation of the development and flowering of Greek art, which created an entirely new style in the course of a few centuries? The decisive constellation is surely the fact that they were able to start with a background provided by several thousand years of tradition – a tradition representing ancient wisdom unencumbered by time or place, while they themselves were primitive beginners, full of strength and vitality, and with undiminished self-confidence backed by their knowledge of the Heroic age of the past. We need merely mention the name of Homer. Moreover, the impulses they received were so straightforward and logical that they were easily absorbed. These Egyptian methods are likely to have been formed by artist-priests, who were at all times cognizant of the fact that the 'law' was a *means of liberation, not of oppression.* It was a medium which might serve as *the tool of the creative imagination*; individuality could flourish, and in the hands of highly talented artists the 'law' could produce the strong and yet refined style whose strict conformity characterises the art of the Greeks as well as that of the Egyptians. At their new start in the 'Geometric period', the Greeks took over this artistic means of expression, gradually developing it with ever-increasing knowledge and exploiting all the possibilities it offered.

Before we present our analyses, some space should be devoted to Greek philosophy, for geometry formed an important part of the Greek way of thinking. We know that Archaic Greek philosophy had already acquired a great deal of knowledge and wisdom in this sphere from Egypt. The Ionian philosophers of the sixth century BC based their systems on Egyptian learning, and tradition has it that Thales of Milet prophesied an eclipse of the sun in 585 BC, on the basis of Egyptian knowledge. Recent research into the history of mathematics shows that Greek mathematics were based on Babylonian and Egyptian learning to a still greater extent than had previously been assumed, and recent finds attest to the very high standard of mathematics in these countries. From this background, which the Greeks had adopted from

these ancient cultures, "arose systematic, deductive mathematics, one of the Greeks' great gifts to the culture of the world" (translated from Viggo Brun: *Alt er tall* [Universitetsforlaget, Oslo 1964]). Eudemos' account, quoted by Proklos, states on the subject of geometry: "But when Thales had come from Egypt, he brought this science to Greece and discovered much himself, and on his work many new discoveries were based . . . Pythagoras' studies concerned the highest principles, and his theoretical work was free from material influence, his mind moved in the realm of pure thought. He created the theory of proportions (or of irrational numbers) and the representation of the five cosmic bodies." In the present context it is interesting to note that mathematics formed part of the faith of the Pythagoreans. God had arranged the cosmos according to numbers, harmony is divine and consists of numerical relations.

We recognise these ideas, for we have met them before: the Egyptians believed that a cosmic order had once and for all been established when the world was created by Ptah (Introduction, *Geometry in Egyptian Art*). Pythagoras (born in 570 BC) is said to have travelled to Egypt as well as to Babylonia. His followers' centre was in Sicily, and shortly after Socrates' death (399 BC) Plato came here, and received enduring impulses. No documents from the hand of Pythagoras have come down to us, but the Phillaos fragment, written by one of his followers, a contemporary of Socrates, states: "You shall not only regard the nature and the power of numbers as being at work in demonic and divine matters, but also everywhere in human works and words, as well as in all technical activity and in music".

While Pythagoras and his followers considered that reality was number, Plato connected numbers with the idea of the underlying reality. All the studies contained in his works concern ideas as such, and as the same law obtains in the world of ideas as in the world with which we are familiar and in which we live, he arrives at a closer determination of the former by means of an investigation of the latter. For "man must be regarded as a small-scale, weak and insignificant copy of the great cosmos, built of the same elements as this, but the elements which are clear and strong in the great world are here weak and insignificant" (from Hoeg's introduction to the Danish edition of Plato's works). This conception of a relationship between the two "worlds" is very close to that held by the Egyptians. A study of Plato's dialogues, especially of *Philebos*, but also of *Gorgias*, *Phaedros*, *The Statesman*, *Timaios*, *The Laws* and others, confirms this view. Plato's philosophy is borne out by the dialectical method he employs. Concepts such as pleasure, sense, goodness, beauty, etc. are explained in terms of examples taken from everyday life, with which everyone is familiar. The conclusions inferred from these examples are then transferred to the realm of ideas. The royal art of statesmanship is illustrated by examples taken from the sphere of weaving, while others are illustrated by examples from medicine, architecture, strategy, etc. These "arts" are thus assumed to be known factors, and after having examined them in detail, Plato states in conclusion that among the arts he has mentioned, those which are carried on by "the true philosopher, in the use of measure and number" (*Philebos* 57 D) are excellent above all the others. For, he says, the mediator between the great world and the small is *number*. In Philebos, Socrates speaks of . . . "that which puts an end to the conflict of opposites (the infinite and the finite) with one another, making them well-proportioned and harmonious by the introduction of number" (*Philebos* 25 D-E). "The addition of these principles will," he continues, "produce limit and establish . . . full perfection" (*Philebos* 26 A); and he devises "a law and order that are marked by limit" (*Philebos* 26 B). We must note that the Greek word here translated by "well-proportioned" and "full perfection" – and elsewhere frequently rendered by "symmetry" – is not synonymous with the English "symmetry". Its English equivalent is balance, harmony between the individual

components of a whole; literally translated, the word symmetrical means "commensurate with". As a noun it can be used in the plural in Greek: we hear for instance, that the Greek painters of the Sicyonian school studied the *symmetries*. Thus matter takes form when numbers are introduced, for by their agency the infinite, the unlimited, receives finitude and existence. Plato then speaks about the third "category", the *cause*, and continues, "Is there anything than a verbal difference between a cause and a maker? Would it not be proper to call that which makes things and that which causes them one and the same? – Quite proper." (*Philebos* 26 E). Number or mathematics, *and* the creative element – both are indispensible: "It seems clear to me that all such arts guard against exceeding the due measure or falling short of it . . . In fact it is precisely by this effort they make to maintain the due measure that they achieve effectiveness and beauty in all that they produce." (*Statesman* LS 4a).

But Plato clearly realised that these words might be misunderstood, and in Phaedros he confounds any would-be contradiction with this pregnant sentence: "But he who, having no touch of the Muses' madness in his soul, comes to the door and thinks that he will get into the temple by the help of *art* – he, I say, and his poetry are not admitted; the sane man disappears and is nowhere when he enters into rivalry with the madman." Here precedence is accorded to the inspired, creative artist – by virtue of the fluency of the madness which possesses him, he is the sovereign ruler in the realm of art.

Plato (427 – 347 BC) lived at a time when Greek culture had in all spheres, not least in that of art, attained an unequalled vitality resulting in a profusion of creativity. His was the century following the period when Greek Naturalism came into existence, a hundred years before Alexander the Great. He was sensible of the tendencies of dissolution, which were to lead to decline during the Hellenistic period, and which finally brought about the fall of Greek art. He is alarmed by this development in more ways than one. The powerful new individualism which had come into being during the preceding century constituted the principal threat to the legal framework of life. Socrates' retort in Gorgias, to the individualist Callicles, "You are neglecting geometry," is quite clear in its purport. In human life as in nature, cosmic symmetry obtains, and his warning is not aimed at Callicles alone.

In *the Laws*, the Athenian describes the Egyptians' excellent legislature and statesmanship. For instance, "outside this official list it was, and still is, forbidden to painters and all other producers of postures and representations to introduce any innovation, whether in such production or in any other branch of music, over and above the traditional forms. And if you look there, you will find that the things depicted or graven there 10 000 years ago (I mean what I say, not loosely, but literally 10 000) are no whit better or worse than the production of today, but wrought with the same art" (*The Laws* 656E – 657A).

To Plato, art and science were one, and he esteems the most exact of "arts" highest, those which "in their use of measure and number (are) immensely superior in point of exactness and truth" (*Philebos* 57 D).

Chapter I

Geometrically Determined Blocks

Plan drawings of six statue bases

When we study the Egyptian way of drawing we notice that a chair, for instance, is drawn in such a way that the seat – seen from above – and the vertical sides lie flush with one another (*Fig. 1*). The drawing of the pool surrounded by trees (*Fig. 2*) is conceived in the same way. Here all the trees are shown as children would draw them, perpendicular to the four sides of the pool. These drawings account for a certain volume or space, limited in the case of the chair by its *depth* and *height*, on the other drawing by the surface of the water and by the height of the surrounding trees. The dimensions of the subject are registered, regardless of how the human eye perceives them. This art is concerned with the true, the absolute measurements; drawings incorporating foreshortening, showing objects as they appear to the human eye, are inconceivable in Egyptian art.

A study of some rectangular caskets from *Tut-Ank-Amon*'s tomb in "Geometry in Greek Art", Tiranti, London 1953 made it quite clear that *the sum of the height and the depth is the basic dimension* from which all the other dimensions of the caskets are derived. It further appeared that this applied to all rectangular blocks. Such a method cannot, however, be employed unless the two planes are shown flush with each other on a plan drawing. If, therefore, we wish to investigate the proportions of a rectangular block, we must lay the vertical plane of the block into the picture surface, in the manner of children's drawings, flush with the horizontal plane of the block (*Fig. 3*), so that the sum of the height and the depth – AB – can be read from the drawing. Moreover, we found that Egyptian sculpture in the round and two-dimensional art were all designed in logical consequence of this way of thinking; we shall later deal in more detail with this phenomenon as it is found in Greek art.

We begin our study of Greek art with an investigation of some statue bases. They are excellent objects for studying the fundamental problems, those concerned with whether the Greeks planned the proportions of rectangular blocks in the same way as the Egyptians had done. There are a number of rectangular statue bases from different periods in the National Archaeological Museum in Athens – the statues disappeared long ago, but their bases are intact, and so well preserved that they yield reliable measurements. True, many of the edges are worn, so that absolutely accurate measurements are not always possible, and we must therefore allow for a certain margin of error.

How then, was the basic dimension AB, the sum of the height and the depth of the base, divided? It appears that the basic principles of this division remained unchanged from the Archaic to the Hellenistic period – and they were the same as those employed by the Egyptians. For this reason we found it unnecessary to arrange the analyses in chronological order – we have instead arranged them according to pedagogic principles, proceeding from the simplest to the more difficult. First, however, we shall study two geometrical diagrams, *Fig. 4* and *Fig. 7*.

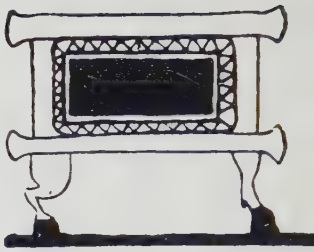


Fig. 1
Egyptian drawing of a chair (from H. Schäfer, Von ägyptischer Kunst, (Leipzig 1930).

Fig. 2
Egyptian garden (From H. Schäfer, Von ägyptischer Kunst, Leipzig 1930).

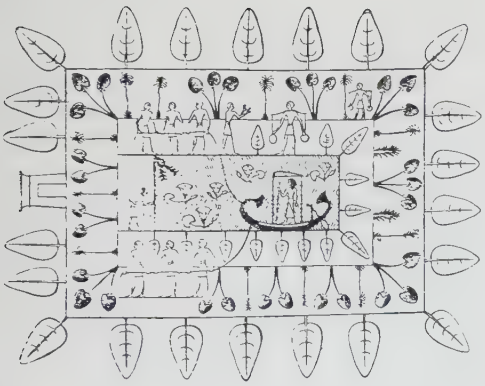
Fig. 3
Drawing of a block.

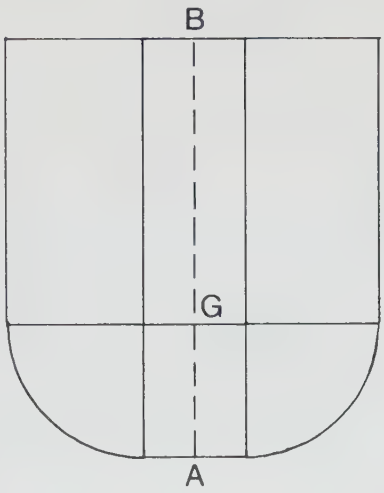
Fig. 4
Drawing of a square with a 'Golden Section' division (φ).

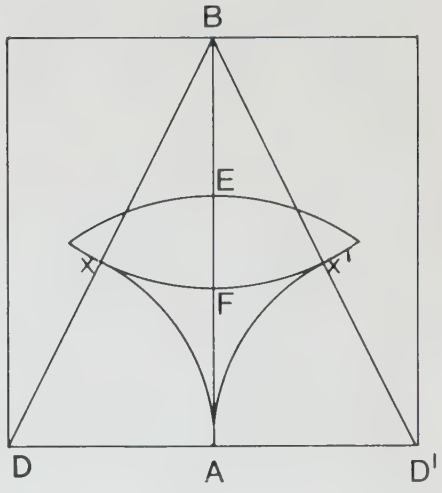
Fig. 5
Construction to find AB according to the φ division when the larger part (major) is known.

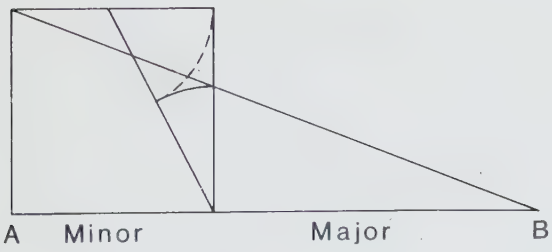
Fig. 6
Construction to find AB according to the φ division when the smaller part (minor) is known.

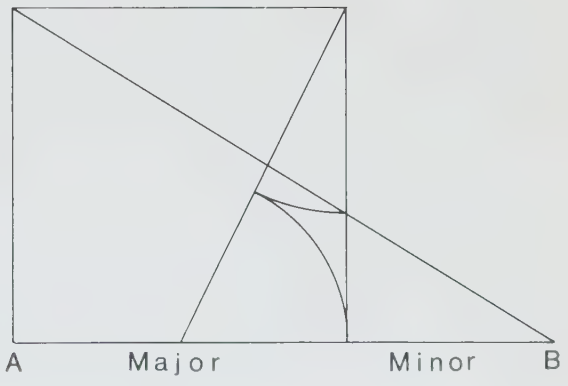
On Fig. 4, AB is the centre line of a square. We construct lines from B to the opposite corners of the square, D and D'. Arcs are described from D and D', through A, and also an arc about B through x and x', the points of intersection between the two first arcs and BD and BD'. F is the point of intersection between the arc about B and AB. An arc with the same radius as that about B is struck from A, and this determines E, its point of intersection with AB. This division is today known as 'the golden section' and is referred to by the Greek letter Φ (a designation which originated during the nineteenth century, and which stands for Pheidias). There are several ways of carrying out a Φ construction; *Fig. 5* and *Fig. 6* show how one can find AB, the sum of both parts, when one of them, either the greater (major) or the lesser (minor) is given. Every line and every point of intersection is derived from AB, the basic dimension. New points of division are determined by combining this construction with the inscribed and the circumscribed circles of the square, by employing diagonals and by joining the centre points of the other three sides of the square to the corners opposite them, *Fig. 7*. The distance between these new points of division produces new dimensions, all determined as functions of the basic dimension, AB. We shall return to this later.











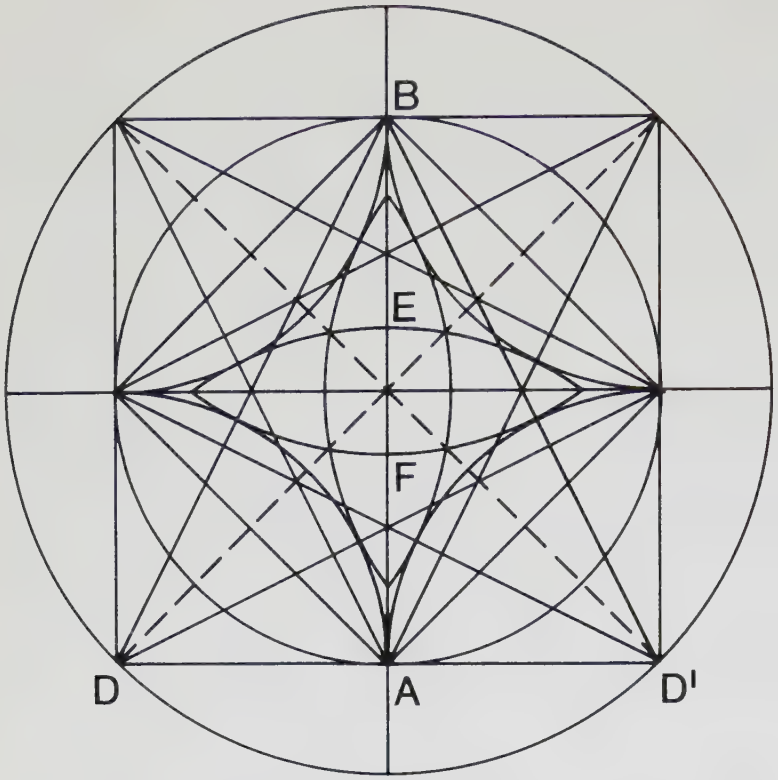


Fig. 7
Fig. 4 repeated, related to the other sides of the square whose diagonals, inscribed and circumscribed circles, are drawn.

PLATE I (Athens N.M. 3476), height 30.0 cm, depth and width 81.0 cm, shows a measured drawing and analysis of a base which is particularly simple, as its top surface is square. AB is equal to the sum of the height of the base, AG, and its depth, GB. DD', is equal to AB. We now have two right-angled triangles, ABD and ABD'. Arcs about D and D' cut the hypotenuses BD and BD' at x and x'. An arc is struck about B through x and x', and an arc of equal radius is described about A. The points of intersection of these arcs, a and b, lie on the sides of the top surface of the base. In other words, the width of the base is equal to ab; we know the depth, GB, which is equal to the width, and thus the height is determined, for it equals AB minus ab. All the dimensions of the base have now been determined geometrically, by dividing the basic dimension AB in Φ.

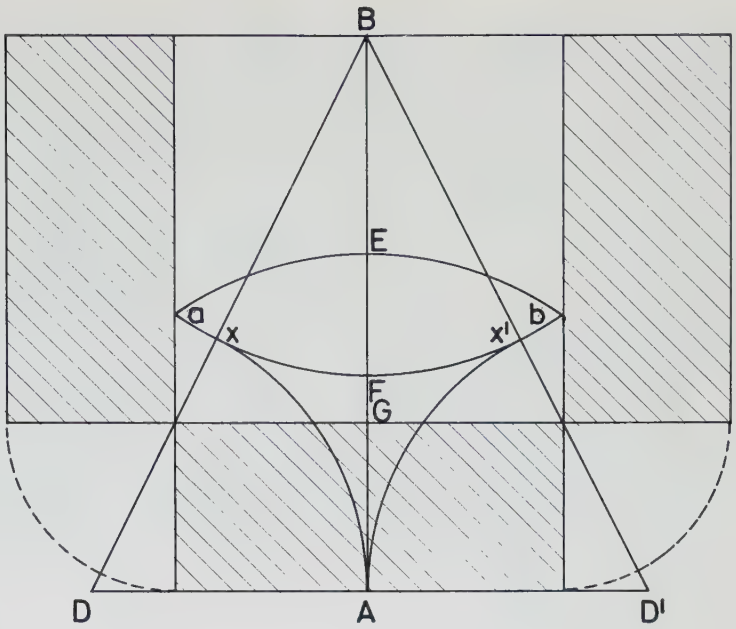
PLATE II (Acropolis Museum 1335). Height 33.0 cm, depth 93.0 cm, width 70.0 cm. Once the width has been determined (here as xx'), only one more determination will yield all the principal dimensions. Here a and b are the points of intersection between the hypotenuses from B and the diagonals of the rectangle whose sides are equal to AB and xx'. The perpendicular distance from ab to B is the height of the base.

PLATE III (Acropolis Museum 1326). Height 33.6 cm, depth 83.0 cm, width 78.0 cm. The analysis of Plate III reveals an unusual feature, for BF, determined by the Φ division of AB, equals half the sum of two heights and one width, CC'. CC' is now divided in Φ, and we arrive at IK, which is equal to the height of the base. Now the width is also determined. (Without going into any detail with regard to the reliefs on the sides of the base, we would draw the reader's attention to the fact that M, the mid-point of AB, determines the position in the right side panel, which is as important a part of the relief as the centre, of the chariot wheel.)

These three examples show that it is possible to *select* from the geometrical dimensions in such a way that the possibilities of varying the proportions of the block are always utilised,

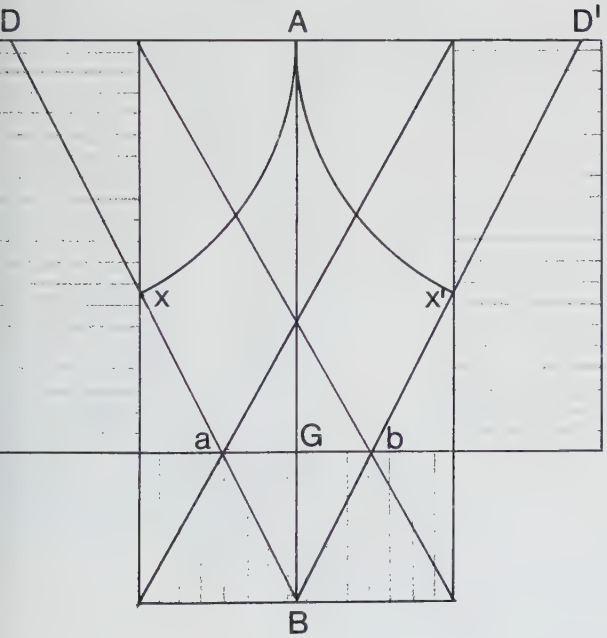
PLATE I

Statue base. Athens N.M. 3476. Vth century B.C. Reliefs on three sides. Analysis based on the author's measured drawing, Scale 1:2. Depth and width 81.0 cm, height 30.0 cm.



LATE II

tatue base. Athens Acro-olis M. 1335. Relief on be front. Analysis based n the author's measured rawing. Scale 1:2. Height 3.0 cm, depth 93.0 cm, idth 70.0 cm.



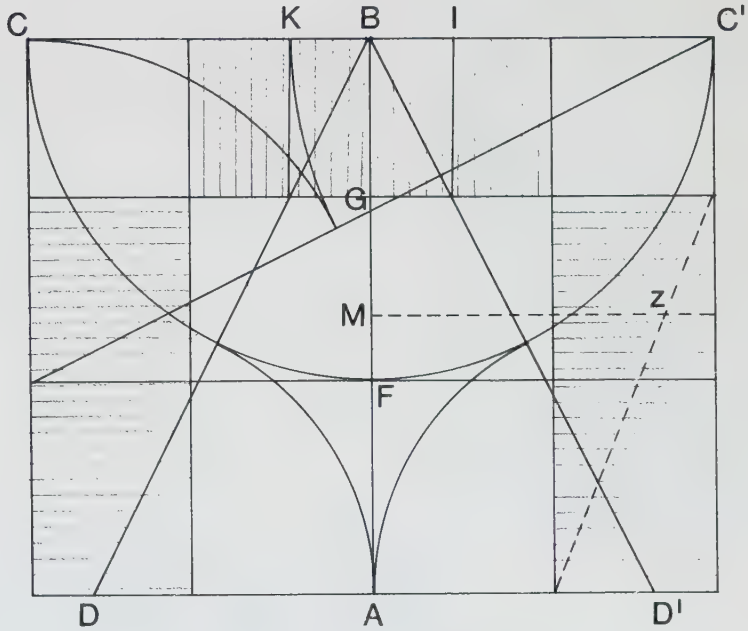


PLATE III

Statue base. Athens Acropolis M. 1326. Reliefs on three sides showing horse chariots and hoplites. Analysis based on the author's measured drawing. Scale 1:2. Height 33.6 cm, depth 83.0 cm, width 78.0 cm.

Fig. 8.
Front relief of Athens N.M. 3477. Hockey players.

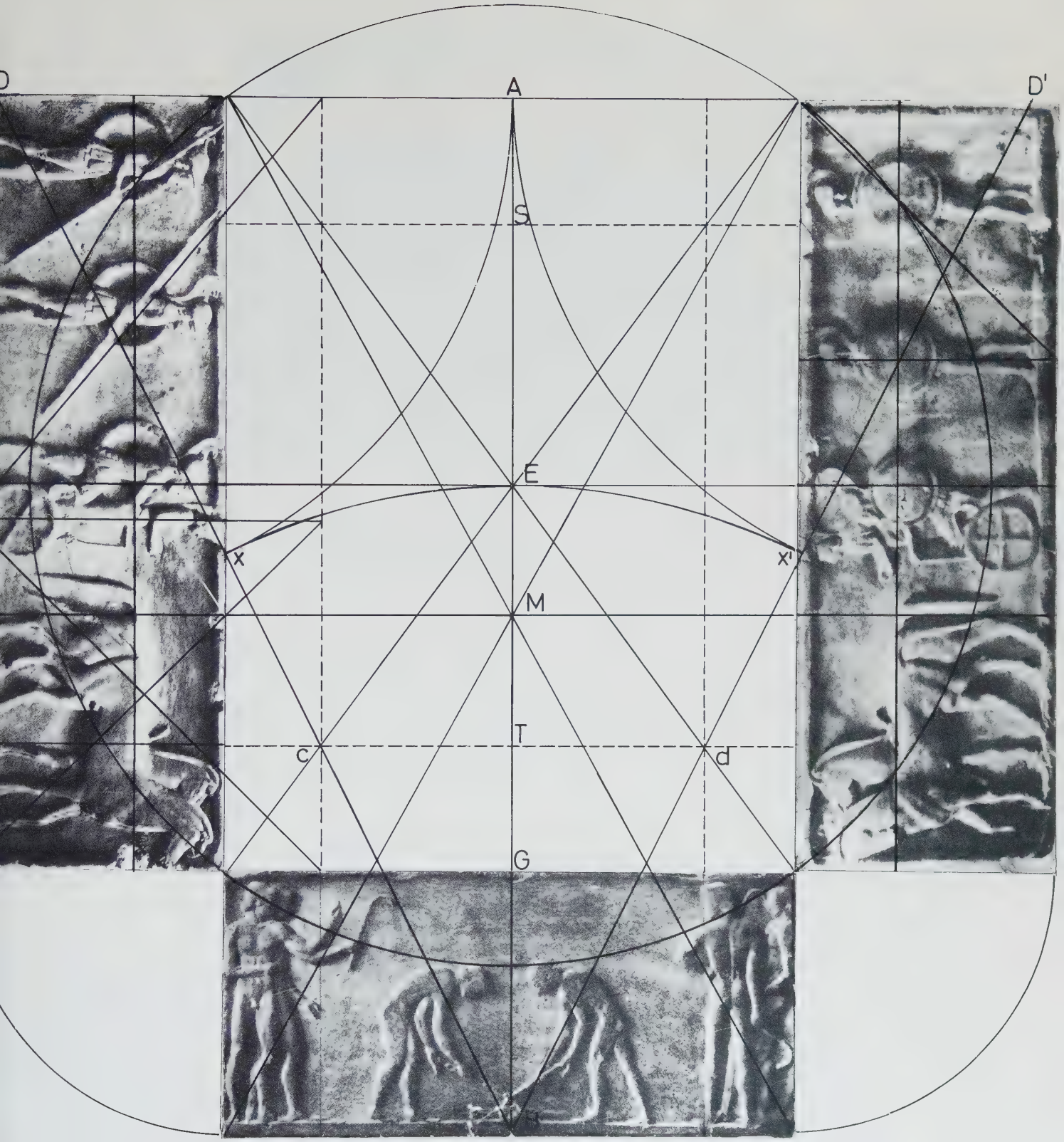
while the basic principle remains unchanged. We shall now by examples demonstrate that the planning of the reliefs on the sides of the block also depends on its geometry.

PLATE IV (Athens N.M. 3477). Height 27 cm, depth 81.0 cm, width 60.0 cm. This base has at the front a relief with hockey players, while reliefs with chariots and warriors decorate the two sides. Four straight lines carved or incised into the top surface of the base form a rectangle which is parallel to the sides of this surface. The proportions of the block are determined on the basis of AB, the sum of the height of the block, BG, and its depth, GA. DD' is set off equal to AB, and the distance between x and x', the points of intersection on the Φ hypotenuses, gives the width of the block (cf. Pl. II). The height of the block is equal to 1/4 AB; now the principal dimensions of the base are determined. The diagonals of the top surface are constructed, and the points of intersection between them and the hypotenuses BD and BD' are c and d. We now see that *the four broken lines on the diagram, which are determined by c and d, are the lines which are clearly and precisely carved on the stone.* Surely this is evidence showing that the proportions of the block were calculated according to the method here described. On the plate we see photographs of the reliefs in position on the sides of the block. Even though a photograph does not provide absolute accuracy, we can see clearly that the design of the reliefs is dependent on the geometric lines. The circumscribed circle on the top surface, for instance, determines the position of the two hockey players at the centre, while the carved lines on the top of the block when produced limit the space available to the players waiting on both sides of the forwards.

At the front of the left relief a square has been constructed, whose side is equal to the distance from the base line to the broken line passing through c. Now a rectangle of the same height forms behind this square, its length equal to the remainder of the base line of the relief. The left upper diagonal of this rectangle determines the lance of the central warrior, while its point of intersection with the Φ line sloping down to the right from D determines the position of the hand which holds the lance.

Whereas the left relief depends on a "greater height" determined by the broken (incised) line on the top surface, the main lines of the right relief are determined by the three squares into which it is divided, and which are of the same height as the block. Each of the two reliefs on the sides thus has its own, distinctive character.

PLATE IV
Statue base. Athens N.M. 3477. About 490 B.C. Analysis based on the author's measured drawi Scale 1:2. Depth 81.0 c width 60.0 cm, height 2



2. Greek art

Fig. 9
Bryaxis' base, analysed on PLATE V.

PLATE V (Athens N.M. 1733). Height 32.0 cm, depth 74.5 cm, width 76.6 cm. This is a mid-fourth-century base, the work of Bryaxis. On three of its sides it has reliefs showing a horseman and an altar; the fourth – the front – has the dedication and signature. From our point of view, however, the round pedestal on the top is most interesting. Its dimensions are clearly defined, but the statue which once stood here is lost. *Fig. 9.*

The basic dimension which enables us to deduce the proportioning of the block is, as usual, the sum of the height and the depth of the block, AB. This is divided at E and F in the normal way, and the arc about A through E cuts that about B through F at x and y. *The distance between these two points is the width of the block.* The diagonals of the rectangle length AB and width xy are constructed. The lower corners of this rectangle are R and R'. AH is set off equal to RR', and this line is divided in Φ at I and K. The arc from H determining K cuts these diagonals at z and u, and *the perpendicular distance between these points and A determines the height of the block.* Thus the depth AG is also determined.

Now we see that the dimensions of the round pedestal are very simply determined by the geometrical pattern resulting from the proportioning of the block. This is true of the circles of the top and the bottom surface, and also of the perpendicular distance between them. M is the centre of the top surface, while N is the midpoint of AH. S and T are the points of intersection between the hypotenuses from B and the diagonals of the top surface. Circles are constructed about M, one with diameter equal to ST and the other passing through v and w. They describe the base and the top of the pedestal as seen from above. A *vertical section* from M upwards shows that the height of its lower part is equal to MN while the height of the upper part equals NF. *This shows that the horizontal projections and the profile projections continue to be determinant.* The planning of the relief, as appears from the plate, follows naturally from these constructions.

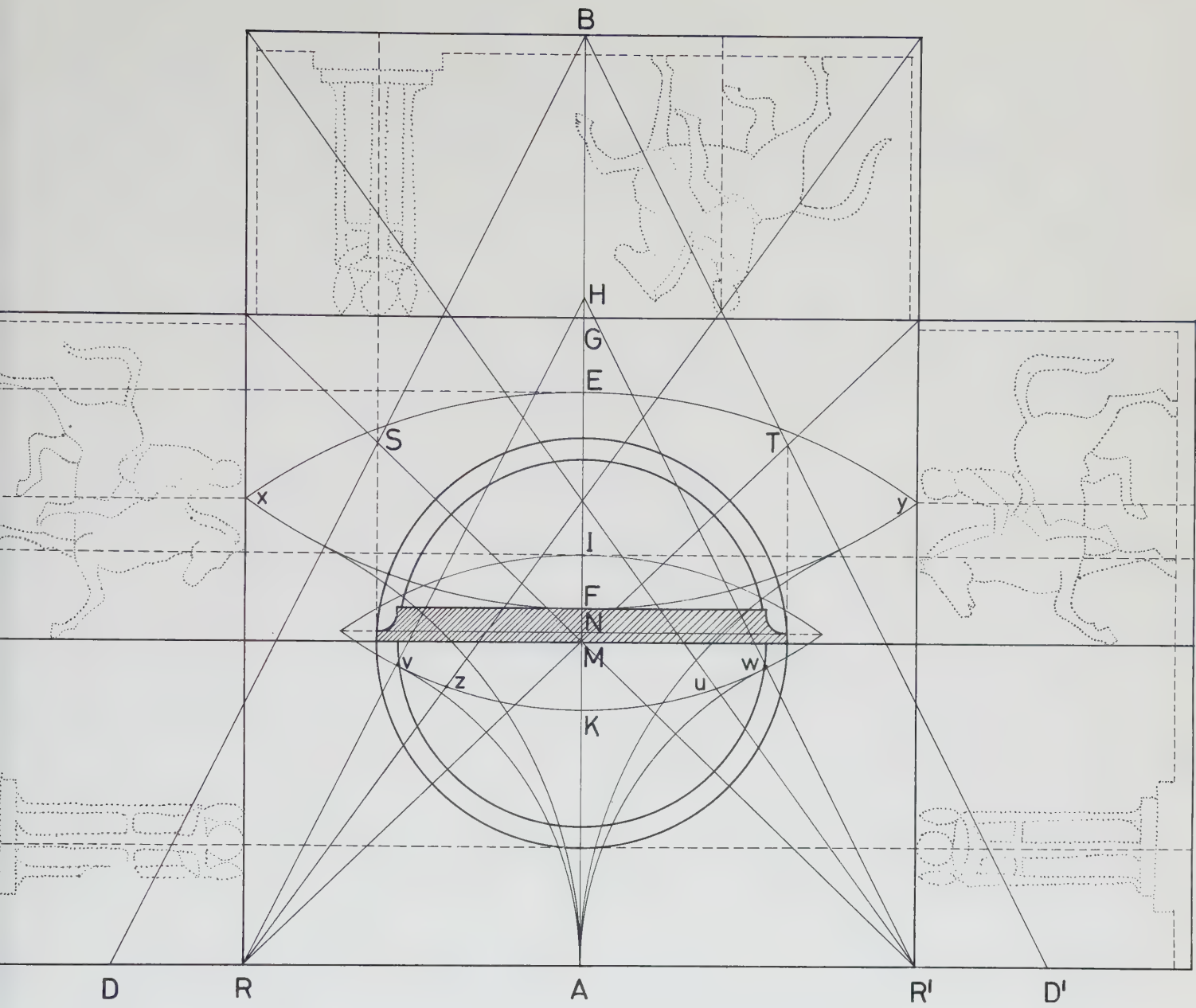


PLATE V

Statue base. Athens N.M. 1733. By Bryaxix. About the middle of the IVth century. Analysis based on the author's measured drawings. Scale 1:2. Height 32.0 cm, depth 74.5 cm, width 76.6 cm.

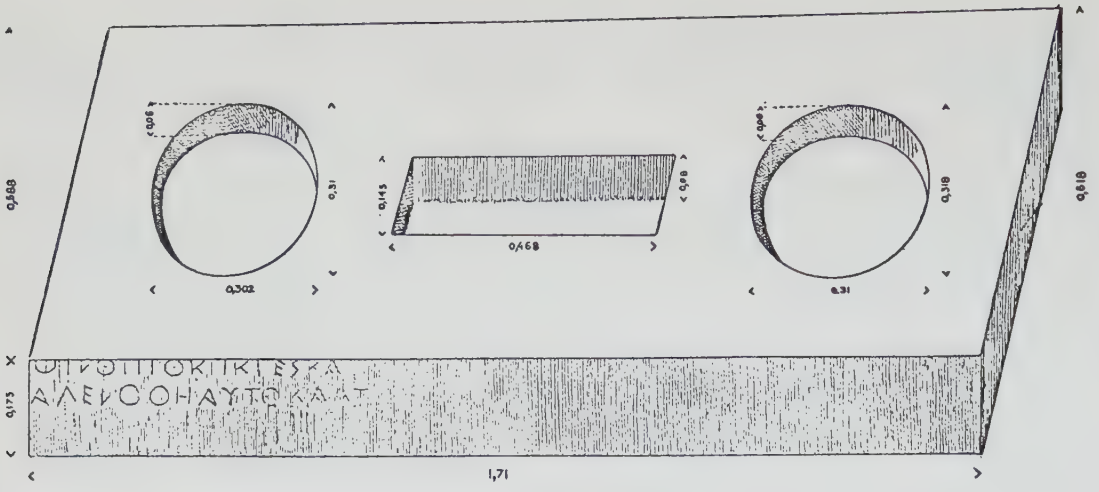


Fig. 10
Measured drawing by K. Rhomaios of the base analysed on PLATE VI.

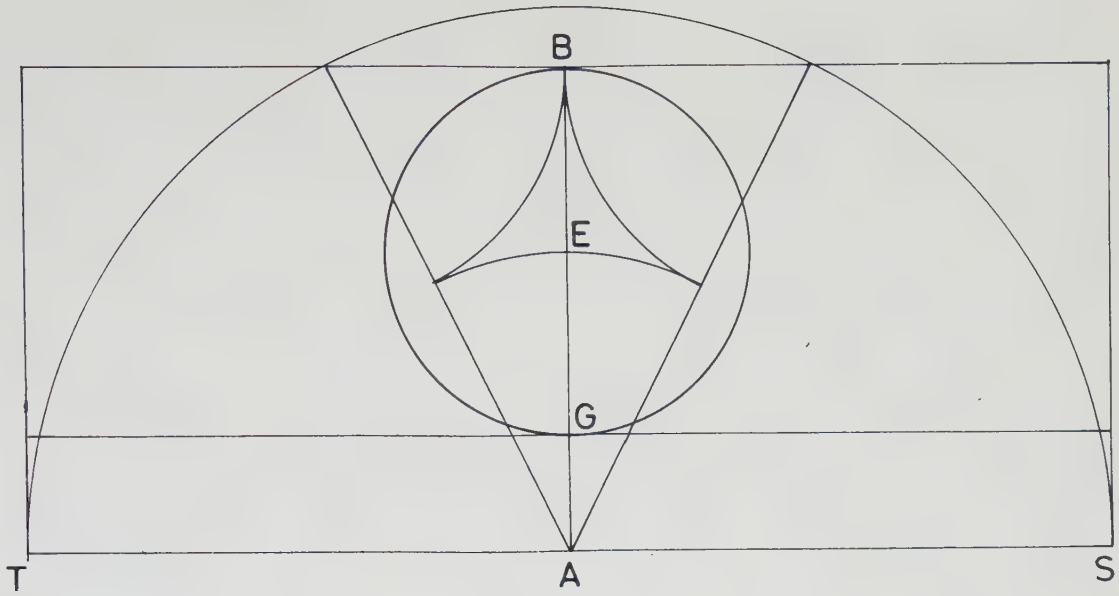
PLATE VI, A, B. Christos Karusos' "Aristodikos" (Kohlhammer, Stuttgart 1961) includes a measured drawing of a base (Athens Epigr. Mus. 13290), prepared by Rhomaios. The base forms part of a memorial to Philoitios and Ktesias. Depressions sunk into the top of the base served for attaching the stele and the two columns flanking it. All these are now lost (*Fig. 10*).

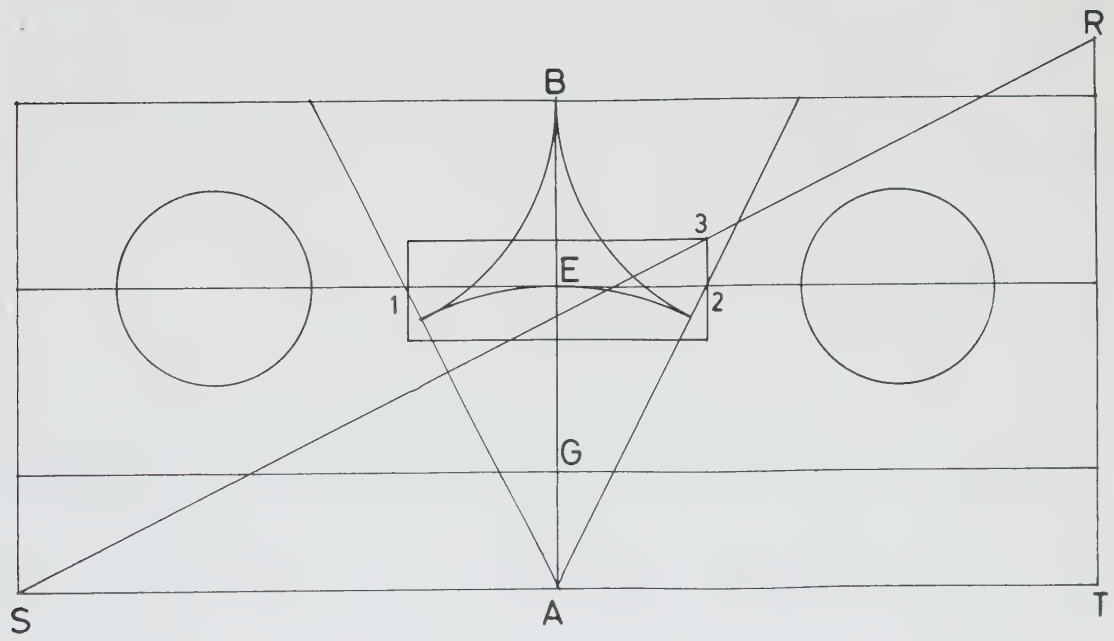
The front of the base bears a funerary inscription, but the tops of the letters have been cut off. This would seem to indicate that this base, which is of Archaic date, was re-carved later, a circumstance which renders it less suitable as evidence on the subject of early art. It does not, however, detract from its usefulness for our purposes, for we have no reason to suppose that a Greek sculptor would rehew a block with less design and less precision than he would apply when working with virgin stone. Rhomaios gives the measurements down to the nearest mm, and they are all indicated on his drawing.

We draw the base in the usual way, to a scale of 1:2, assuming that the basic dimension again is the sum of the height and the depth of the base, AB. Plate VIA shows that *the width of the base is determined by the sum of the two hypotenuses from A.* AB is divided in Φ in the normal way at E, and as E is the centre of the top surface, G is also determined. *Now the height of the base, AG, and its depth, GB, are determined.* On Plate VIB, the E horizontal has been drawn, and its points of intersection with the hypotenuses from A, 1 and 2, determine the length of the rectangular depression in the top surface of the base. TR = ½ ST, and RS cuts the perpendicular through 2 at point 3. Twice the distance from 2 to 3 is the short side of the rectangle marking the depression for the stele in the top surface of the base. The round hollows for the columns lie centrally between the short sides of the base and the rectangular depression, and the diameter of the circles is equal to half of this distance. All this can be controlled arithmetically, though we must bear in mind when carrying out an arithmetical control that the depth on the left side (0.588) is 3 cm less than that of the right (0.618); this is a fairly common phenomenon. If we work with a depth of 0.588, we arrive at a width of 1.70, which would appear to be a reasonable figure.

PLATE VII (Athens N.M. 1464). Height 37.0 cm, depth 42.0 cm, length 154.0 cm. This is a statue base with a horse relief in front. The measured drawing of the base is to a scale of 1:2. It is much higher than the preceding base, and not as deep. The drawing shows the same main features as those explained in connection with the preceding analysis: *the length (width) of the*

PLATE VI A, B
Statue base. Athens Epigr. M. 13290. Analysis based on measured drawing by K. Rhomaios, scale 1:1. Published in Christos Karusos' Aristodikos.





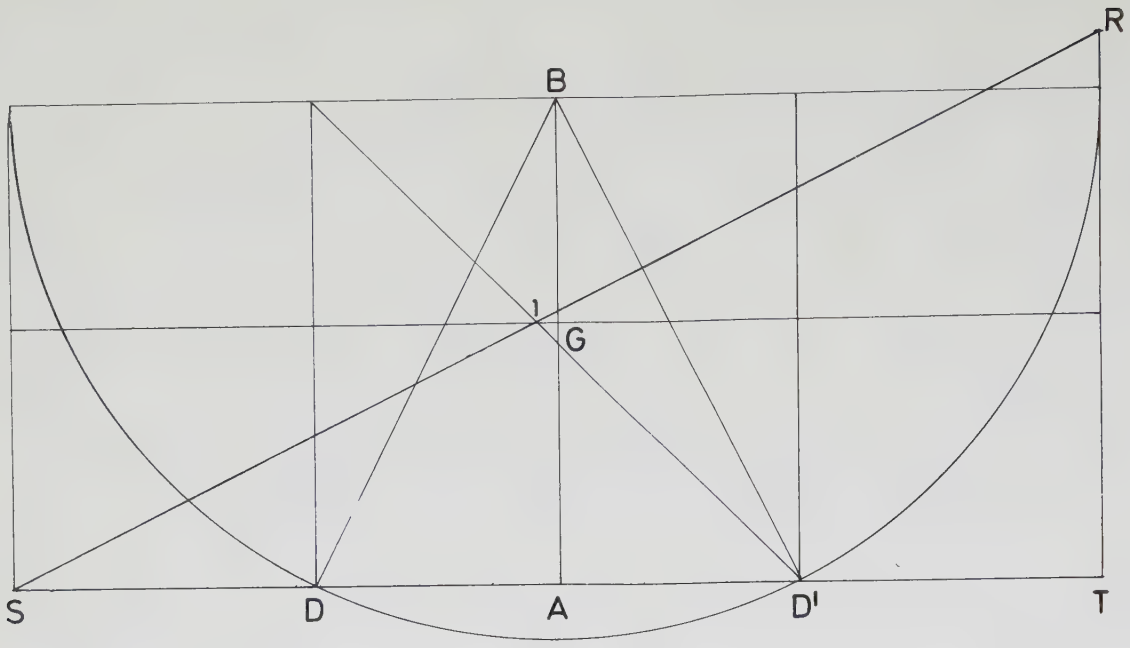


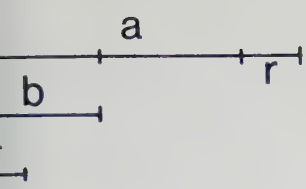
PLATE VII
Statue base. Athens N.M. 1464. Hellenistic. Analysis based on the author's measured drawing. Scale 1:2. Height 37.0 cm, depth 42 cm, length 154.0 cm.

base is determined as twice the length of the hypotenuse of the triangle on AB. AB is, as usual, the sum of the height and the depth. This analysis shows that a Φ triangle (TSR) has also here been constructed on the base line, TR equalling ½ST, and TSR is of decisive importance for the further planning of the proportions of the base. In this instance, the G horizontal, the line of demarcation between the height and the depth, is determined by the triangle. A square is constructed on DD', and a diagonal is drawn from D'. Point 1 on the G horizontal is the point of intersection between this diagonal and SR. Thus the height, AG, and the depth, GB, have been geometrically determined.

Euclid and Plato on 'The Golden Section'

The proportions of these seven statue bases are determined according to an extremely simple and practical method, a method which moreover offers the artist a wide choice of possibilities, as all the geometrical points arising from the construction are in contact with the entire unity. The Egyptians worked in this way from the time of the Old Kingdom, about 3000 BC, and our further investigations should confirm the theory that Greek artists employed the same basic principles from the Archaic period throughout the different ages. We have emphasised the childish simplicity of the method, obtaining in the way in which the picture surface is conceived in terms of planes — in actual fact perpendicular to one another — lying flush, and also in the extremely simple and practical method of division employed.

At this point we would quote Plato (*The Laws*, 819a–820b, Bury, Loeb Ed.), The Athenian: "One ought to declare, then, that the free-born children should learn as much of the subjects as the innumerable crowd of children in Egypt learn along with their letters. First, as regards counting, lessons have been invented for the merest infants to learn, by way of play and fun . . . The next step of the (Egyptian) teachers is by lessons in weights and measures, to clear away



. 11 a
e 'Antanairesis Method'
'alternate subtraction'
om Viggo Brun, Alt er
l.)

a certain kind of ignorance, both absurd and disgraceful, which is naturally inherent in all men touching lines, weights and measures. By supposing that some of them are neither absolutely nor moderately commensurable, some not, whereas you regard them all as commensurable, – what do you think of your mental state with respect to them? Evidently it is a sorry state." In his book *Alt er tall* Viggo Brun adds: "It seems extremely probable that Plato meant that the Egyptians did away with ignorance in the matter of irrational ratios, perhaps already at the school level."

It is possible that not all who read our analyses realise that they are based on irrational numbers, as our divisions are founded on the geometrical ratio which is usually known as 'The Golden Section'. Euclid writes about these matters (Book VI, Definition 3): "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser." This may be expressed thus (a being the greater part, b the lesser): $(a+b):a = a:b$. Today we should say that a is the mean proportional of $(a+b)$ and b. If we assume the length of b to be 1, the value of Φ may be calculated as follows: $(a+1):a = a:1$. The length a and the value of Φ may be calculated from this equation; the result is 1.61803398 . ., an irrational number. If we, on the other hand, assume the length of the greater part, a, to be 1, the corresponding equation yields the result that $b = 1/\Phi$, the reciprocal of Φ which, expressed in decimals, is 0.61803398 . . . Φ is the only positive number which is 1 greater than its own reciprocal.

The peculiar properties of proportions and ratios have given rise to a literature of their own. Mediaeval and Renaissance mathematicians were practically obsessed by the properties of Φ. Kepler, for instance, wrote: "Geometry has two great treasures. One of these is Pythagoras' theorem. The other is the division of a line into 'extreme and mean proportional'. The first of these may be compared to a bar of gold, the second can be likened to a costly jewel."

Plato's Timaios has the following sentence, which has formed the subject of many discussions: (Timaios, 31 BC) "But two things cannot be satisfactorily (beautifully) united without a third, for there must be some bond between them drawing them together. And of all bonds the best (the most beautiful) is that which makes itself and the terms it connects a unity in the fullest sense; and it is of the nature of a continued geometrical proportion to effect this most perfectly."

It is not unreasonable to connect this description with the Φ proportion. We must bear in mind, however, that while Euclid employs the word *divide*, Plato uses the opposite concept – *join*. If we let the terms of Euclid's equation change place (a still being the greater part), $a:(a+b) = b:a$, we literally have as the middle term "the bond between them . . . the best (the most beautiful) . . . which makes itself and the terms it connects a unity in the fullest sense." Moreover, this is effected by the "continued geometrical proportion". Euclid and Plato are both referring to a mean proportional, and it seems most likely that the mean proportional in question is that which we here designate Φ.

In an article: "Når har man først oppdaget de irrasjonale forhold?" (*Nordisk Matematisk Tidsskrift*, Vol. 16, Oslo 1968), Viggo Brun draws attention to the so-called "antanairesis" method, *Fig. 11a*, which, according to him, probably derived from an extremely ancient method employed by craftsmen. The word "antanairesis" must in this context be translated by "alternate subtraction". If one needs to establish the ratio between the length of two sticks, a and b, one compares them by placing a alongside b. If this can be done a number of times, n, without remainder, the answer is clear. If there is a remainder r, the ratio between r and b is now established by placing r alongside b. This leads to "alternate subtraction" – the antanai-

resis method. (Our modern method of trying to establish the ratio between a and b in terms of a decimal fraction is not as obvious).

Applying the train of thought exemplified by Brun's example to our problem, we can say that we have two sticks, and that the ratio between their lengths is Φ. If we now (like Euclid) call the longer stick (a + b) and subtract a, the greater part, from it and continue the comparison of the lengths of the sticks in the way illustrated on *Fig. 11b*, the ratio between a and b will remain unchanged ad infinitum. Kepler called the Φ division "a costly jewel", because of its many remarkable properties. Plato, as we saw above, spoke of the most beautiful of bonds, a description not necessarily based on an aesthetic evaluation, but rather on one of philosophy: the most beautiful ratio must be that incorporating the greatest possibility of creating a unity, a cosmos.

Plato's characterisation of Φ as the "most beautiful" ratio requires the existence of other ratios, as shown by Egil A. Wyller's *Tübinger Vorlesungen* 1965, where he discusses the line parable in the final part of "*The Republic*" (*Der späte Platon*, Felix Meiner Verlag, München 1965). Here Plato deals with a simple ratio between two lines, with a mean proportional between them. There is an infinite number of such ratios, and Φ is one of them. Plato describes it as the most beautiful, while Matila Ghyka in *Le Nombre d'Or* (Gallimard 1931) speaks of it as the most economical, because the Φ ratio can be expressed by means of two terms only, a and b, while all other ratios incorporating a mean proportional require three terms. The purely practical properties of this ratio are of great importance, as will appear from a study of the further analyses of the present book. Far more works of art of all types were investigated than the number finally selected for publication. Nearly a hundred vases were analysed, and the method of Φ division in every instance, varying its combinations from one case to the next. This does not, of course, exclude the use of other symmetries, perhaps in combination with Φ. On some plates, for instance, we note the use of a division into four, and a division into three.

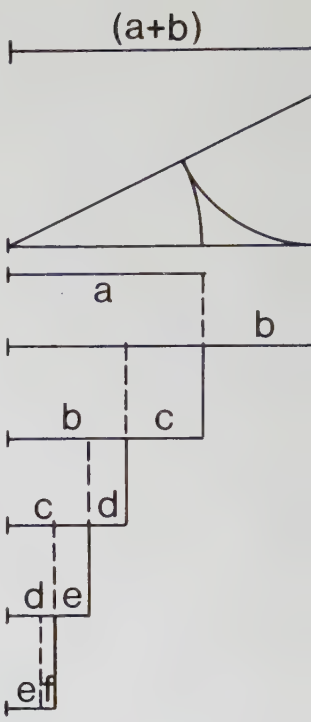


Fig. 11·b
The Antanairesis Method combined with the φ proportion.

Geometrical planning as the primary moment of architecture. Vitruvius.

The plan drawing of rectangular blocks, executed in the manner exemplified in the seven analyses of statue bases, is an essential prerequisite for the further analysis of Greek sculptures. We shall attempt in the following section to account for the methods employed by sculpture. First however, we must make some reference to the enormous amount of material represented by architecture, although we cannot devote our undivided attention to this subject within the framework of the present book. We shall confine ourselves to three architectural analyses — first of the Arsenal Inscription, an original text carved on a stone block, then of the small pottery temple from Perachora, while the third concerns the principal dimensions of the Theseion in Athens.

The Arsenal Inscription explains in detail the dimensions of the planned great Arsenal of Peiraeus. We do not know the building, but on a block all the dimensions of the building are specified in feet and fractions of a foot. The question now arises: how do the geometrical calculations for which we have accounted above compare with this unit of measurement? That the foot was in common use is an established fact, and it is also frequently mentioned by one of the most important Roman sources, Vitruvius.

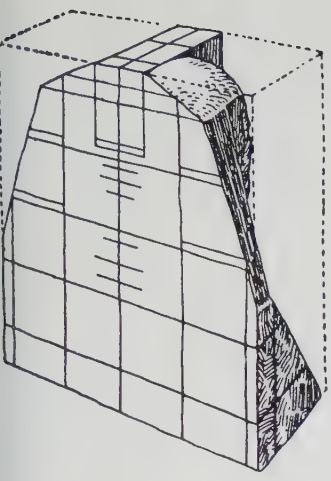


Fig. 12
Egyptian royal head with geometric marks (From H. Schäfer, Von ägyptischer Kunst.).

Can the *foot* have been used side by side with *geometrical planning*, by a method analogous to that accounted for in "*Geometry in Egyptian Art*"? In 1882 Perrot and Chipiez realised that Egyptian art combined two methods in the design of one work of art. There are some Egyptian sculptor's models with marks representing a grid, and other marks quite unconnected with these, *Fig. 12.* In *Geometry in Egyptian Art* we showed that these latter marks represent a method which corresponds to that elucidated in the present chapter. If calculation in feet were the only method employed, what did the Greek philosophers mean when they described geometry as the fundamental element? The foot is no more than a unit of measurement, it determines the size of the building. Geometry, on the other hand, is *absolute*, and entirely independent of size or scale.

PLATE VIII. We shall first analyse the little pottery Hera temple in the National Museum, Athens, *Fig. 13.* The ware is fairly coarse, the roof and the columns of the porch are missing, except that a base with plinth for a double column remains of one. The workmanship is by no means perfect, and there is a certain amount of distortion, but the measurements are nevertheless clear enough to justify a sketch of an analysis.

The following measurements were taken:
Greatest length of cella 28.5 cm
Width of cella 20.0 cm
Projection of sides beyond front of cella 0.9 cm
Distance from front of porch (G) to side walls 7.0 cm
The base of the column lies within the porch, its outer edge 1 cm from the front of the porch, its side aligned with the outer edge of the outer cella wall.

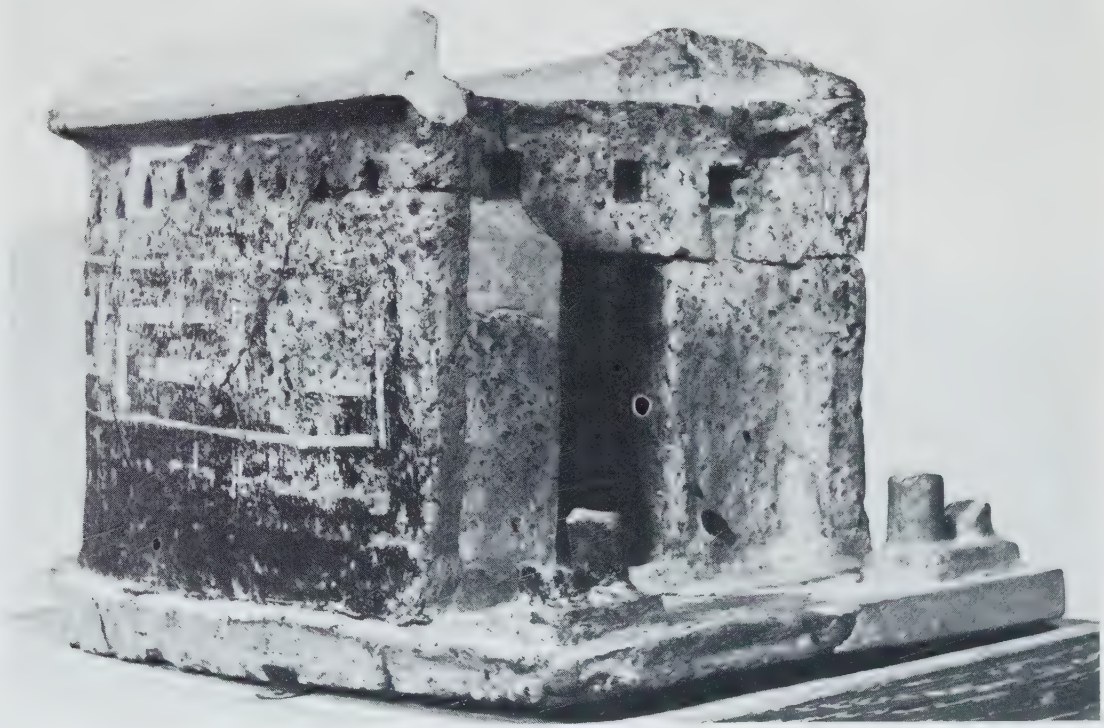


Fig. 13
Pottery temple from Geometric period, analysed on PLATE VIII.

Width of column base 4.0 cm
Depth of column base 2.7 cm
Height of front wall to lower edge of roof 15.5 cm
Height of door 10.5 cm
Width of door, above 4.0 cm
Width of door, below 5.0 cm

PLATE VIII
Pottery temple dedicated to Hera in her sanctuary of Perachora. Athens N.M. 16684. Analysis based on the author's measured drawing. Scale 1:2.

Because of the small size of this temple, we are able to illustrate it by a half-size measured drawing, Plate VIII. With the exception of the apsidal end, this is again a rectangular block. We must visualise this block with its sides placed flush with one another in the picture plane, in the manner employed in the analyses of the statue bases.

As usual we start our analysis by ascertaining the basic dimension, the sum of the height AG and the depth GB of the temple. As we are concerned with finding out whether Egyptian methods were employed here, we calculate the height of the temple *without* the roof, in the manner we were able to demonstrate in *Geometry in Egyptian Art*, in the analyses of caskets from *Tut-Ank-Amon*'s tomb. Since a Greek temple resembles a casket in shape, as well as in its function as a shrine for the divinity, an analogous investigation seems called for. The drawing should not need explanation.

One of the most remarkable documents in the history of architecture is the so-called Arsenal Inscription. This marble tablet, bearing 97 lines of inscription, was found in Peiraeus in 1884. The text gives an exhaustive and precise account of the measurements to which the Arsenal of Peiraeus was to be built. The building, one of the Seven Wonders of the World, was completed in 329 BC, and the responsible architect was Philon, who was later to build the porch of the great Telestorion in Eleusis. The Greek word for arsenal, literally translated, means 'store-house for tackle', and this particular store-house was of great importance for the Athenian navy. Plutarch, c. AD 50 – 120, wrote that "Sulla burned most of Peiraeus, including 'Philon's arsenal', that admirable structure". Although the building is lost forever, the inscription has again brought it to life, at least theoretically.

In his book *Arsenalet i Piraeus og Oldtidens Byggeregler* (Copenhagen 1922), the Danish scholar Wilhelm Marstrand has made a contribution of fundamental importance to the study

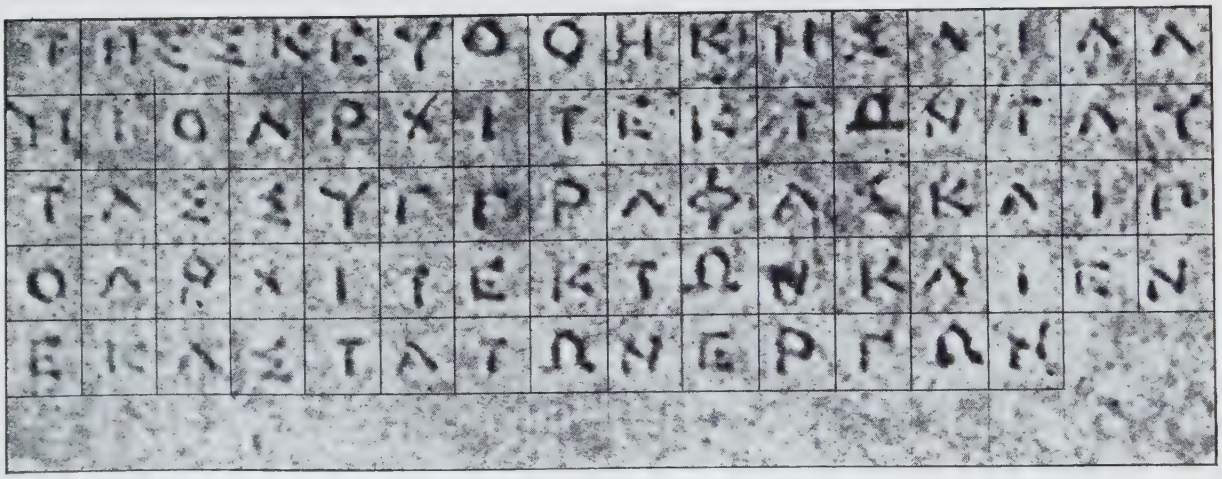
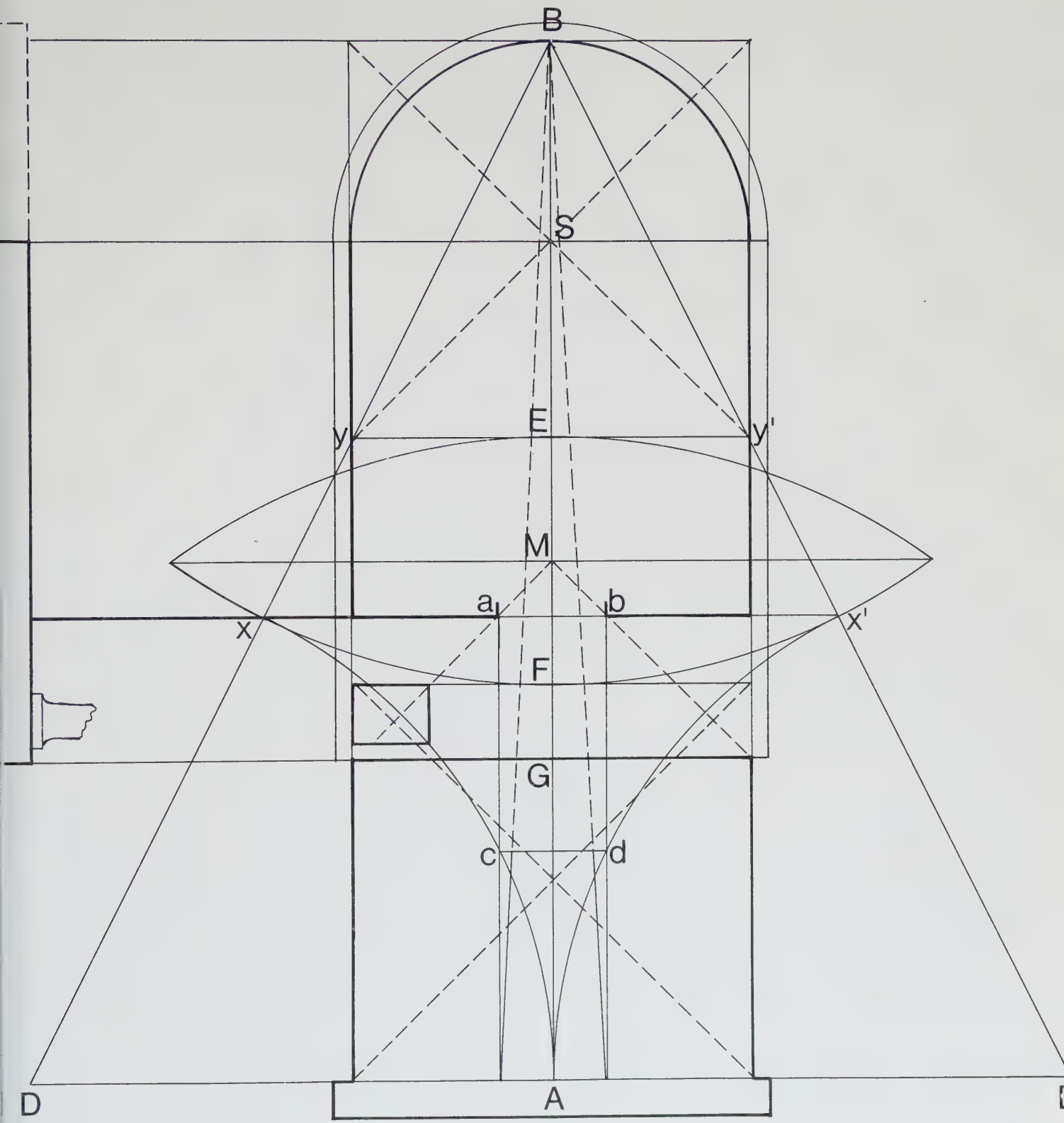


Fig. 14
The Arsenal inscription. Grid drawn by Marstrand


B
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y'
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x'
F
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D
A

and understanding of this inscription. We have used his translation of the inscription, and have also obtained various items of information from his book, which unfortunately exists only in the original Danish version.

The inscription gives all the measurements important for our purpose, first and foremost the length, height and width of the building. We quote from the inscription: "From the front of the ship sheds, which have a common roof, to have *a length of 400 feet, a width of 50 feet.*" The length and the width being given, we must now find how the height to the lower edge of the roof is indicated. We employ the Egyptian way of thinking here, regarding the height of the piers bearing the inner room as the "greatest" height. The inscription states that the piers were to be *30 feet high.* These measurements produce an extremely long, narrow and low building, and the most natural method of analysis is the one we employ on Plates VI and VII, where we called the long side of the block its 'width', and the end the 'depth'. We shall use the same terminology here, which means that our basic dimension AB is equal to the sum of the height of the piers and the depth of the building (GB) or, in figures, 30 feet + 50 feet = 80 feet = AB. *It now appears that 5 AB is equal to the "width" of the side of the building, which is stated to be 400 feet.* These measurements form part of the so-called Fibonacci series (1, 2, 3, 5, 8, 13, 21, 34, 55, etc.), each of whose terms is equal to the sum of the two preceding terms; in other words, a Φ series. The ratio between them increases in accuracy the further we proceed in the series, and our figures are early terms; a Φ division of AB results in a height of 30 feet for the piers (the minor of 80 feet). Seeing that the inscription also mentions a *levelling course* of 1.5 feet, the precision of the result must surely be considered satisfactory.

It seems justifiable to assume that the architect, Philon, who had devised so detailed a plan for this important building, must have calculated all measurements, but he did not publish the drawing showing his calculations. Instead he prepared a new plan and perhaps submitted plan and sections separately, in the manner of modern architects. These drawings showed the unit of measurement in normal use, the *foot*, and his instructions were based on this second set of drawings. Two systems of measurement were thus employed side by side, each with its own particular sphere. There is nothing new in this, the same practice had been followed in the design of Egyptian sculptures (see *Fig. 12*). In short: geometry was employed in *the composition* of a work of art, while calculation in feet (or, in Egypt, the use of the squared grid) relates to its execution, to *the carrying out* of the geometrical plan.

The earliest written sources dealing with the Greek attitude towards art are Roman, and the first of these is Vitruvius. There was a Greek literature dealing with architecture, though only fragments remain. Vitruvius, a contemporary of Caesar's, took part in the campaigns in Gaul, and he constructed war machines for Caesar. By profession he was an architect, and his *Ten Books on Architecture* is the only antique work on architecture which is preserved in its entirety. To the modern reader, Vitruvius' work seems confused, long-winded and, at times, extremely naive. Not surprising, then, that the great Renaissance theoretician, Leon Battista Alberti, was very critical in his judgement of Vitruvius.

We shall first quote from Vitrivius' chapter II "The fundamental principles of architecture" (transl. M. H. Morgan, New York 1960):

"1. Architecture depends on order (Greek ταξις), arrangement (Greek διαθεσις), eurythmy, symmetry, propriety, and economy (Greek οικουομια).

2. Order gives due measures to the members of a work considered separately, and symmetrical agreement to the proportions of the whole. It is an adjustment according to quantity (Greek ποσοτης). By this I mean the selection of modules from the members of the work itself,

and, starting from these individual parts of members, constructing the whole work to correspond . . .

3. Eurythmy is beauty and fitness in the adjustment of the members. *This is found when the members of a work are of a height suited to their length, of a breadth suited to their length, and, in a word, when they correspond symmetrically.*

4. Symmetry is a proper agreement between the members of the work itself, and the relation between the different parts and the whole general scheme, in accordance with a certain part selected as a standard . . ."

A great deal of this should be of interest to us in our present study, but unfortunately it is difficult to know exactly what the various terms used by Vitruvius imply. He had, as we know, taken them from Greek, and already during the Renaissance Alberti complained of the lack of clarity of the language. According to Alberti, Vitruvius' style was so poor that "the Latins took him for a Greek, and the Greeks for a Latin". Moreover, modern languages lack terms which express these concepts exactly. A few points should, however, be quite clear: Vitruvius' point of departure is based on "modules from the members of the work itself." This is immediately preceded by the words, "*Order* gives due measures to the members of a work considered separately, and symmetrical agreement to the proportions of the whole." We should from this be able to infer that "Order" is the primary factor – an apparent self-contradiction. He says not a word about *how* his points are to be put into practice.

The final sentence of his Note 17 runs, "It is true that it is by arithmetic that the total cost of buildings is calculated and measurements are computed, but difficult questions involving symmetry are solved by means of geometrical theories and methods." Not a word of information about these methods, however.

We know that the geometrical design was put into execution on the basis of feet – the Arsenal inscription provides conclusive evidence of this practice. But what is the connection between the geometrical measurements and the foot? It seems not impossible that the unit of measurement, *the foot*, may have been applied already when the basic dimension, AB, was decided on. We may note that the basic dimension, AB, of the Theseion is 29.909 m, which equals 100 feet of 0.299 m each. This is an entirely plausible length for a foot. The absolute size of the foot employd in Greece varies with time and place, spanning over a range from 0.297 m to 0.355 m. If the basic dimension AB was determined by the foot unit, and if all other dimensions were then derived from AB by means of Φ division, it is obvious that these latter measurements cannot be expressed in terms of whole feet. We must remember that they represent irrational numbers, and if we express these dimensions in terms of feet, our figures can be no more than approximate. *Only geometry will yield the absolute measurements.* We know, however, that the length of the foot employed by the Greeks – and to a lesser degree also that employed by the Romans – varied. The Oxford Classical Dictionary 1961 mentions several different values for the foot: 320.5 mm (the Olympic), 333 mm (the Aeginetan), 330 mm (the Pergamene), 350 mm (the Samian), 295.7 mm (the Athenian) and the Roman foot, 296 mm. The British Museum has two foot rules, measuring 292 mm and 294 mm respectively. Herbert Koch also gives a Solonic foot (note, p. 74), of 296 mm, which is equal to the Roman foot; he finds it strange that the Romans should have adopted this foot rather than the "Pheidonic".

Obviously such widely differing measurements cannot provide a secure basis for further conclusions. It seems not unlikely that the measurements of a building or a piece of sculpture were determined in terms of the basic dimension, AB, measuring a given number of feet. But

Fig. 15
Carpenter's Square. London B.M. (W.T. 966).

the objection that these figures are vague and inconclusive, as a result of the varying values of the foot, is perfectly legitimate. For the geometric construction this is of no importance.

We cite one more of the investigations we have conducted, in an attempt to throw more light on the connection between geometrical calculations and the unit of measurement, the foot, which we know to have been used when buildings were erected. The Greek and Roman Department of the British Museum have a collection of measuring instruments arranged in one of its show-cases. One of these is most peculiar — we are referring to the "*Carpenter's square*". *(W.T. 966). Fig. 15.* It consists of a two-armed, vertical bronze plate, 4 mm thick, let into a horizontal foot-plate. The square stands firmly on the foot-plate without any kind of support, in spite of its irregular form — one arm extends far beyond the edge of the foot-plate. The museum catalogue does not, unfortunately, supply any information about this strange object — it is said to have come from Italy, possibly from Pompeii or Herculanum, judging from the patina of the bronze. The same show-case also holds two foot rules, one which gives the length of a foot of 30 cm, the other indicating a foot of 29.4 cm. The latter (Hamilton Coll. 56) is described as "Foot-rule, measuring 0.9675 English feet, 11.6 inches; hinge in the middle; apparently not divided into inches". Each half measures 14.7 cm, so that the length of the foot is 29.4 cm.

PLATE IX. We examined the square more closely, and found that the height of the vertical side was 14.7 cm, or half a foot, according to the second of the foot rules mentioned above. The same length is repeated from the extreme point of the long side of the square, measuring vertically down, and also from the inner point of the same side vertically down to the height of the upper edge of the foot-plate. This is very handy: if one wishes to measure the distance from, for instance, the lower edge of a fillet to a point above it, one can place the upper edge of the foot-plate along the lower edge of the fillet, and measure ½ foot to the inner point of the long side. If one wishes to take a measurement from the upper edge of the fillet, one can

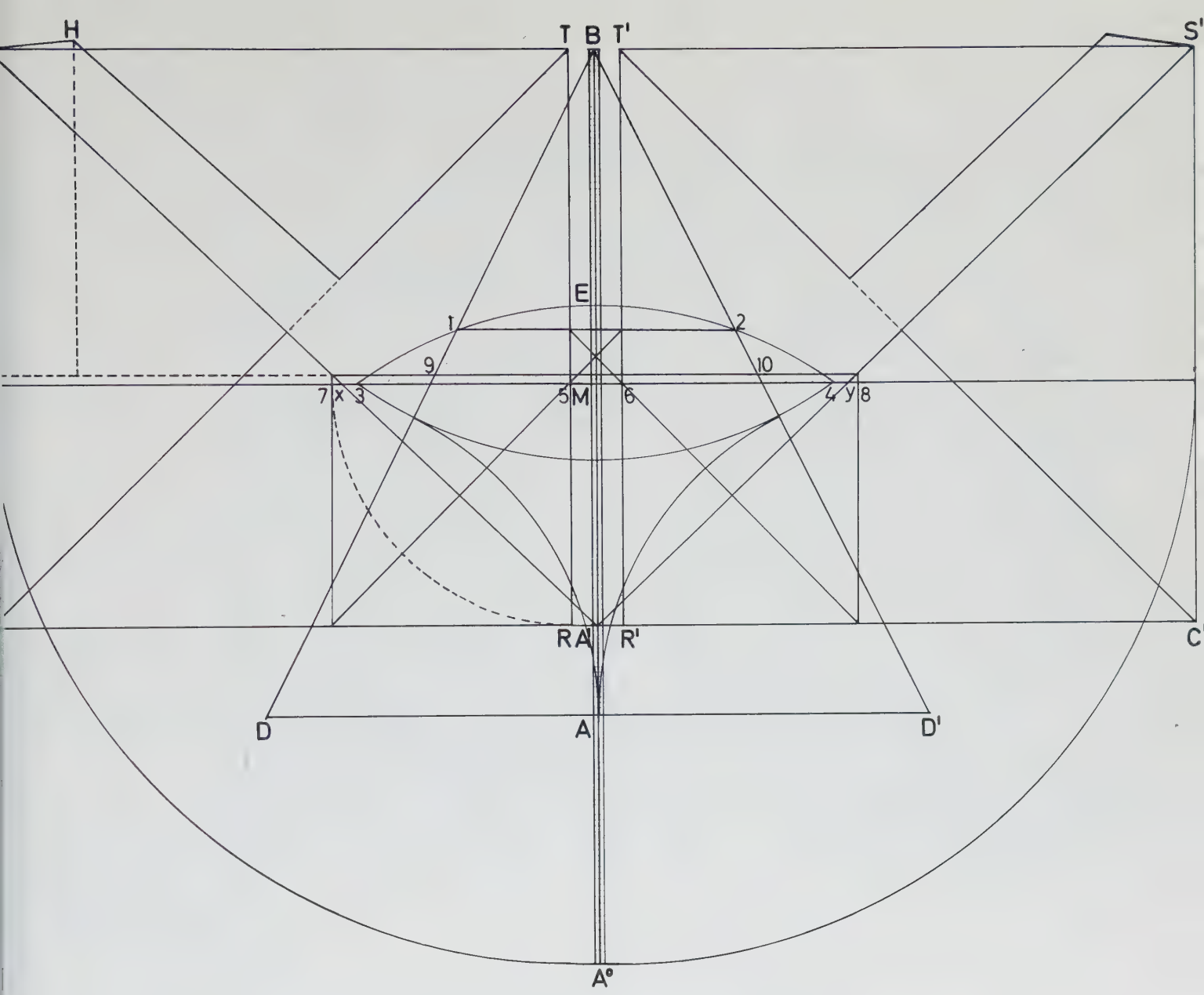


PLATE IX
'Carpenter's Square'. London B.M. (W.T. 966). Analysis based on the author's measured drawing. Scale 1:1.

employ one of the two other means of measuring offered by the square, selecting that which is most natural for the particular purpose.

But this does not explain the remarkable shape of the instrument. It must conceal yet other measurements, probably geometrical ones, which the craftsman was not required to know or understand, but which he was expected to follow. If the foot had been the only unit of measurement of any importance, a foot-rule – perhaps with a few simple divisions – would have served the purpose admirably, and a strange instrument such as this would have been quite unnecessary.

The square, a three-dimensional instrument, is here shown from the front, from the two sides, and from above, in the same manner as that employed in our drawings of rectangular

blocks. The analysis has been carried out in full scale. The narrow front is shown from M to B, the horizontal view from M to A°, while the foot-plate extends from M to A', and width from 5 to 6. The horizontal through M represents the base line on which the foot-plate stands, and TS (T'S') is equal to BA°.

AB is a foot of 29.4 cm, and AM = MB. We divide AB in Φ at E, and 1 and 2 are the points of intersection between the E arcs and the Φ hypotenuses. It appears that the perpendicular distance between these points and the M horizontal determines the width of the foot-plate, seen from above. The Φ arc about A through E, and about B through F, cut each other at 3 and 4, and half the distance between these two points determines the greatest length of the foot-plate, measured along its upper edge. (On the drawing we have indicated the distances between 5 and 7, and between 6 and 8. These are repeated, seen from above, from 5 to R, and from 6 to R'.) It appears that RT is the side of a square whose upper horizontal side is TS. Its left bottom corner is C. The diagonal TC determines the angle of the instrument at T, 45°. From S a line is drawn to A' determining the lower edge of the long arm which extends beyond the foot-plate. The line determines the oblique front edge of the foot-plate (as seen from the side) where it cuts the perpendicular through 7, and it interesects with the horizontal line through 7 at x. Thus the height of the foot-plate is determined. We realise that a number of geometrical dimensions, all of them based on the Φ division of the foot – in the manner accounted for in this chapter – are contained in this Carpenter's Square. They are all closely inter-related. As long as we restrict our measuring activities to the geometrical lines and points, the dimensions are functions of the whole, and no irrelevant remainder will occur. Thus a craftsman who adhered strictly to the measurements indicated by his tool worked in close harmony with the whole, as long as the foot on which his tool was based, formed part of this.

Statue bases, caskets, and monumental buildings – all were executed on the basis of these simple methods. Common to them all is the fact that horizontal and vertical sections are both in the same drawing thereby securing correct measurements. They were placed flush in the picture surface, and the sum of the height and the depth was divided in the way we have demonstrated in our analyses. In *Philebos*, 56A, Socrates says: "Building, however, makes a considerable use of measures and instruments, and the remarkable exactness thus attained makes it more scientific than most sorts of knowledge." He continues by saying that, in his opinion, this is achieved because "it (building) employs straight-edge and peg-and-cord, I believe, and compasses and plummet, and an ingenious kind of set-square."

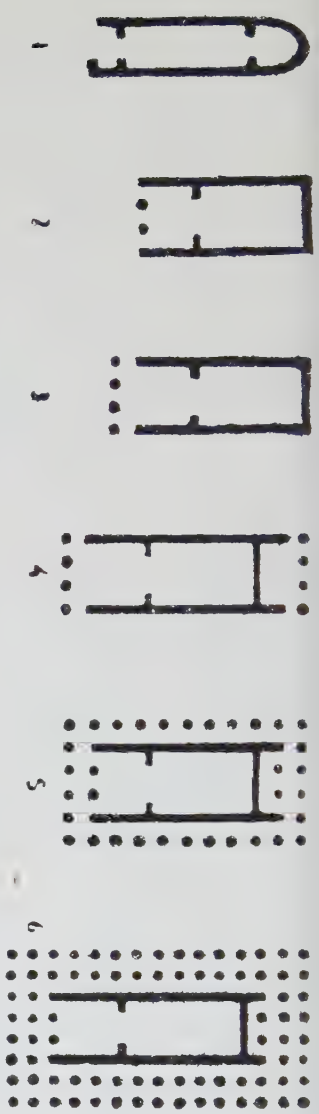


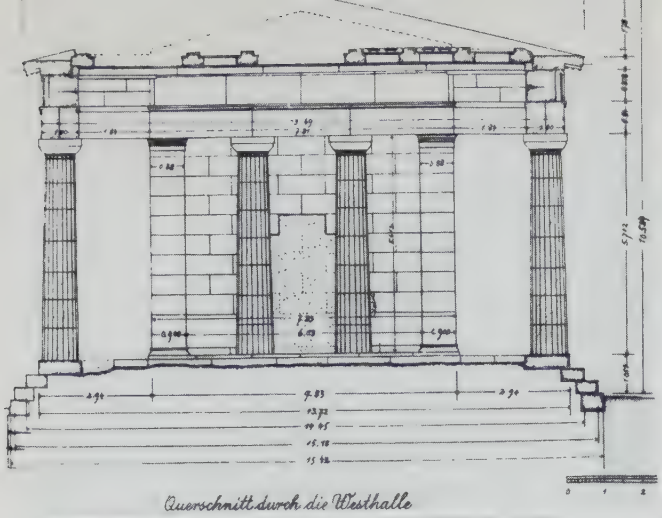
Fig. 16
Various types of Greek temples (From Kjellberg and Säflund, Grekisk och Romersk Konst. Stockholm 1964).

Fig. 17
Theseion in Athens.

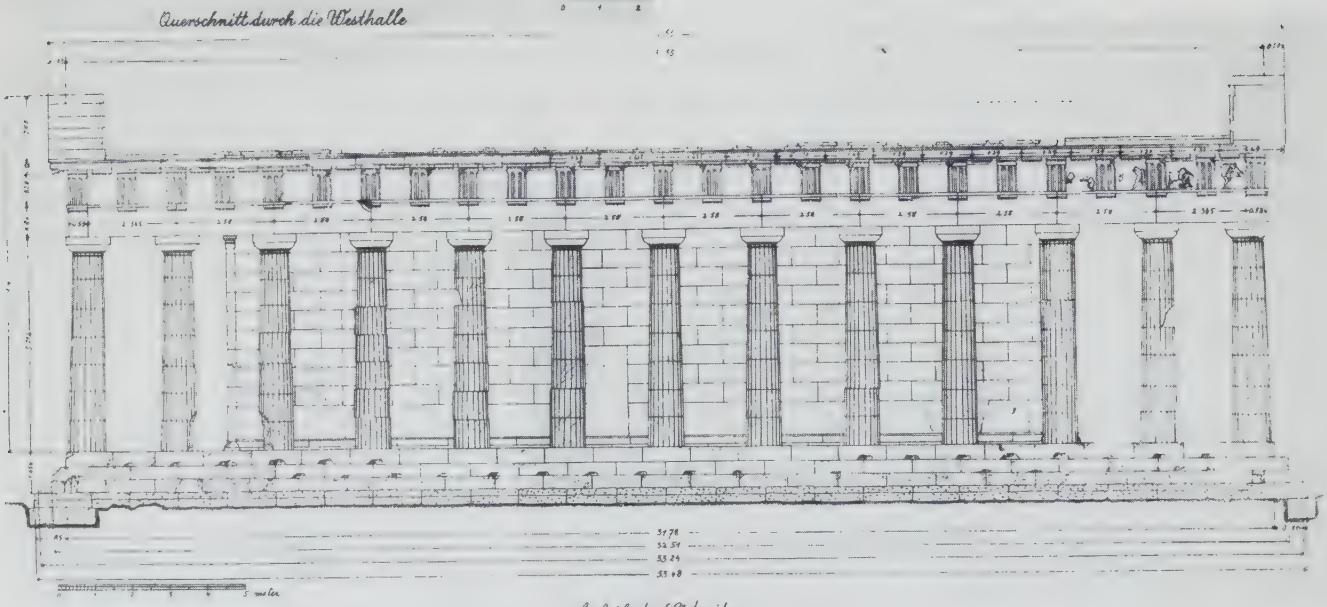
Three sacred architectural monuments

We now turn to the Greek temple to investigate the dimensions of one of the best preserved and most extensively studied Greek temples, the *Theseion in Athens*, also known as the Hephaisteion (ca. 450 BC). This temple has been measured in great detail by both American and German archaeologists, and their results have been published in: William Bell Dinsmoor, *Observations on the Hephaisteion*, American School of Classical Studies at Athens, 1941 (Hesperia, Supplement V), and Herbert Koch in his *Studien zum Theseustempel in Athen 1955* (Abhandlungen der Sächsischen Akademie der Wissenschaften zu Leipzig).

Violent earthquakes have done considerable damage to the Theseion. Moreover, when the temple was converted into a Christian Church, extensive alterations were carried out in its interior. The floor of the cella has been opened a number of times, most recently during the



Querschnitt durch die Westhalle



Aufriß der Südseite

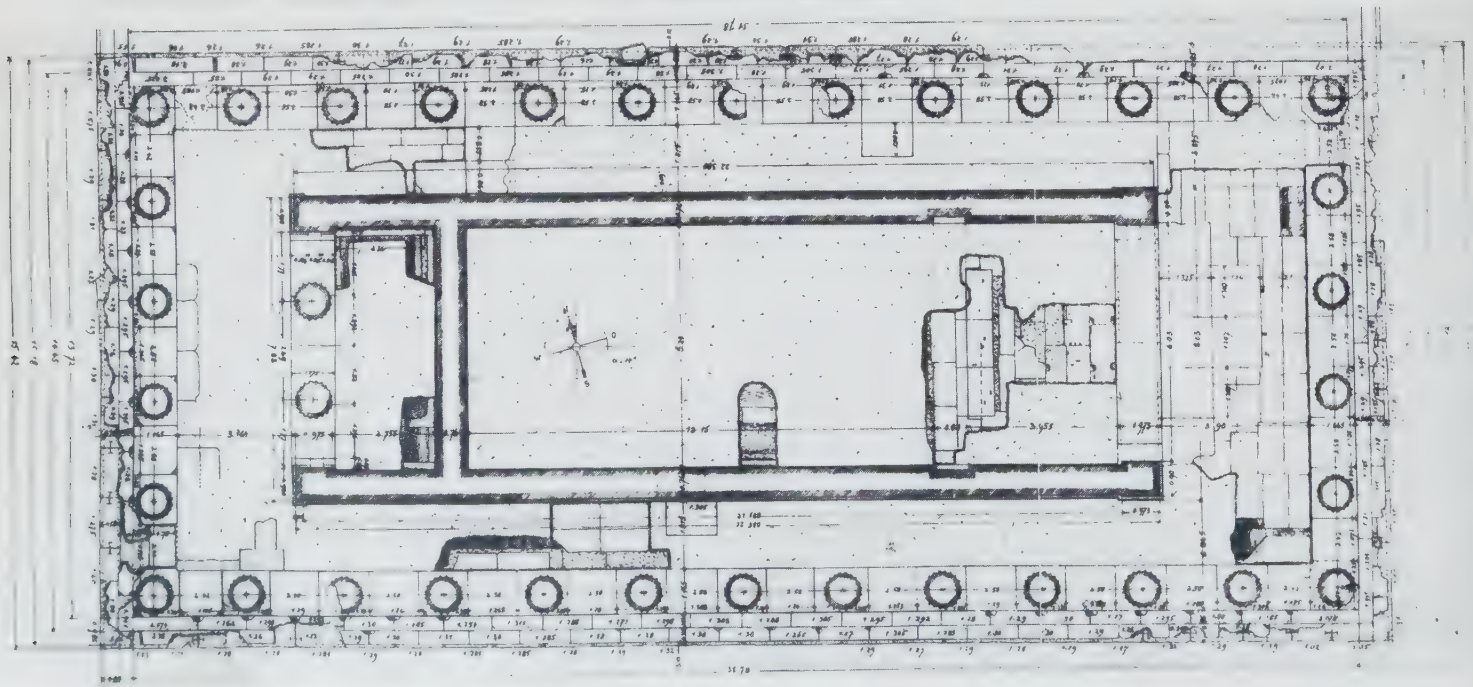
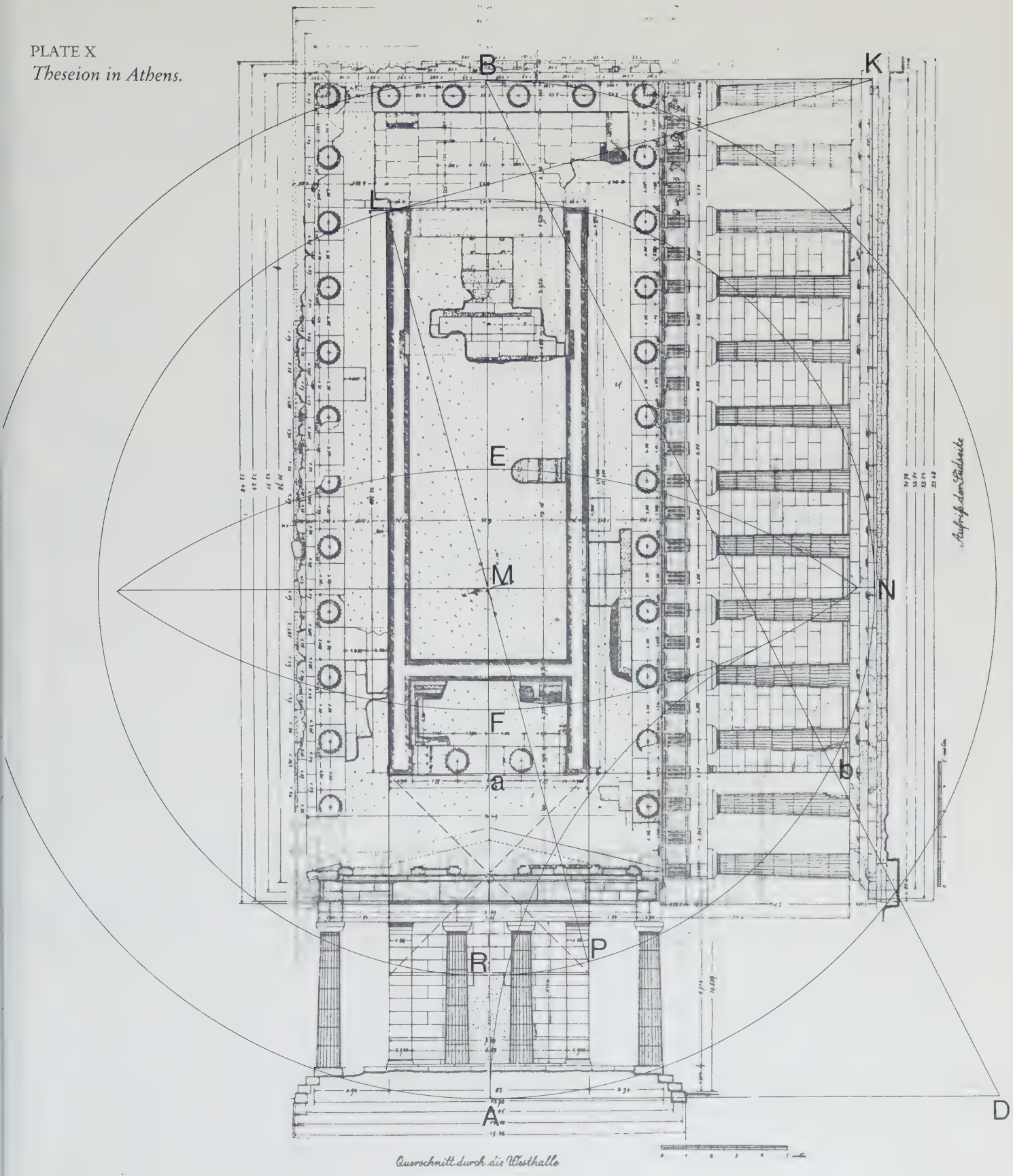


Fig. 18 a,b,c
Measured drawing by Herbert Koch of Theseion in Athens. Analysed on PLATE X.

PLATE X

Theseion in Athens.



nineteenth century, in order to accommodate Christian burials. As a result, large blocks had to be fitted together during the restoration of the temple, as though they had been the minute parts of a jig-saw puzzle. Dinsmoor's plan agrees completely with Koch's dimensions, and it is included in Koch's report. Our investigation is based on Koch's publication only, as this includes drawings of the plan and of all elevations and sections (*Fig. 18 a,b,c,d*). Moreover, the number of measurements here given is so great that a complete arithmetical control of the geometrical analysis can be carried out on the evidence they supply. Basing his remarks on his exhaustive study of the temple, Herbert Koch himself writes, "Unsere Untersuchungen am Theseion haben mit voller Sicherheit ergeben, daß das Werkmass allein zum Verständniss der Proportionen keinesfalls ausreicht, daß man vielmehr wirklich mit geometrisch ähnlichen Figuren, mit Festlegung wichtiger Punkte durch Diagonalen und mit dem Goldenen Schnitt zu rechnen hat." He also refers to a number of other scholars, all of whom hold views similar to his own, and he contributes important geometrical observations. These are not, however, of immediate concern to us, and we shall therefore concentrate our attention on the plan and sections, viewed as a whole.

If we look at the various types of Greek temples (*Fig. 16*), the importance of the cella is immediately apparent. It housed the image of the god, and was the centre of the building, its nucleus, in every respect. The earliest temples were of wood; they consisted of a cella only, or of a cella with a small porch.

The geometrical connection established between the principal dimensions of the temple is based on the method for which we have accounted in the present chapter. Although it seems probable — as shown by Koch — that the analysis could be continued, we shall stop at this point. A quotation from Herbert Koch may suitably conclude this analysis: "es erscheint auch wirklich einleuchtend, daß nur grosse, den Gesamtmessungen des Baues entnommene Verhältnisse seine Proportionen bestimmt haben."

We shall now proceed to the geometrical investigation of the Theseion, applying the method we are familiar with from previous analyses. We consider the temple as a block limited on all four sides by rows of columns. Inside we find a smaller block, the cella, with a sun mark carved in one of the stones of the floor. This mark is neither placed in the centre of the cella nor in the centre of the great temple floor. We see that the west front has only corner columns, the central portion marks the front of the cella. In PLATE X we can see how the front of the temple is placed flush with the big floor in the picture plane. The Greeks measured the height as the distance from the second step (A, N) to the top of the triglyphs forming a border over the row of columns (visible on the right hand side) where this is placed flush with the floor. AB, the sum of the temple heigth and length, is divided in Φ at E and F with M on the centre line. A circle about M with radius EB through N determines the position of the second step on the south side. A vertical is constructed through N. This meets the B-horizontal at K, a tangent at L is drawn to the circle about M. L determines the width as well as the length and, in addition, the sun mark. If we consider the temple and its cella as three-dimensional blocks, we see that both have M as their centre. The whole analysis may be regarded as an example of Greek simplicity and monumentality.

Pursuing the geometric way of thinking further, we return to Vitruvius as he is presented by Ceasario Ceasariano.

The Cathedral of Milan. PLATE XXI.

Unfortunately, none of Vitruvius' original architectural drawings are preserved, but the first Italian edition with a commentary by Ceasario Ceasariano, which was printed in Venice in

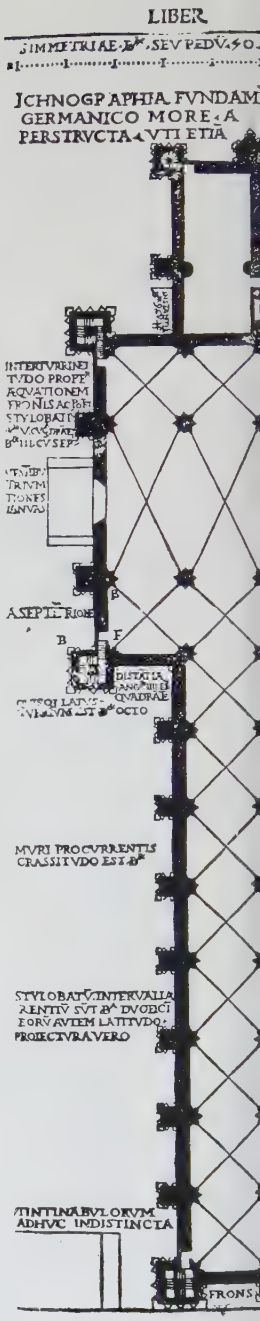


Fig. 19
Ceasario Ceasariano's plan-drawing of the Cathedral in Milan.

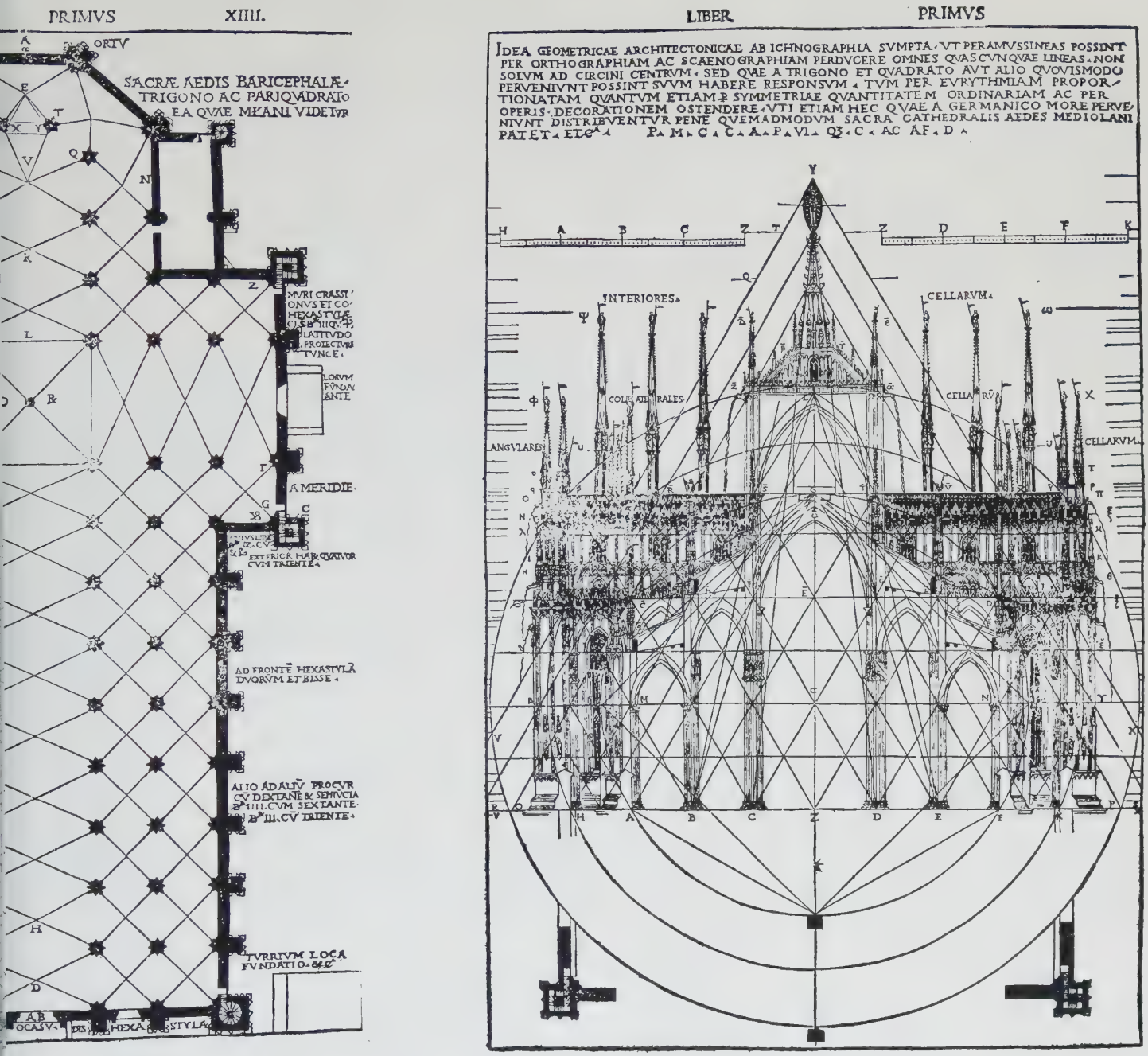

PRIMVS XIIII.
ORTV
SACRÆ AEDIS BARICEPHALÆ
TRIGONO AC PARIQVADRATO
EA QVÆ MEANI VIDETVR
A MERIDIE
AD FRONTE HEXASTYLA
DVORVM ET BISSE
TVRRIVM LOCA
LIBER PRIMVS
IDEA GEOMETRICAE ARCHITECTONICAE AB ICHNOGRAPHIA SVMPTA VT PERAMVSSINEAS POSSINT
PER ORTHOGRAPHIAM AC SCAENOGRAPHIAM PERDVCERE OMNES QVASCVNQVAE LINEAS NON
SOLVM AD CIRCINI CENTRVM SED QVAE A TRIGONO ET QVADRATO AVT ALIO QVOVISMODO
PERVENIVNT POSSINT SVVM HABERE RESPONSVM TVM PER EVRYTHMIAM PROPOR
TIONATAM QVANTVM ETIAM PER SYMMETRIAE QVANTITATEM ORDINARIAM AC PER
OPERIS DECORATIONEM OSTENDERE VTI ETIAM HEC QVAE A GERMANICO MORE PERVE
NIVNT DISTRIBVENTVR PENE QVEMADMODVM SACRA CATHEDRALIS AEDES MEDIOLANI
PATET ET
INTERIORES
CELLARVM
ANGVLARES
CELLARVM

Fig. 20
Ceasario Ceasariano's drawing of the vertical section of the Cathedral in Milan.

1521, is illustrated with drawings. Whether these are Ceasarianos' originals or whether they are based on older drawings, we do not know.

Ceasariano's Vitruvius edition includes two drawings of the Cathedral in Milan, a Gothic building, *Fig. 19* and *Fig. 20*. For centuries scholars of architecture have been intrigued by these drawings for a variety of reasons. They cannot possibly have come from Vitruvius – Ceasariano must have prepared them in order to illustrate some of Vitruvius' points, in which

case the same way of thinking underlies both. — A violent dispute arose while the cathedral was being built. The master-builders were afraid that the construction according to the original plan was too weak, i.e. they believed the structure vould not safely bear the projected height. The account of a meeting convened in 1398 to discuss these matters is still in existence; we quote from the English translation in Macody Lund's *Ad Quadratum*: "The Italian masters at a critical point during the work had summoned Master Jean Mignot from Paris. In their answer to his criticism of the constructions, they said, rather unfortunately, 'that the Science of Geometry was of no importance here, because Science is one thing, Art another', to which the Frenchman answered irrefutably, 'Art without Science is nothing'." This reply shows that Plato's philosophy was still remembered by the learned church architects.

On Ceasariano's drawings, the plan is covered with a squared grid, while the sect on is divided into isosceles triangles. The plan, in other words, is designed 'ad quadratum', while the section is designed 'ad triangulum'. But from our point of view this is not the most striking aspect of these drawings. Of exceedingly great interest to us is the fact that plan drawings of two corners appear below the drawing of the section of the church. This would seem to indicate that plan and section had been calculated on the same drawing; in fact, it appears that *these corners on the section drawing agree completely with the two western corners of the transcepts!*

The two drawings were transferred on to transparent film, and then superimosed on each other so that the two corners mentioned above coincided.

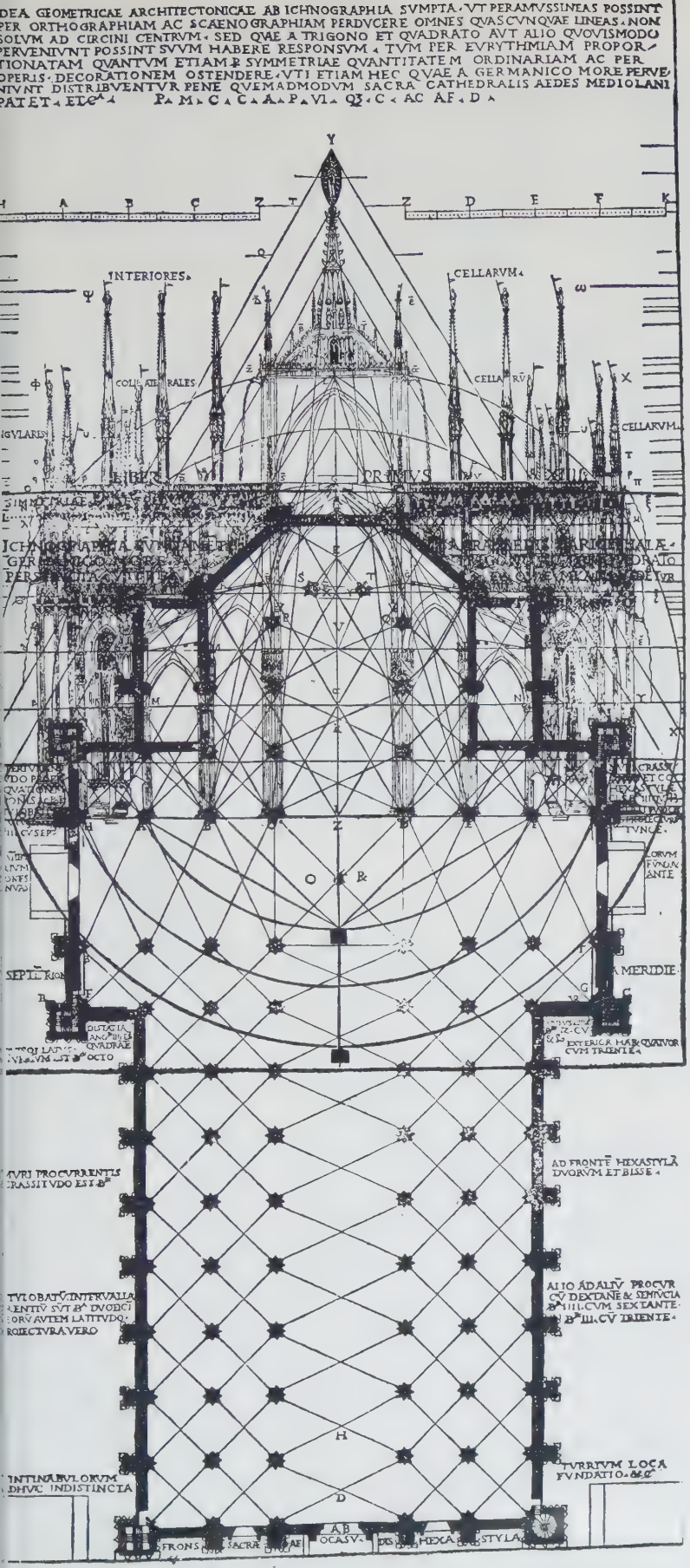
The length of the church is AG, the height equals GB, which equals OG, the vertical height of the church up to the roof. (The roof starts at G, the level marked by the two uppermost gargoyles, level G.)

We may say that the church has been designed in a way we know from the preceding examples. This applies to both plan and section. The Φ constructions give point H, the Holiest of the Holy of the cathedral, shown on the ground plan.

Our investigations show that Ceasario Ceasariano desired to camouflage a secret. In addition to the two corner points on the horizontal projection already mentioned, we find a mark across the highest spire of the tower. This mark is purely geometrical with no other significance than the one we have mentioned, as GB equals twice the height of the front. The mark on the spire determines point B, evident to the constructor who is let in on the secrets of the guild, secrets we have undoubtedly touched.

The grids on CC's drawings are a function of the geometrical analysis which remain a secret. Only the two corners mentioned and the mark on the highest spire are sufficient to the reader familiar with the secrets of the guild.

PLATE XI
Analysis of Ceasario Ceasariano's drawings of the Cathedral in Milan in his Italian Edition of Vitruvius. Venice 1521.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SOLVM AD CIRCINI CENTRVM. SED QVAE A TRIGONO ET QVADRATO AVT ALIO QVOVISMODO
PERVENIVNT POSSINT SVVM HABERE RESPONSVM. TVM PER EVRYTHMIAM PROPOR-
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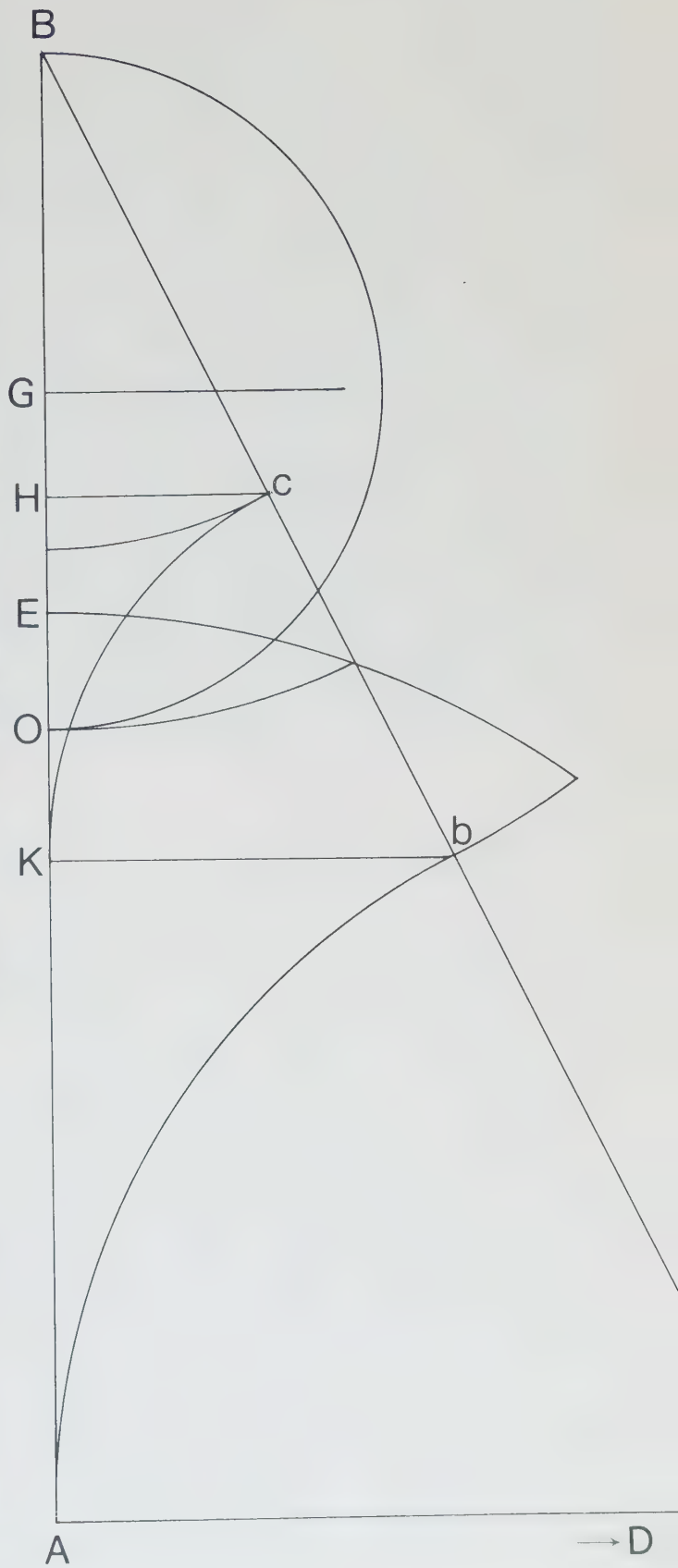
OPERIS DECORATIONEM OSTENDERE. VTI ETIAM HEC QVAE A GERMANICO MORE PERVE-
NIVNT DISTRIBVENTVR PENE QVEMADMODVM SACRA CATHEDRALIS AEDES MEDIOLANI
PATET. ETC.
INTERIORES
CELLARVM
CELLARVM
AD FRONTE HEXASTYLA
DVORVM ET BISSE
ALIO AD ALIV PROCVR
CV DEXTANE & SEMVNCIA
TVRRIVM LOCA
FVNDATIO
AB
OCASV
HEXA
STYLA


B
G
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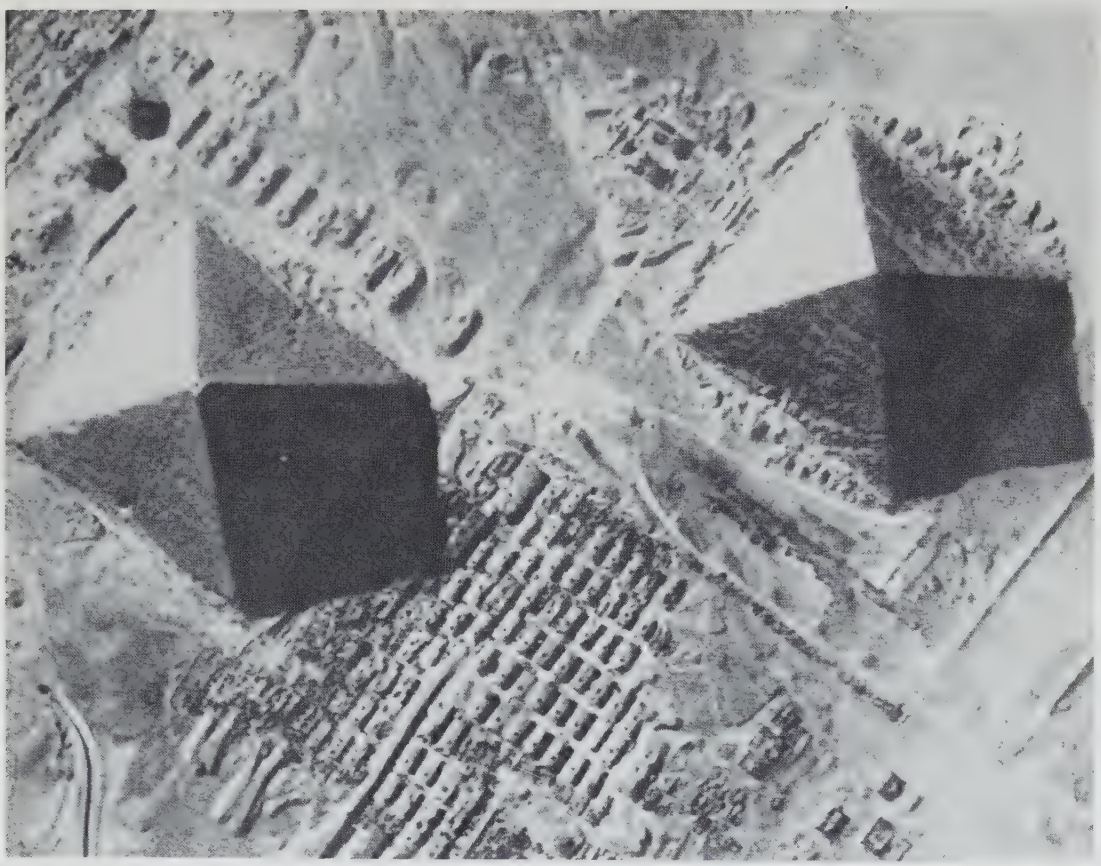


Fig. 21 a
Aerial view of Giza. Cheops pyramid to the left.

We would remind the reader that this book is the second volume of a trilogy entitled *Depth and Movement.* The first volume, *Geometry in Egyptian Art* (London 1955), showed through analyses of the procedure of forming human figures how the third dimension, the depth (depending on the plinth) was placed flush with the height in the picture plane. AB, the sum of these two quantities, was divided in φ and further partitions gave all other sizes as shown in the analysis of the Theseion. An analogous analysis of an Egyptian construction, the Cheops pyramid, is given here in order to emphasize the line from Egypt to Greece and further to the Nordic countries in the early Middle Ages.

The sum of the height and width of the pyramid is AB, which is divided in Φ at E and F, M is the centre. An arc from B through M cuts the sides of the pyramid at G and G'. The points of intersection between the E arc and the F arc indicate the length of the base on which the pyramid rests and the lower edge is given by M. GG' lies lower then E, a fact explaining the incline towards the centre (E) on the vertical section of the base line. The E horizontal cuts BD at x. From this point vertical lines are constructed on the two hypotenuses marking the «Grand Gallery» as well as the «Descending Gallery». The determination of these shows clearly the decisive importance of the Φ division also with respect to cult activities.

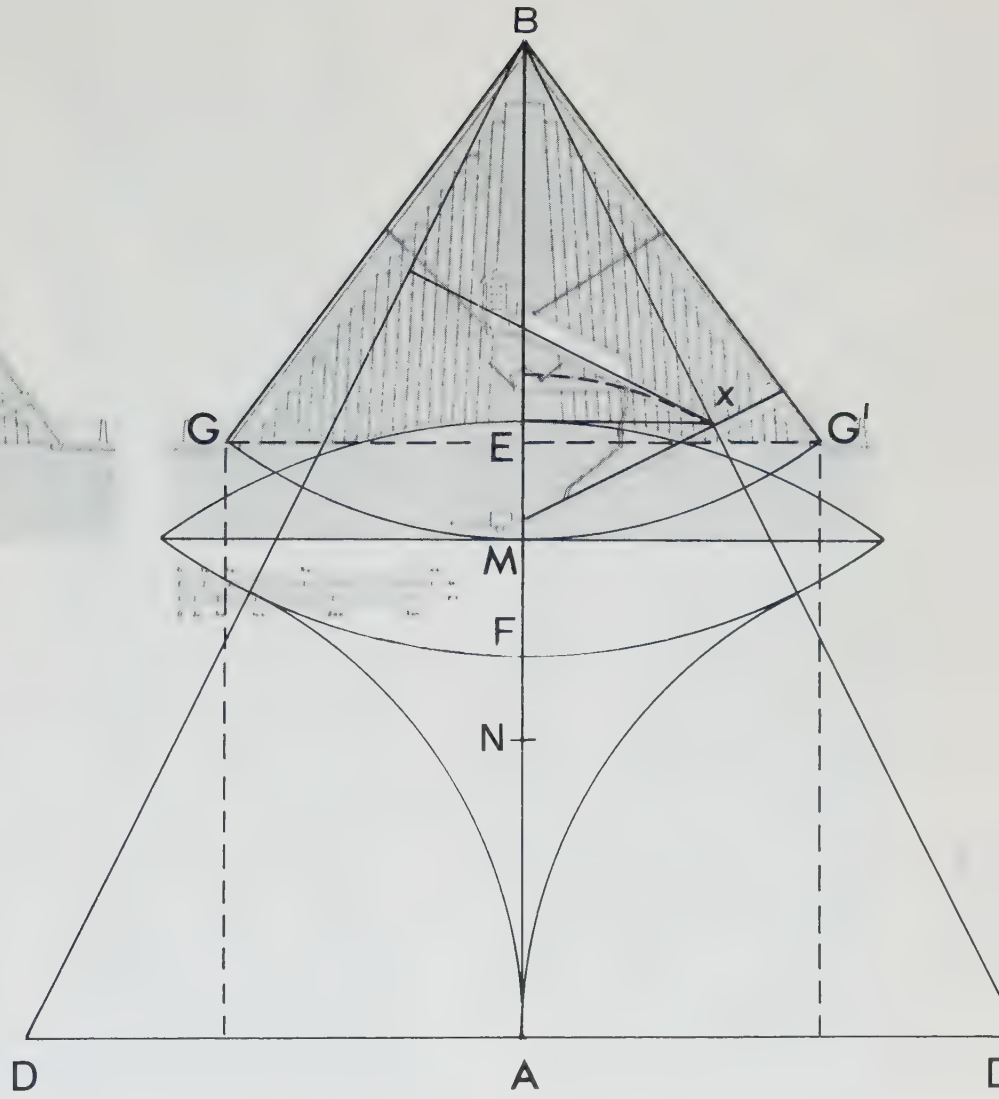


Fig. 21 b
Cheops pyramid, Giza, built c. 2600 B.C. Rising to a height of 146.5 m on a base measuring 230 m by 230 m, this building, the largest in history, measures more than 2.5 million m³. From Henri Stierlin: Architektur der Welt. Munich 1977.

PLATE XII
The Cheops Pyramid.

Chapter II

From the Egyptian to the Greek Way of Representing the Human Figure

Statuary based on Egyptian methods

When one has spent many years studying Greek art, the task of accounting for the results of the research appears staggering. Words cannot explain the way in which the artists thought and worked. Nor can mere words convey to the reader an adequate impression of the topic, they cannot give him a true understanding and experience of Greek art. Film would perhaps have been the best medium, a series of pictures unfolding before the eyes of the audience. But we must content ourselves with presenting our investigations and compiling a kind of picture book – the necessary explanations being provided by the text which accompanies the analyses.

The investigations presented in this chapter are concerned both with sculpture in the round and with flat pictures, for we found that these can explain and illuminate one another. This is not surprising; it is in fact extremely logical, for both branches of art are – as they were in Egypt – based on the rectangular block, drawn as we showed in the preceding chapter (Fig. 3).

Walking through the first galleries of the National Museum in Athens where the Archaic sculptures stand, we soon become aware of the close relationship with other ancient cultures. Julius Lange rightly classified both Archaic Greek and Egyptian art under the heading "pre-Greek", he was mistaken, however, when he described all the "pre-Greek" figures as being "frontal". Some of them are frontal, it is true, such as the Artemis from Delos (Athens N.M.1), c. 650 BC, *Fig. 22*. She is in complete repose, there is not the slightest hint of any turning or motion. The Egyptians had represented the dead and the gods entirely frontally – was this Greek practice, too? The upper part of the body of the large, seated figure from Arcadia (N.M.57), *Fig. 23*, clearly turns slightly towards the side, while the corner posts of the throne represent absolute frontality. This is no new feature, however; it was well known in Egyptian art. The same may be said of the somewhat later double sculpture (N.M.56), *Fig. 24*, with its characteristically connected bodies and block. Both the figures take a step forward with the leg nearest the middle, and from the hip down, the bodies turn away from each other. The upper parts of the bodies, however, are in absolute frontality, and together with the heads they form part of the architectural compositon of the sculpture.

The entire Archaic period provides opportunities for observations of this kind. *Fig. 25* shows a kouros (N.M.8), whose legs are missing. The upper part of his body is clearly turned, but his head is frontal. His hands are close to the sides of the body and their lowest points form an oblique line, perpendicular to the centre line of the figure. One might perhaps say that the figure is in a state of transition between repose and movement. The profound impression this sculptural style makes on the spectator is beyond question.

Fig. 22
Artemis from Delos. About 650 B.C. Athens N.M. 1. Height 1.90 m.

Fig. 23
Seated goddess or 'Heroized Mortal'. About 630 B.C. Late Daedalic work. Athens N.M. 56. Max. width 0.52 m.

Fig. 24
Sepulchral monument. Boeotian work from about the middle of Vth century. Above there would have been a sphinx. Height of figures 1.47 m.

Fig. 25
Kouros. Athens N.M. 8. Found on Thera. Height 1.24 m.

The splendour of this early period of Greek art grows out of the purity of form and the expression evinced by these sculptures. From a naturalistic point of view this art may be termed "primitive", but once we have experienced the series of sculptures in the Acropolis Museum, we realise that this epithet is absurd and entirely misleading, *Fig. 26*, *Fig. 27*. Here the Greek sculptors carried the possibilities inherent in their style to the utmost limits, with a sense of imagination and refinement which never becomes superficial, and nowhere breaks the simple, fundamental principles on which their art was based. We experience the magnificent "freedom" of their art in correlation with its own, innate "law".

We are here concerned with the following two questions: did the Greeks during this early period consciously work according to the same lines of thought as the Egyptians had done (cf. *Geometry in Egyptian Art*)? Can we, further, explain parts of what History of Art terms "the Greek triumph" on the basis of this heritage from Egypt? We shall attempt to elucidate these problems by means of a number of analyses of Greek works of art.

The little Archaic Hermes stele (Athens N.M. 3728) may be regarded as a "missing link" between the rectangular block and the human figure, *Fig. 28*. The museum catalogue describes it as "small square Hermaic stele, ithyphallic, and with a head of a bearded Hermes. Excellent island work abut 520 BC, from Sifnos. Height 0.66 m. A sculpture of mighty simplicity!"

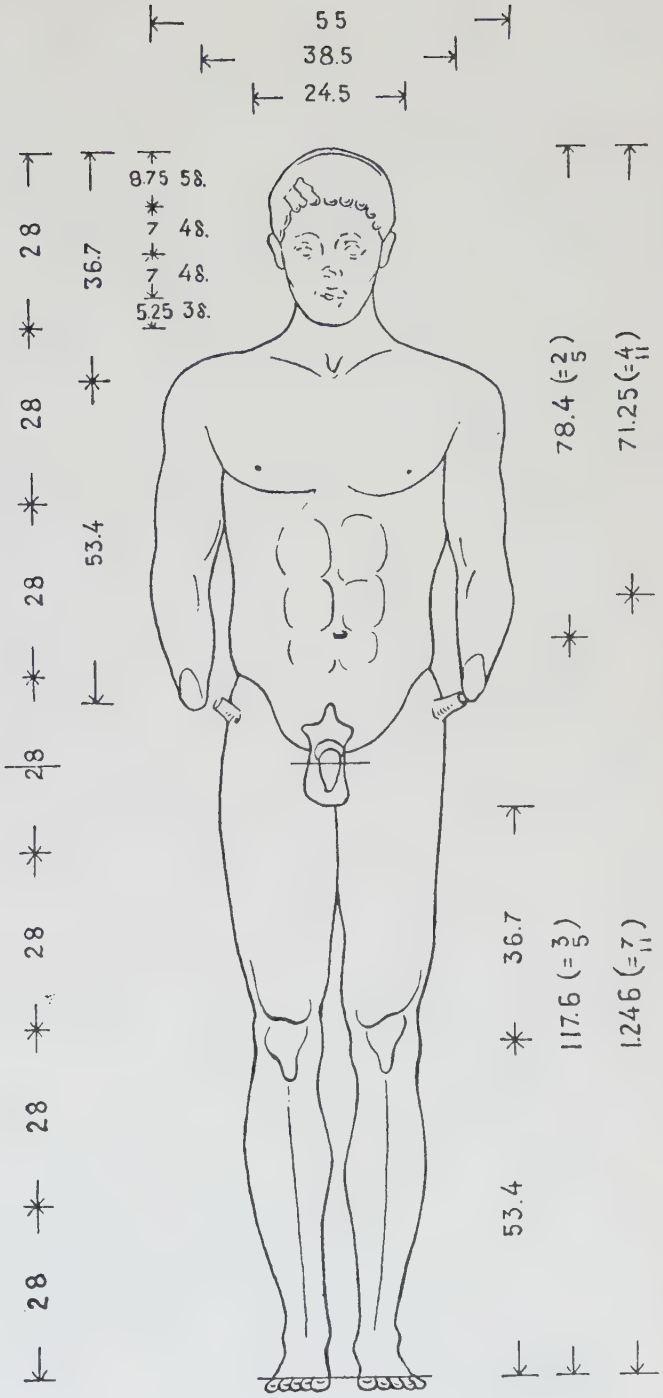
The block (placed on a modern base) is unbroken up to shoulder level. The head, which dominates the upper part of the stele, is very slightly tilted to one side, so that the left eye is a little higher than the right. A large phallos lies at the centre of the vertical central axis of the block. It is shown in high relief on the vertical front of the block, pointing upwards — an elementary form of symbolism. An investigation of the dimensions of the block, according to the methods

Fig. 26
'The Peplos Kore'. About 530 B.C. Athens Akropolis M. 679. Height 4 ft.

Fig. 27
Archaic kore. About 490 B.C. Island marble. Athens Akropolis M. 675. Height 4 ft.

g. 28
all square, Hermaic
le, ithyphallic, and with
head of a bearded Her-
s. About 520 B.C. Island
rk. Athens N.M. 3728.

g. 29
istodikos. Attic work.
out 500 B.C. Athens
M. 3938. Height 1.96 m.

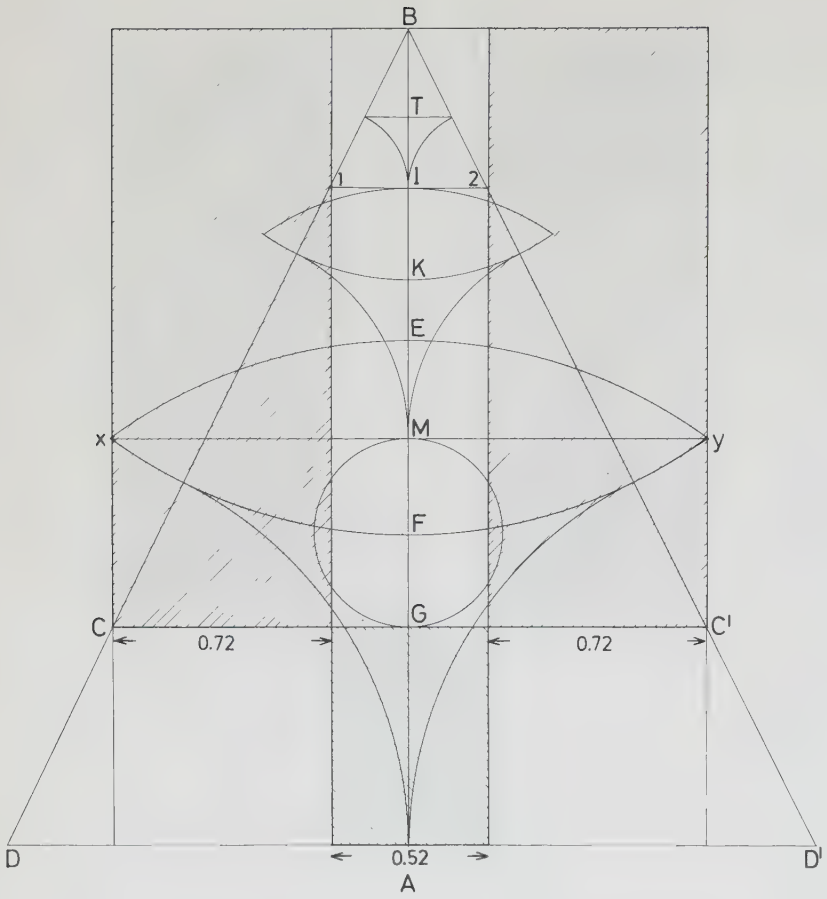


g. 30
easured drawing of Ari-
dikos by K. Rhomaios.
ublished in Christos
arusos, Aristodikos.
uttgart 1961).

employed in the preceding chapter, would seem to show that this half-human figure was planned in the same way as the rectangular blocks.*

We are fortunate in that Christos Karusos' book *Aristodikos* includes a complete set of measurements of the statue of this name (Athens N.M. 3938), *Fig. 30.* This is a tomb statue from c. 510 BC, and all its main dimensions are preserved; moreover, the plinth is also intact.

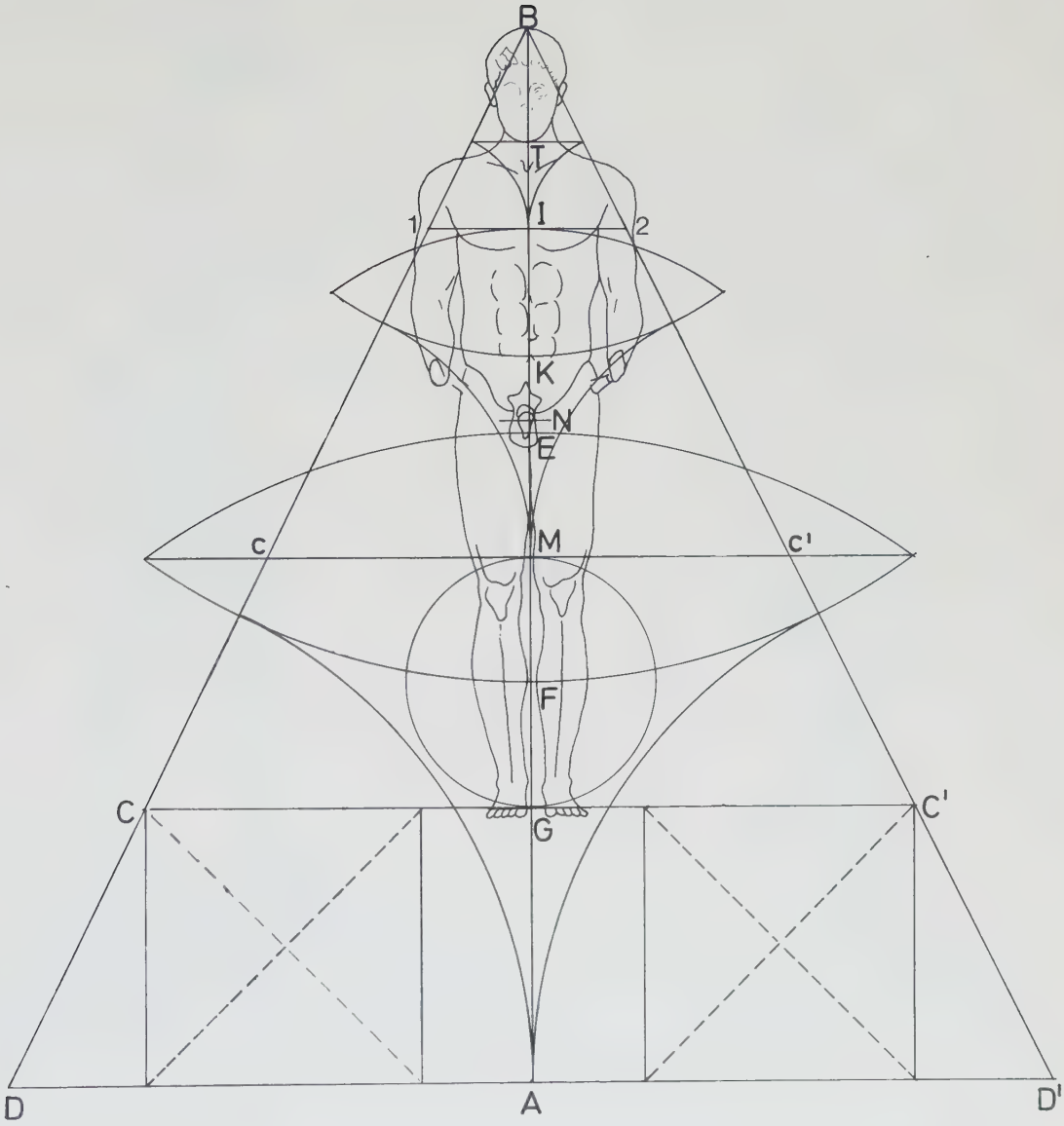
* The sculpture was exhibited at the time and it was not possible to obtain correct measurements.



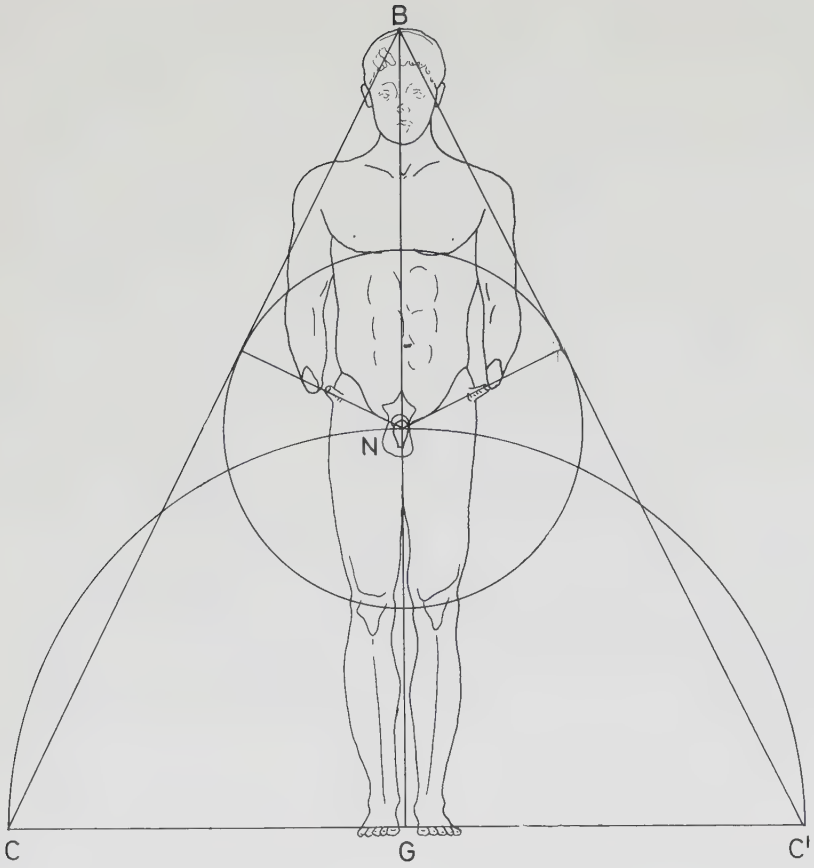
The measured drawing, the work of K. Rhomaios, is an extremely painstaking piece of work, and it gives a great number of measurements. According to Rhomaios it seems fairly certain that the sculptor worked with a module of a foot of 28 cm. He finds this measurement in the size of the head, which in turn equals 1/7 of the height of the whole figure = 1.96 m. The measured drawing, *Fig. 30,* does not include the measurements of the plinth, but these are given with exact figures in Karusos' text. Here the author has some very interesting reflections concerning the relationship between the slight deviations from the strictly mathematical rectangle apparent in the plinth, and the suggestion of movement perceptible in the figure. But he concludes: "Sie in die Reihe von Denkmälern, die das für die Plastik und die Architektur festgestellte Bedürfnis nach latenter Bewegung zeigen, einordnen und ihren Platz darin bestimmen zu wollen, ist freilich verfrüht, da darauf bezügliche systematische Untersuchungen kaum begonnen haben."

PLATE XIII A, B, C Athens N.M. 3938. Employing Karusos' and Rhomaios' measurements, we determine AB as the sum of GB, the height of the figure (1.96 m) and AG (0.72 m) = 2.68 m. Drawing this in the usual way to a scale of 1:10, we arrive at the figure shown in PLATE XIII A. AB is divided in Φ at E (above) and at F (below), while M is the mid-point. A circle (centre F) through M determines G, and *the depth of the base AG, and the height of the figure, GB, are now geometrically determined.* A horizontal is constructed through G, and it intersects with the Φ hypotenuses at C and C'. From C and C' we mark off AG, the depth (length) of the base (0.72 cm) along the G horizontal towards its centre, and we find that

PLATE XIII, A, B, C
Aristodikos. Athens N.M. 3938. Analysis based on the measured drawing by K. Rhomaios, published in Christos Karusos' Aristodikos. Stuttgart 1961.



the distance between these two sides of the base is equal to its width (0.52 m). The very simplicity of this construction should provide good evidence of its correctness. This impression is further supported by more detailed division. (PLATE XIII A makes it appear as though the distance between the points of intersection of the Φ arcs, x and y, is equal to CC'. A larger scale drawing shows that this is not the case, as is also proved by geometrical control.) We return to quote Karusos, who points out that not all the dimensions of this tomb sculpture are preserved, and he therefore feels that one can go no further in trying to reconstruct the process of creation from the artist's point of view.



He also explains that there must have been at least one more step below the plinth. In the face of these reflections, we would point out that the basic dimension AB is determined as the sum of the *height of the figure and the depth of the base*, and that the *height* of the base therefore is of secondary importance. *This measurement, as well as that of any step below it, is derived from AB, as are also all the other dimensions of the statue.* We must expect similar contacts to have existed between every base (with which we worked in Chapter I) and its lost statue. We shall later show by analysis examples of the measurements of a lower base below the top base or plinth being derived from the sum of the height of the figure and the depth of the plinth. Later on we shall also show examples of a base of considerable height being derived from the same dimension AB (Fig. 48).

Further divisions are shown on PLATE XIIIC. M is the mid-point of AB, and MB is divided in Φ at I (above) and at K (below). Arcs through I struck from 1 and 2, the points of intersection between the I horizontal and the Φ hypotenuses, cut these hypotenuses at level T. The following now appears: TB determines the height of the head, and according to Rhomaios this is equal to the foot of 0.28 m which is said to form the module of this sculpture. Our geometrical determination of the measurements can be verified arithmetically, and we now find that *in this statue, the size of the foot is a function of the geometrical division of AB, the sum of the height of the figure and the depth of the base.*

Fig 31, 32
Epiphany of Apollo. Bronze. End of VIth century B.C. Lakonia. Athens N.M. 16365. Front view and side view.

On his drawing, Rhomaios has indicated the mid-point of the figure by a horizontal mark through the penis. On PLATE XIIIC we have struck a circle with this point as its centre (N), and tangential to the Φ hypotenuses (compare PLATE X). The perpendiculars from N to the hypotenuses pass straight through the arm supports projecting from the lower edge of the pectoral muscles.

Rhomaios and Karusos were aware of the fact that in archaic statues of this type the shoulder, chest and hip of the statue are lowered in keeping with the advanced foot – and this the Egyptians did not permit. – The basic proportions of the sculpture are, nevertheless, calculated according to the Egyptian tradition.

PLATE XIV, Athens N.M. 16365. Next we shall investigate the little bronze "*Epiphany of Apollo*" made about ten years later, *Figs. 31, 32.* The following measurements have been taken: height of figure from right heel to top of head 17 cm. Depth (length) of top plinth 5.9 cm. Depth (length) of lower base 8.5 cm. Height of lower base to top of plinth 1.6 cm. The analysis has been carried out full scale. We referred to this bronze in connection with the preceding analysis, because its plinth rests on a base in such a way that a step is formed between the upper plinth and the base below. Investigations show that the dimensions of the top plinth and also those of the base are determined by divisions of AB, *the sum of the height of the figure and the depth (length) of the top plinth.* By a Φ division of AB we arrive at the points of intersection on the Φ hypotenuses x and x', and thus at O. OA' is marked off equal to OB, and A'B is divided in Φ at I and K (dotted). We see that *the depth of the plinth AG is equal to IK.* The sketch of the figure, as seen from the side, shows that it is standing on the G horizontal, the top of the plinth sloping slightly towards the front, an apparent help to the figure in its forward movement. NM determines the width of the plinth, M being the mid-point of AB, and N that of GB. The length of the lower base is determined by A'G and, as seen from the side, the front edge touches the sides of the plinth, as seen from above. GK is the length of that part of the base which projects beyond the plinth at the back. The way in which contact is obtained between the geometry of the front and that of the profile of the figure is interesting: the circle centre B through S (radius minor of MB) touches the nose and forehead of the profile. This is achieved by letting the lower base touch the side of the plinth as seen from above.

The man is in intense movement, advancing his left foot in a long stride while putting his right shoulder and the upper part of his body forward. At the same time his left shoulder is lowered. This is most easily seen from behind, and from this position we can also see that the head is turned towards the left. Thus we here have a figure in fairly complicated movement, and it is difficult to ascertain how conscious of a theoretical background to his composition the artist actually was.

The figure is the work of an artist feeling his way at a time of transition, and whether it is unique, we do not know. Karusos, in his *Ein lakonischer Apollon, Charites*, Studien zur Altertumswissenschaft (Bonn 1957), considers it to be the product of a Laconian workshop intensely interested in just these problems. The design abounds in vital strength, and the figure is executed with a vivid understanding of the muscular movement of the human body. Marks on the bronze show that *such small figures were cast over cores of wood*, and the artist must have known how deeply he was to cut the various planes, etc., and he must have marked his fixed points on the block. Then he could carve the wooden model over which the bronze was cast, using these marks as his point of departure.

In *Geometry in Egyptian Art* we gave an account of the Egyptian method of determining or explaining a figure taking a long stride forward. This method cannot, however, have made possible the design of this figure, which also bends *sideways*, for its front view would *not* show the true measurements. As soon as the Greek sculptors no longer found the stiff, Egyptian way of representation satisfactory, they were forced to find new solutions, new methods. In our attempts at understanding the results they gradually achieved, the way of thinking on which their work was based, we found it necessary to ascertain how they represented movement on a flat surface, in painting or on reliefs. We shall deal with this in the following section, before returning to sculpture in the round at a later stage. — Again we must turn to Egypt.

PLATE XIV
Epiphany of Apollo. Bronze. Athens N.M. 163665. Analysis based on the author's measured drawing. Scale 1:1.

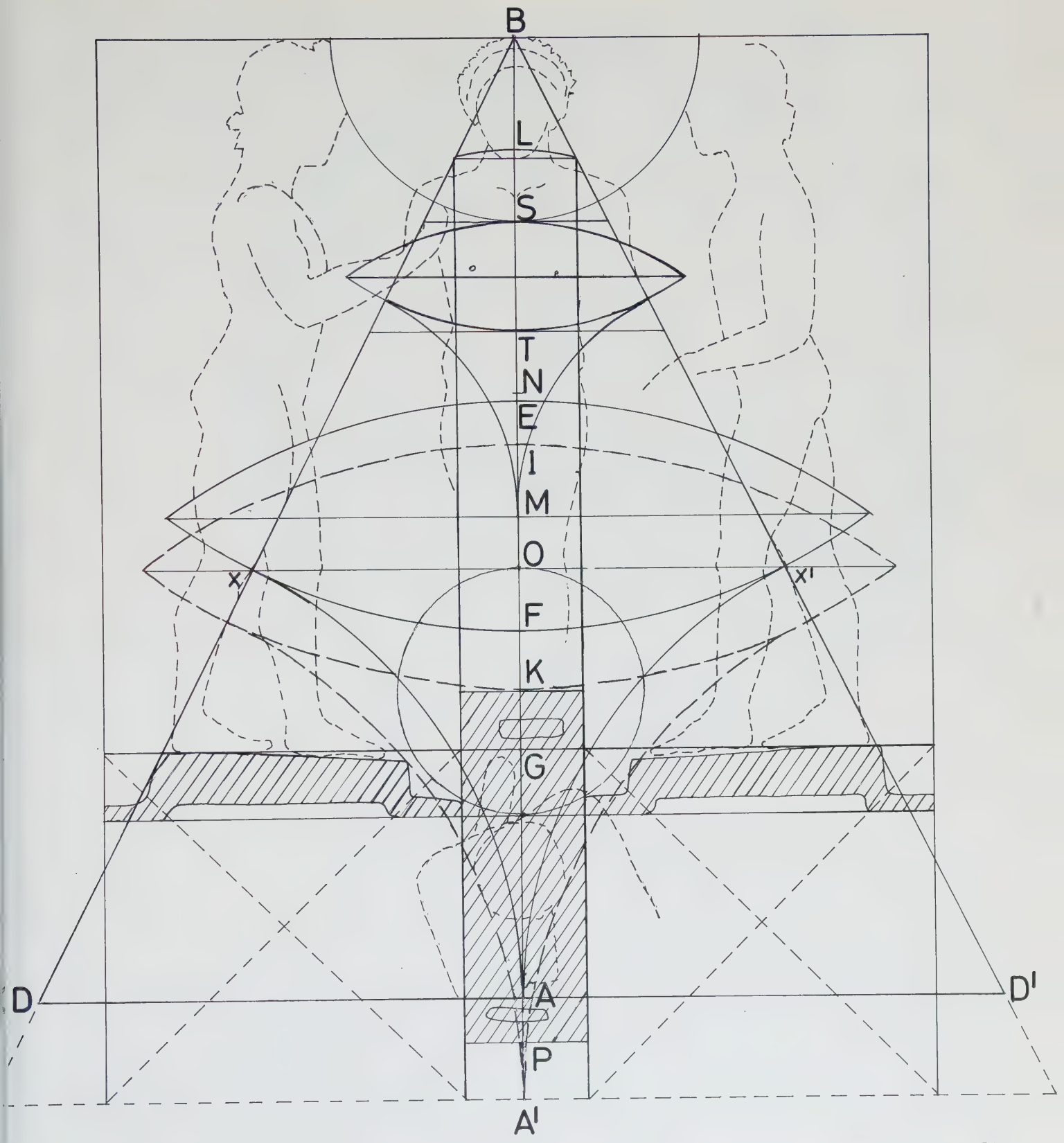

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Fig. 33
'Dionysos Sailing on the Ocean'. Kylix by Exekias. München Staatl. Antiken Samml. 2044. About 540 B.C.

The Egyptian method of representing movement on flat pictures known in Greece from Archaic times

Egyptian art was not as one-sided as tradition would have us believe. The large chariot compositions, regardless of whether they are carved on the temple walls in low relief or painted on a casket in Tut-Ank-Amon's tomb, bear witness to a remarkable power of expressing dramatic movement. Here we see Pharao standing in his chariot, his bow drawn, his horses galloping ahead. Whether the king is aiming his arrow at a target or at the enemy, the artist's method is the same. We have accounted for this method in *Geometry in Egyptian Art*, and we shall now see that Greek artists were familiar with it and employed it as early as the Archaic period. By means of a number of examples we shall trace the use of this method into the Classical Age.

The basic Egyptian principle may be summarised thus: since the figure cannot be shown in several places, representing movement from one position to another (like modern film), the artists *let the geometry of the picture carry out the movement*: the geometrical points, lines and dimensions to which the figures are bound are assigned a new place, and are woven into the scene represented.

PLATE XV A1, A2, B, C (München Staatl. Antiken Sammlung 2044). In the analysis of *Dionysos Sailing on the Ocean*, we let the picture provide the implications. *Fig. 33.* Dionysos is sitting in his boat, cup in hand, and the boat is sailing in a fair breeze. Dionysos, the boat and the mast are all shown from the side (the upper part of Dionysos' body is turned, so that it appears frontally in the Egyptian manner). At the same time we look straight down on to the sea, which fills the surface and provides a background for the vines which grow out of the mast, carrying bunches of grapes, and for the dolphins, "the mutinous sailors", swimming in the sea. Thus Dionysos in his boat, in profile projection, is outlined against the sea, which is rendered in horizontal projection. This is precisely what the Egyptians did. They conceived the objects as solids, three-dimensionally, and they placed the different representations of them on the picture surface as seen from above, from the side or from the front.

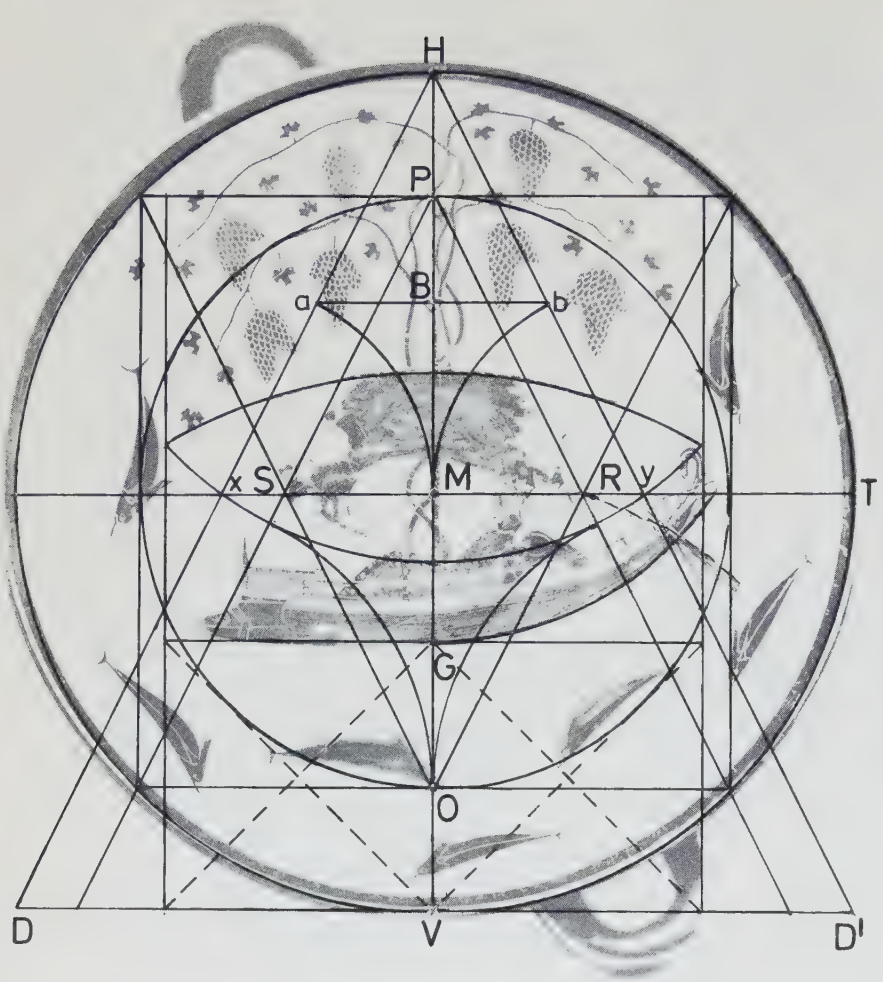
This picture, painted on the slightly concave base of a wide open kylix, is the work of the well-known vase-painter Exekias, and dates from c. 540 BC. The limits of the picture are given by the circular edge of the vase, but we must assume a slight margin of error due to its concavity. The photograph forms a horizontal projection of the slightly concave scene.

We are struck by the fact that the mast, which passes through the mid-point of the scene, shows such a strong backward slope in relation to the axis of the handles. This makes for a feeling of tension which requires relief and explanation, and it also gives us a hint as how to approach the analysis.

It appears that the boat, with its strictly limited dimensions, depends on the dimensions of the picture surface. This is a fundamental feature with which we are familiar from the Egyptian flat pictures. In PLATE XV A1, we have constructed the vertical diameter, VH, of the circular vase, orientated by the mast. Further, we show the inscribed square of the circle centre M, and the inscribed circle of this square, whose vertical diameter is OP. Lines are drawn from O and P to the opposite corners of the square (Φ hypotenuses), and the points of intersection between these and the M horizontal are termed S and R. DD' is set off equal to VH, the vertical diameter of the vase, and the Φ hypotenuses DH and D'H are constructed. The points of intersection between these hypotenuses and the M horizontal are x and y. MH is divided in Φ, and the arcs struck from x and y through M cut the hypotenuses at a and b. A line drawn through these points *determines the height of the mast at B.*

Moreover, VG is equal to the horizontal distance from R to T, which lies on the edge of the vase, and 2 VG is equal to the length of the boat, measured at right angles to the mast. An arc struck from B, the top of the mast, and passing through G describes the lower edge of the boat from G to the rudder (note the unmotivated stroke which follows the arc to the right of the rudder). *An arc about V, with the length of the boat as radius, describes the upper curve of the sail,* and a Φ division of OP further shows that the distance between the points of intersection of the Φ arc from P, and the curve of the sail, is equal to the length of the boat. Thus the relationship between the measurements of the boat and the geometry of the picture surface should be clear.

But the boat is moving, the artist has depicted it with full-spread sails. If it had been a sculpture, we should now have been able to draw the rectangular block from which it had been car-



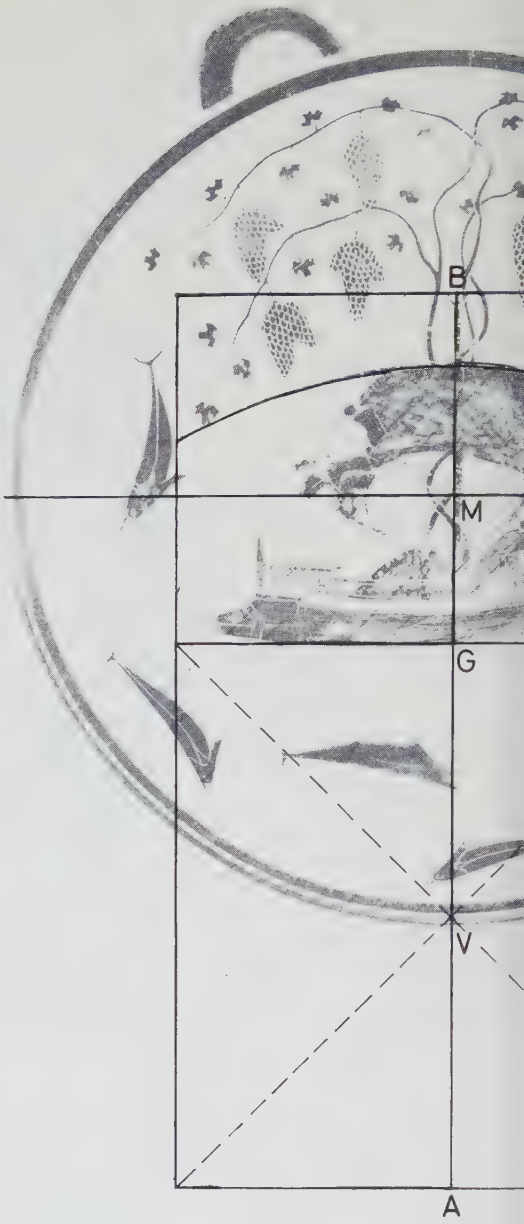
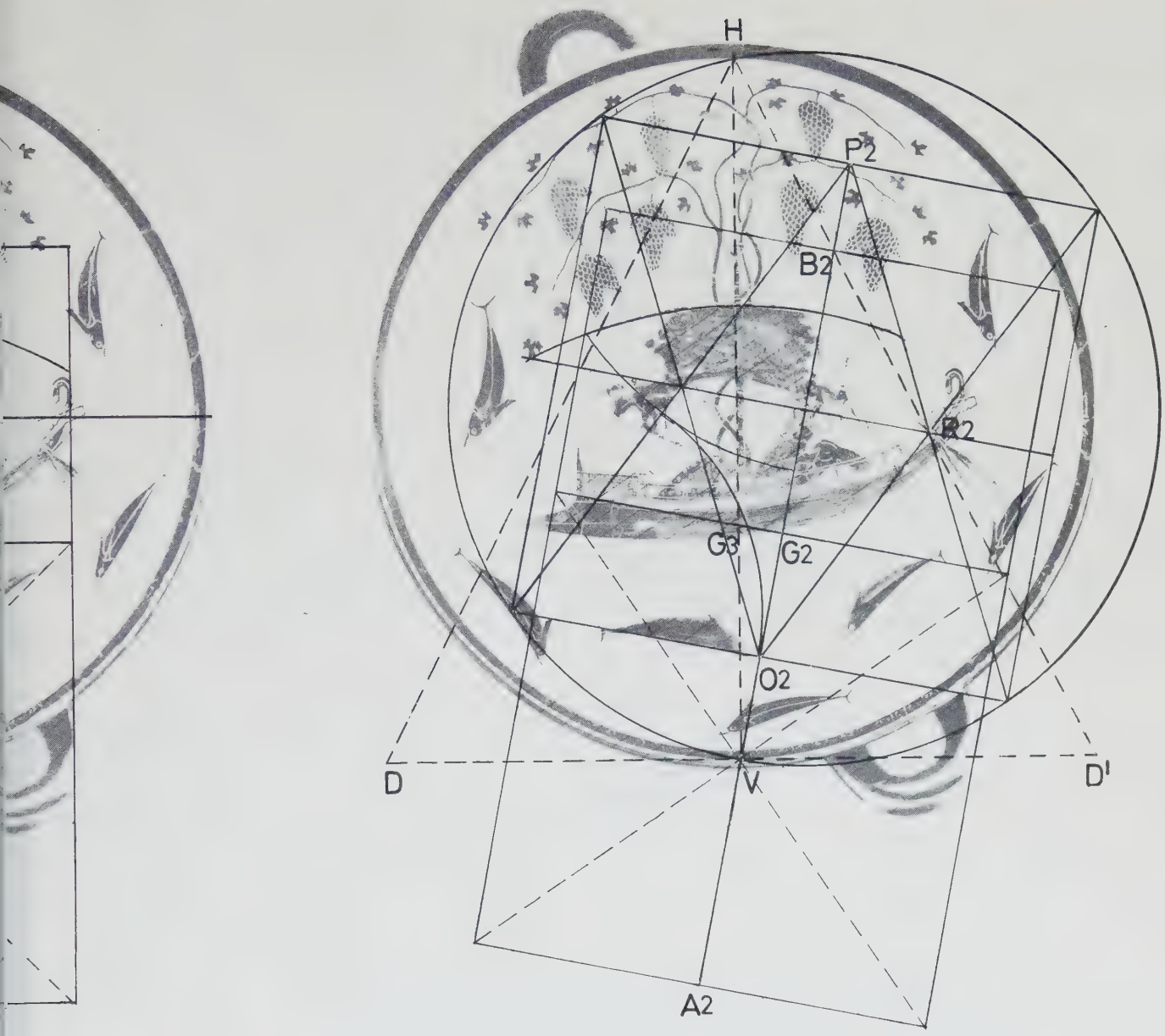


PLATE XV A1, A2, B, C
Dionysos Sailing on the Ocean

ved or hewn, for we know two of the perpendicular dimensions, its length and its height. If we view this imaginary block from the front, looking at the prow of the boat, the depth of the block equals the length of the boat and its height is equal to the height of the boat, up to the top of the mast. This is shown on PLATE XV A2, where the length of the boat has been set off from G to A. As the height is equal to GB, we have now found the basic dimension AB.

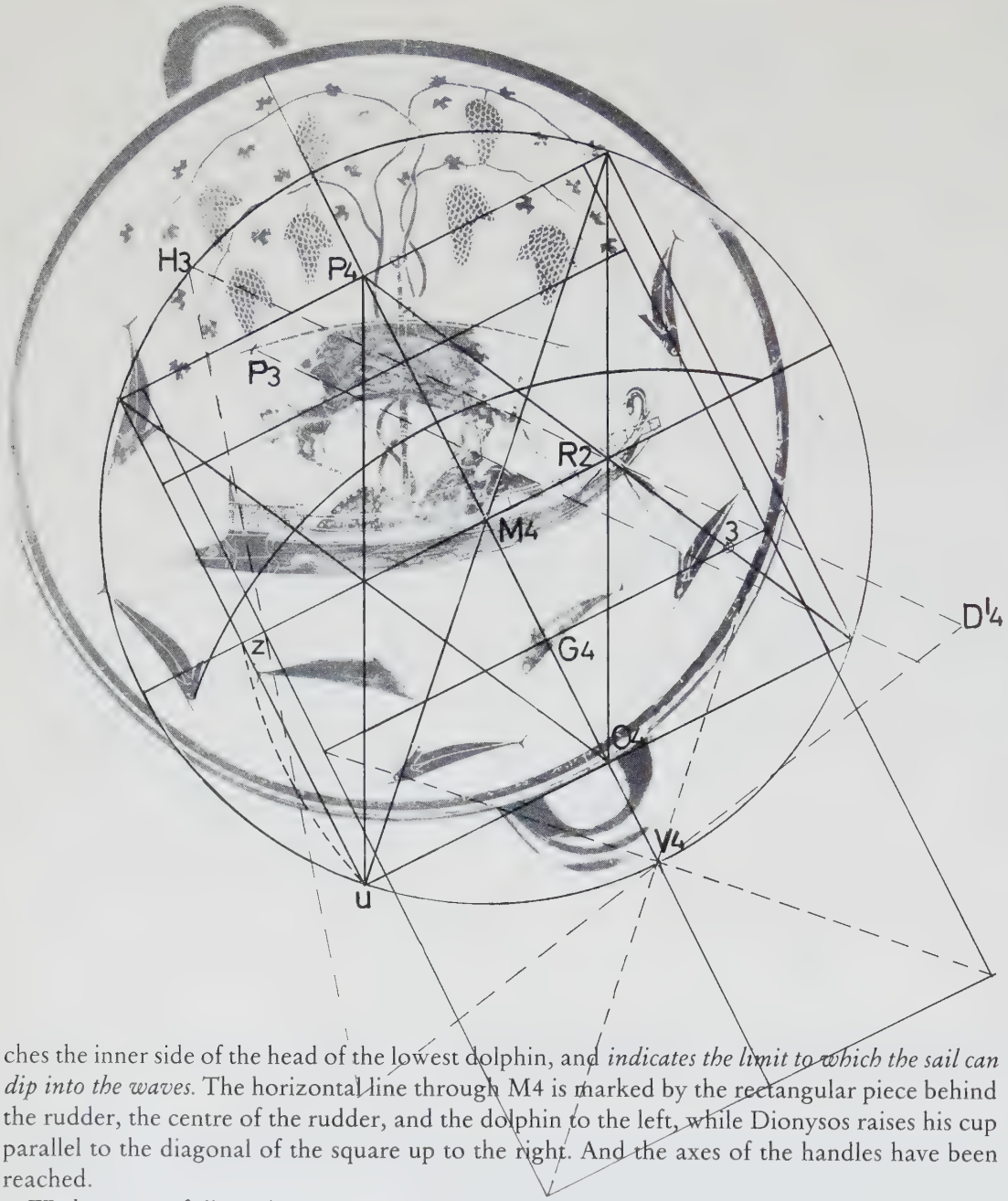
We already know that V lies on the edge of the vase, and we see that it moreover lies at the centre of the square bisected by AG. The fact that V is the centre of AG means that, according to the Egyptian way of thinking, it must be regarded as *the horizontal projection of the mast.*

Now we shall trace the different stages of the boat's movement from the start when its mast was parallel to the axis of the handles, and to the place shown. This means that we have to go backwards and thus find the way Dionysos *has* sailed. PLATE XV B is based on a pivoting



movenment of the geometrical pattern about V. It stops when point R (PLATE XV A1) lies on HD', and thus the centre of the rudder is determined (R2). The "horizontal" direction of this position is clearly marked by the lower edge of the dolphin on the O2 horizontal just below the boat. The "horizontal" centre line is clearly indicated by R2, and passing through the head of the god on the left, it cuts the curve of the sail below the lowest leaf of the vine: this sets a limit to the pivoting movement of the boat.

The geometrical lines on PLATE XV B pivot about R2, the centre of the rudder, until the *left* Φ hypotenuse from P4 is parallel with the mast as determined on PLATE XVC. The rear of the two-piece rudder clearly marks the *right* Φ hypotenuse from P4, while the left Φ hypotenuse from H3 aligns with the inner edge of the point of the prow. The curve of the sail tou-



ches the inner side of the head of the lowest dolphin, and *indicates the limit to which the sail can dip into the waves.* The horizontal line through M4 is marked by the rectangular piece behind the rudder, the centre of the rudder, and the dolphin to the left, while Dionysos raises his cup parallel to the diagonal of the square up to the right. And the axes of the handles have been reached.

We have now followed Dionysos' journey through three stages, so to speak backwards, to where it started. This way of representing movement is based on Egyptian tradition and, as we have seen, the Greeks mastered this method already during the Archaic period. The motif is simple enough – how many of us have not drawn a man sailing a boat in our childhood? And do not children strew some fish or a few waves over the picture? And yet – what a difference! Dionysos' journey is eternal, because the scene incorporates a new dynamic element, which assigns a function to each part of the picture and which determines a "unity of time, space and action". This is the main difference between Exekias' Dionysos and simple primitivism.

Fig. 34, 35
Lakonian kylix. 'King Arkesilas. Paris, Bibliothèque Nat. 47, C.2866. VIth century.

PLATE XVI A1, A2, B, C, D, E, F, G (Bibliothèque Nationale, Paris. 47 C.2866). The representation shows King Arkesilas of Cyrene (560–555 BC) controlling the weight of silphion. *Figs. 34, 35.* This is also a scene of movement. Weights are being added to one of the scales, silphion to the other; the balance moves as the action, directed by the King, progresses. There is an effect of space unfolding around and above the balance well known to us from Egypt, the birds seem to fly in from the very firmament. Such expressions of space are of a purely artistic nature, and they could hardly be created by a representative of any primitive phase of art (see PLATE XV in Geometry in Egyptian Art). True, from a naturalistic point of view the figures appear primitive, but the way in which they are placed, their dimensions as seen in relation to the total scene, the drawing, the variation between the figure and the background, the extent of the intervening spaces, the play of the lines – by virtue of all these, the composition assumes the form of a profoundly pregnant entity. We have every reason for investigating whether the methods here employed are the same as those used by the Egyptians – the methods we were able to trace in the scene of Dionysos on the Ocean.

The positions of the men and their movements make it clear that all interest is centred on the King and on the turning of the scales. Inscriptions between the figures explain their tasks: on the left "he who guards the cargo", the next is a single word "balance", while the third has not been interpreted with any certainty, but it may mean something like "crusher of silphion", one who kneads or presses the silphion plant (whose sap was used for perfume, medicine and spice). The inscription in front of the figure on the right has been read as "stop" or "stand firmly" or "shall I empty?". Another reading suggests that it is the name of one of the men on the right. Bottom left we read "overseer", while the word to the right of it has so far defied interpretation.

While the picture surface of the scene of Dionysos Sailing on the Ocean represented the ocean as seen from above, so that it formed a background for the boat, the picture surface in this case is a vertical projection: the deck of the ship, the scene of action, is shown in section and represented by a horizontal line, and the figures are outlined against a vertical background. We shall, however, see that the dimensions of the figures and the space at their disposal – as seen from above – have been determined according to the normal Egyptian method.

The investigation is based on Ernst Pfuhl's measured drawing, published in *Malerei und Zeichnung der Griechen* (München 1923); this seemed preferable to a photograph, because of the pronounced curvature of the vase. The photograph, however, shows that the axis of the handles is parallel with the base line, the section of the deck of the ship *(Fig. 36).* Thus the turning of the scales, the principal motive of the composition, is accentuated. The King's height, seated, we shall call GB (see PLATE XVI A1). This line passes through the rivet where the legs of the

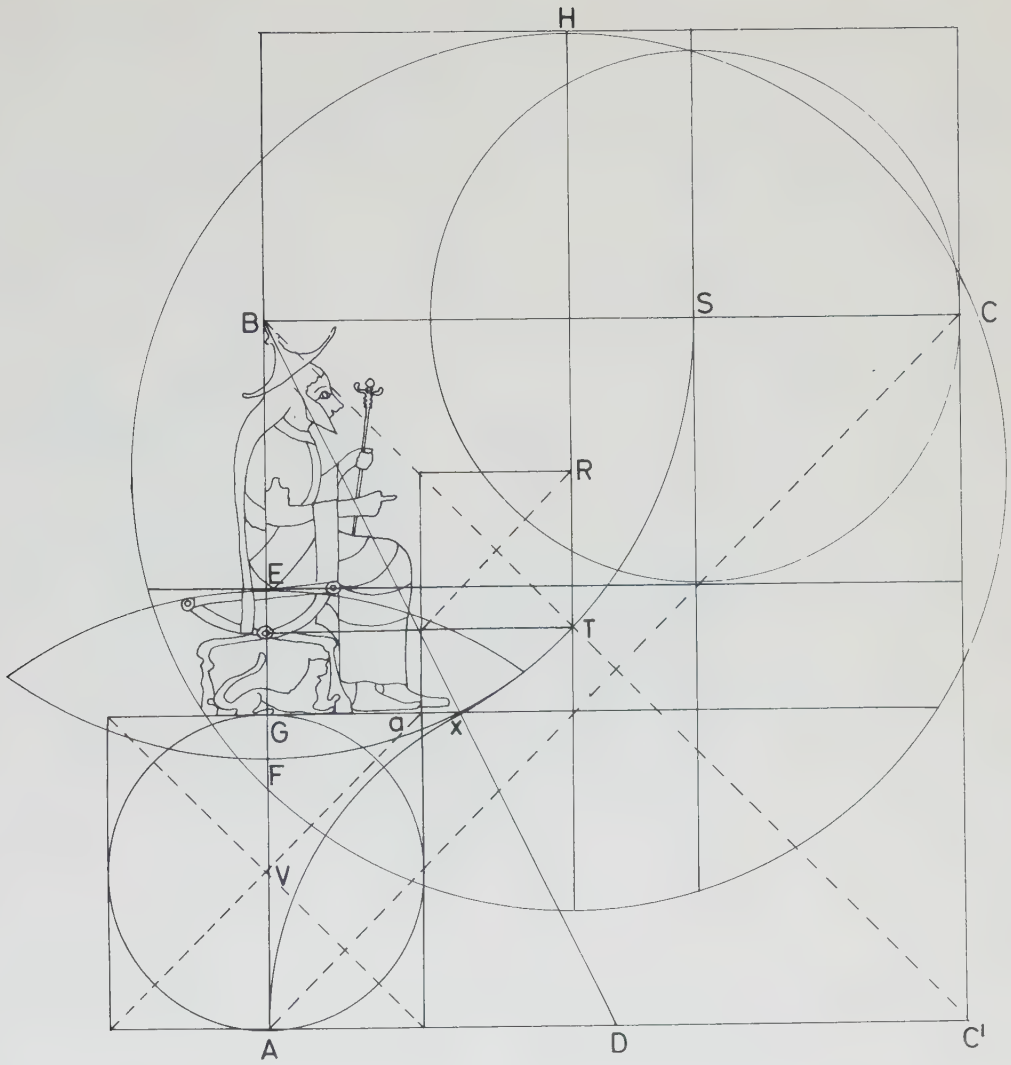
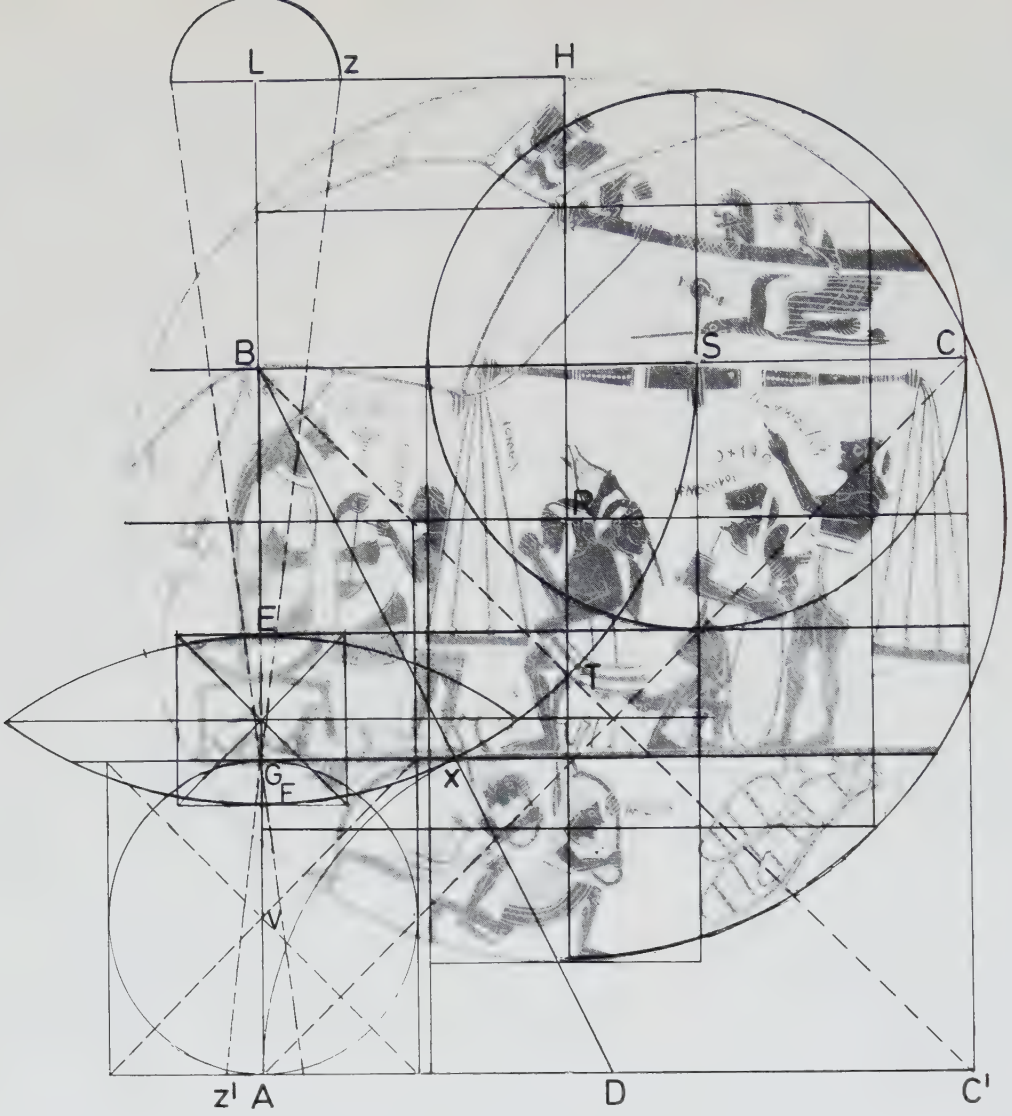


PLATE XVI A1, A2, B, C, D, E, F, G.
'King Arkesilas of Cyrene'. Round picture from a kylix, Bibliotèque Nationale, Paris. 47 C. 2866. Analysis based on measured drawing in Ernst Pfuhl's Malerei und Zeichnung der Griechen. Munich 1923.

folding stool cross, and we may think of him seated and at rest, about GB, before he lifts his sceptre obliquely forward in his left hand, while raising his right arm. The distance from this vertical axis to the tip of his toe, we may, in the Egyptian manner, regard as "half his width". By this we mean that a circle around V (the horizontal projection of the vertical axis), its radius equal to this distance, marks the space at his disposal, seen from above. The circumscribed square of this circle is drawn, and we have now determined point A. Following the reasoning of the preceding chapter, we can call AB the basic dimension, equal to the sum of the height and the depth of the block. The only difference is that we are here working with the *side* of the hypothetical block.

AB is now divided in Φ at E (above) and at F (below), AD = ½AB. An arc from D through A determines x on the Φ hypotenuse. A number of points are now given:



1) The x horizontal determines the base line of the King (at G), and the square bisected by AG is determined.
2) The E horizontal gives the height of the folding stool.
3) The arc about B through F determines S (on the B horizontal), and the circle centre S, radius BE, which is the minor of AB, marks the greatest width of the balance.
4) The diagonal BC' of square ABCC' is cut by the arc centre B through F at T, and the level of the point of intersection of the legs of the stool is determined. TR is set off equal to the horizontal distance from T to BD. *R is the centre of the circular field, and its radius is equal to the major of AB* (*BF*).

When we transfer the construction from PLATE A1 to PLATE A2, we note that the diameter of the circle about S is equal to the distance between the ends of the balance; we therefore term it the "circle of the balance". This circle marks the horizontal projection of the space at the disposal of the balance, in keeping with the Egyptian way of thinking. Moreover, the length of the sceptre is equal to the radius of the circle, centre V, and thus this circle marks the space at the disposal of the King in this way also.

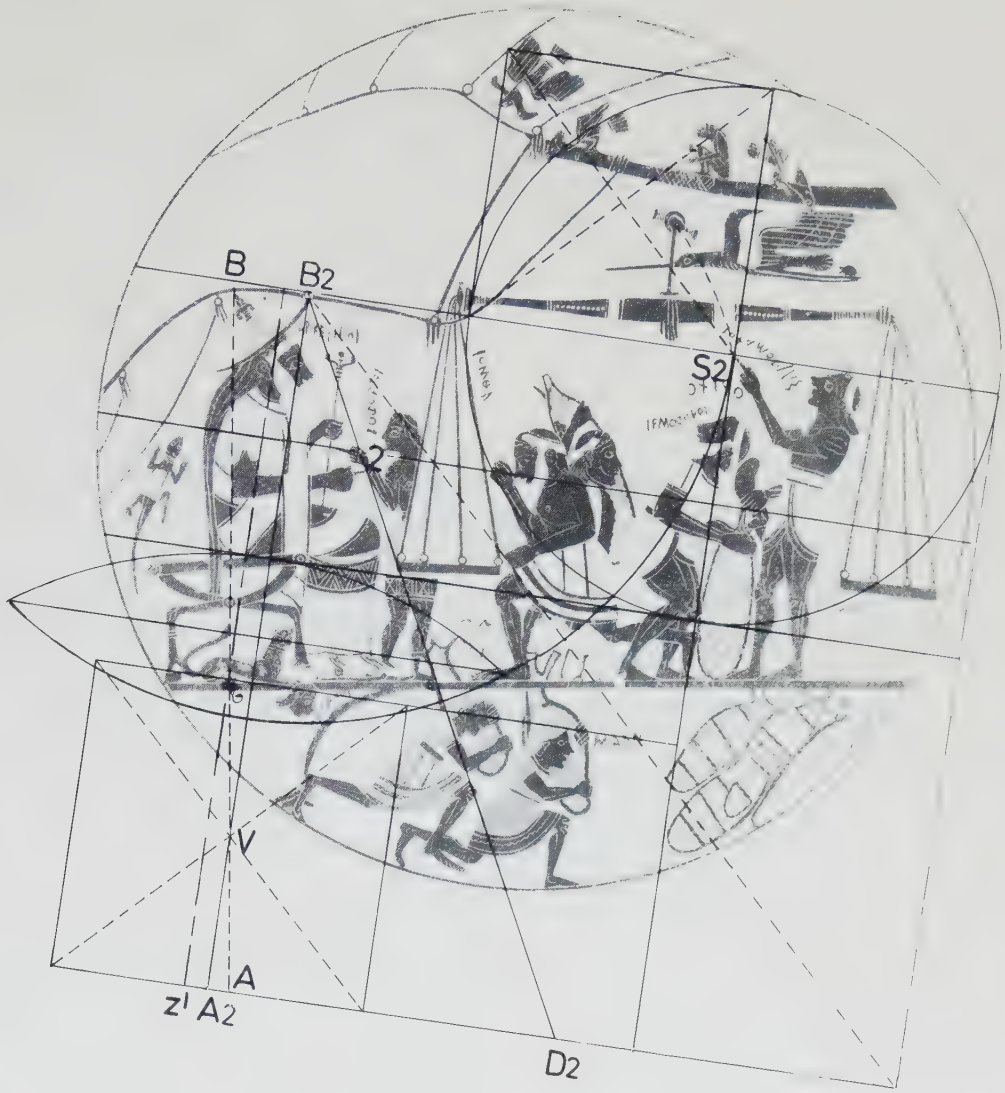
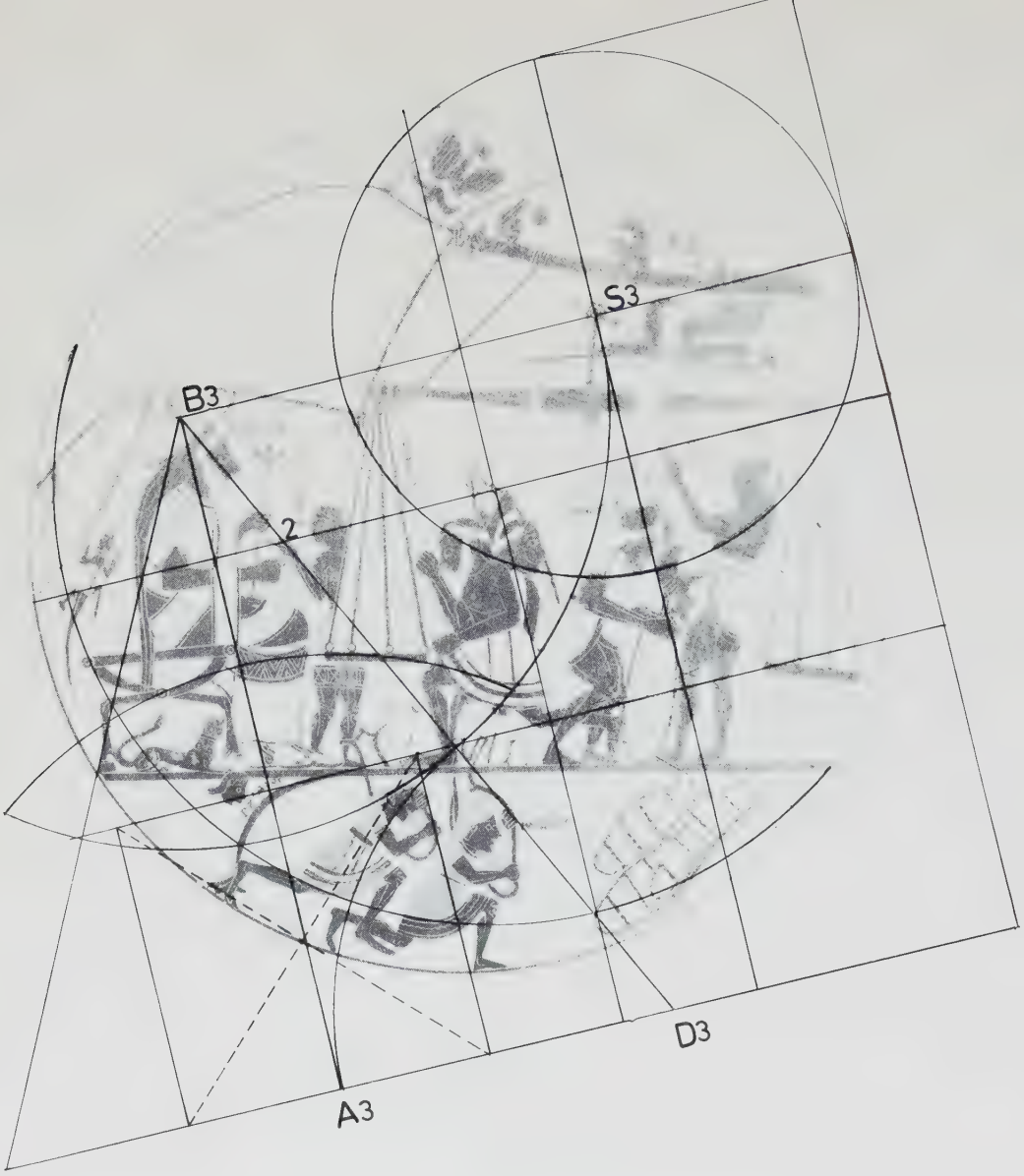


PLATE XVI A2 shows oblique axes through G and going up to the H horizontal, indicated by a dotted line. The axes are tangential to the circle centre L, diameter equal to EF, and thus their angle is determined. EF presumably expresses the width of the King (the Egyptian way of thinking), seen from the front. In the square bisected by EF we see the King represented by a panther, his symbol, and the overseer guarding the cargo below deck.

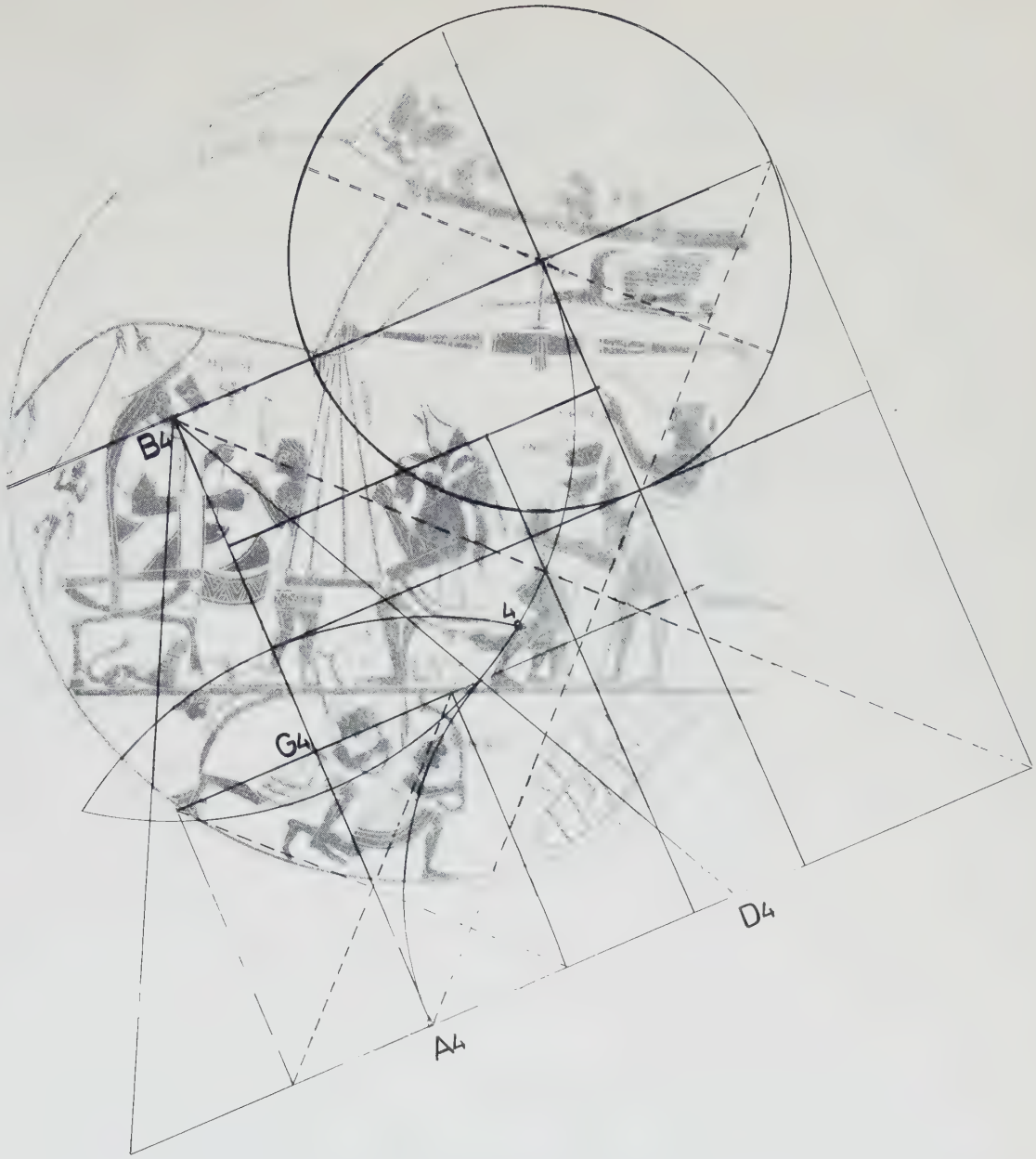
The geometrical lines now swing to the right about V (the horizontal projection of the king's vertical axis), until AB is parallel with the axis which slopes to the right. All the other geometrical lines follow suit, and we arrive at PLATE XVI B. On this we see that the sceptre is "vertical" in the new positon of the geometrical lines in what we may term the B system. Moreover, the B2 horizontal is tangential to the first finger of the man farthest to the right, as well as to his forehead. The bag of silphion in front of him is "vertical", as are also the white stripes on the large bird above the balance. The centre of the "circle of the balance" lies at the tip of the first finger of the man on the right, while the man in front of the king has his pointing



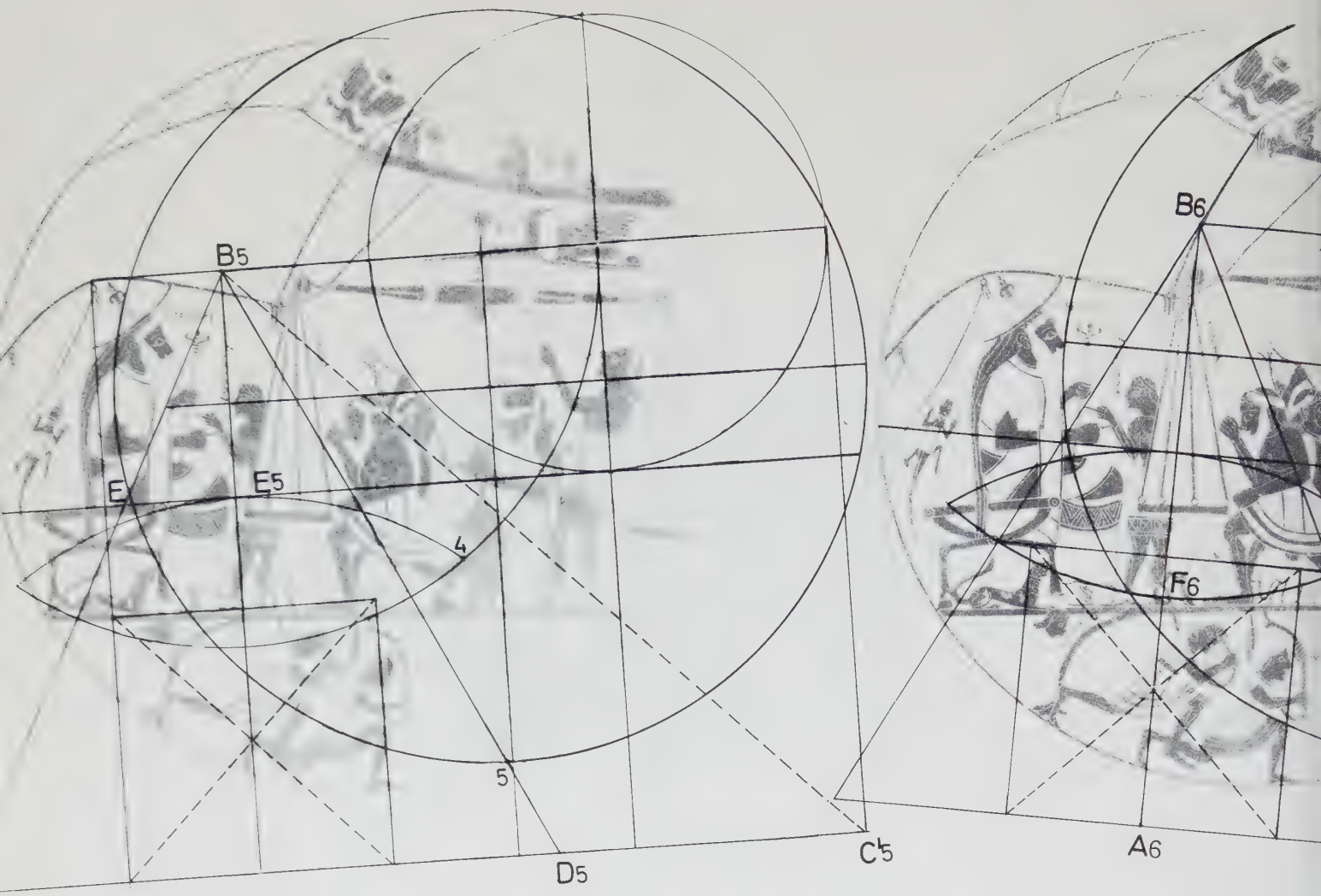
finger in the horizontal diameter of the picture. These points give a clear indication of the B position of the geometrical lines.

The further pivoting movements are of an entirely different nature. They are governed by the turning of the balance as weights and silphion are added. Now the principal figure is the man closest to the King, "he who guards the cargo", for the King has given his orders and lifted his sceptre. The geometrical lines turn about point 2 within the hand of this man. This point is geometrically determined as the point of intersection between the "horizontal" diameter of the circular field as shown on PLATE XVI B and B2D2. The geometrical lines of PLATE B pivot upwards, in an anti-clockwise direction. Now the "centre of the balance" lies at the centre of the ring with the cross peg, immediately above the bird.

PLATE XVI C shows a number of significant points marking the geometrical lines. The King and the man closest to him are clearly dependent on B3D3. This line can be traced below

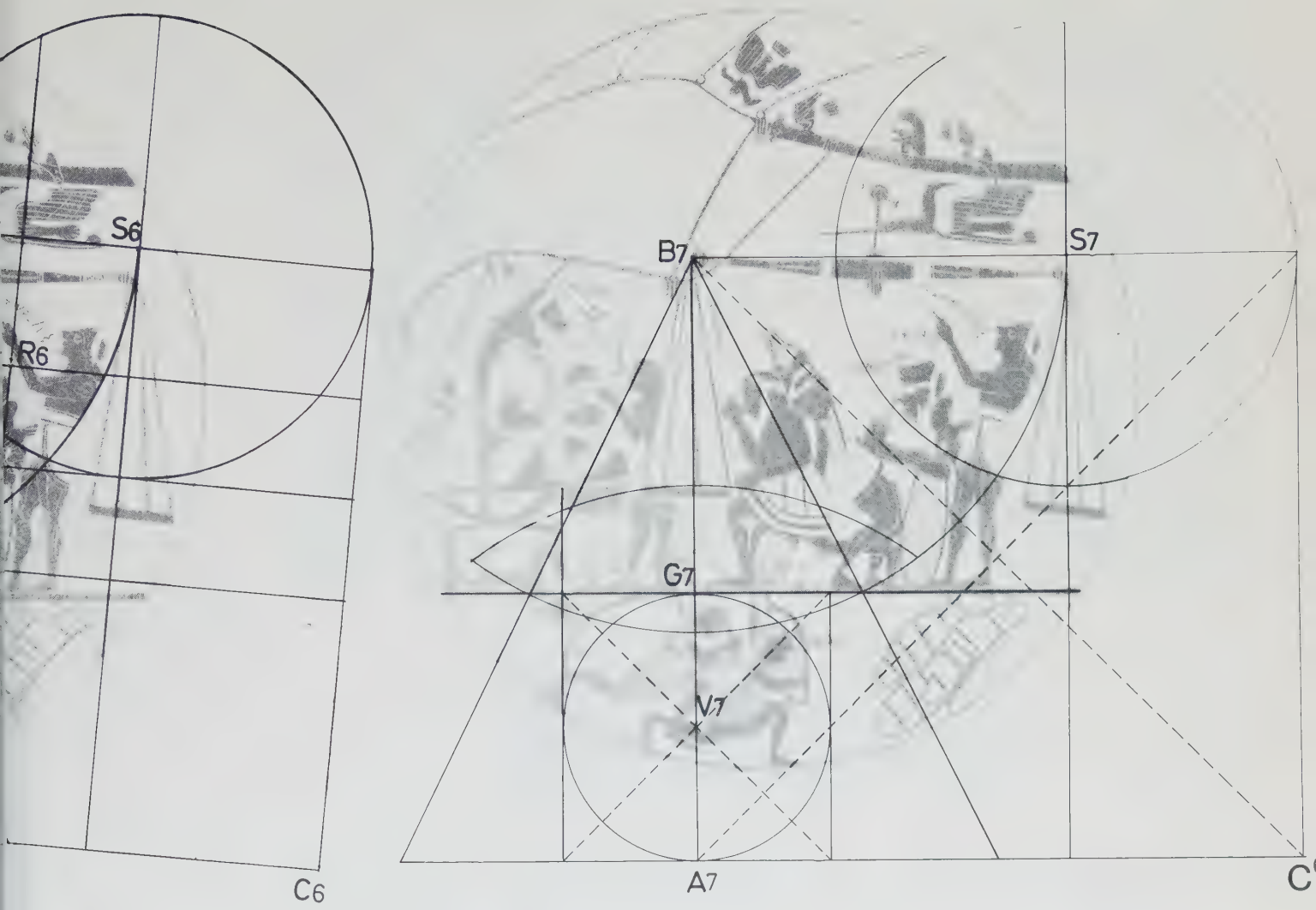


deck, where the first of the men carrying sacks touches it. The Φ arcs from B3 and A3 are also important. It seems as though the great arc about B4 may have inspired the way in which the man with bent knees, who turns to look up at the balance, is depicted. The geometrical lines continue in their anti-clockwise movement, now about S3, the centre of the "circle of the balance", until the left upper corner of the square bisected by A4G4 lies on the circumference of the circular field – at the feet of the overseer, PLATE XVI D. Here the pivoting action stops. The man farthest on the right points with "vertical" lower arm and first finger up at the balance and says «stop», and the peg of the ring of the balance is in a diagonal position. This is the maximum extent of the movement, an extremely important stage. The gesticulation between the King and the man closest to him also indicates its importance; moreover, the three men in the hold below deck have now clearly attained contact with the horizontal projection of the King. The right scale, surely heavily laden with silphion, is hanging far down.



Now the geometrical lines swing back, PLATE XVI E, around the right point of intersection (4) between the arcs about A4 and B4 until the E5 horizontal goes through E (on PLATE A1). The bird with the long beak pointing towards B5 marks this position, and also the top edge of the stool. The geometrical pattern has moved far over to the right, so that the face of the man farthest on the right aligns with the vertical centre line of the balance. In this position, the two sacks on deck are tangential to the "circle of the balance". Small weights are added to the left scale, and the balance is adjusted in two more positions. First the pattern pivots around point 5 on PLATE XVI E, the point of intersection between the perimeter of the circular field and B5D5. The geometrical lines swing to the right so that the "vertical" lines are parallel with the sceptre, PLATE XVI F B6S6 is shown by the big bird, while the verticals through B6 and S6 cut the ends of the balance arm. The Φ arc from B6 is strongly marked near F6, and we specially note the standing man to the right. The man in front of him bends down, so that the R6 horizontal is tangential to the top of his head.

Now the horizontal projection of the King has attained complete contact with the men below deck. We must note the upward extension of the left Φ hypotenuse from B6, and the circle



of the picture surface immediately to the left of it, tangential to the king. Both these lines are of importance for the cordage by the birds at the top of the scene.

The geometrical lines pivot downwards about C6, PLATE XVI G, until the G7 horizontal lies in the base line of the original position. The left scale pan now lies on A7B7, the vertical axis of the King, determined by the upper Φ division of A7B7, while the square centre V7, which marked the horizontal projection of the King, now encloses the face of the overseer and the two silphion bearers. The inscribed circle of the square, with radius equal to the length of the sceptre, explains the otherwise inexplicable curve behind the figures. The King is no longer in the scene – everything is left to his servants. The upper edge of the arm of the balance coincides with B7S7 for half its length. Towards the right it turns down, and the right scale pan hangs a little farther down than the left. The King has given good weight.

These two analyses of the work of Archaic painters should make it abundantly clear why the Greek vase painters were highly esteemed artists, and why the names of many of them have come down to us.

Fig. 36 a, b, c
Reliefs on the sides of the base Pl I.

PLATE XVII A1, A2, B, C, D, E. On PLATE IV we showed how the reliefs on the sides of this statue base were dependent on the proportioning pattern of the base. After our two last analyses it seems natural to investigate if the composition of similar reliefs also were dependent on *movement* of the geometrical pattern of the base. We therefore return to PLATE I, *Fig. 36.* Photographs of the reliefs have been mounted on the sides of the base making it possible to study whether the movement of the figures, the action of the scenes, is determined by pivoting movements of the geometrical pattern of the base.

On PLATE XVII A1 we show the pattern in the original position, and also in its first swing to the left (dotted). As usual, the pivot, point 2 (bottom right), lies in the *horizontal projection*, for V is the horizontal projection of AB. The extent of the movement is determined by V2B2 being tangential to the circle about Z, the left lower corner of the top of base, its radius equal to the height of the block. These two positions together are of decisive importance for the design of the front relief – this should appear from the plate. A Φ division of AG (the height of the front relief) is shown and the importance of the Φ arcs from A and G is very clear, as well as the circle about N (GN = NB) circumscribing the top of the base, and also the circles about z and u, tangential to the A horizontal.

Surely the entire geometrical pattern in both its positions has left its traces also on the side reliefs. Unfortunately the story told by the scenes on these reliefs is not as familiar as that of the

vase painting investigated above. This makes for much greater difficulty in determining the pivoting movements. However, we decided to risk an investigation because it seems that the clear marks in the right relief, first and foremost shown by the men's staves, represent traces of the geometric pattern in various positions. The position of the staves is by no means accidental — it is obvious that the men are marking some important feature by this means. On this theory, for which we have found support in a number of analyses of Greek two-dimensional art, we shall attempt to explain the reliefs on this base.

It appears that on the right relief, a horizontal line through x on BD' (determining O) marks the vertical centre line of the chair on which the man is sitting, left on the relief. Like King Arkesilas, he has placed his staff (the sceptre) in front of him, and we shall investigate whether this is due to a pivoting movement in the horizontal projection. On PLATE XVII B, the same

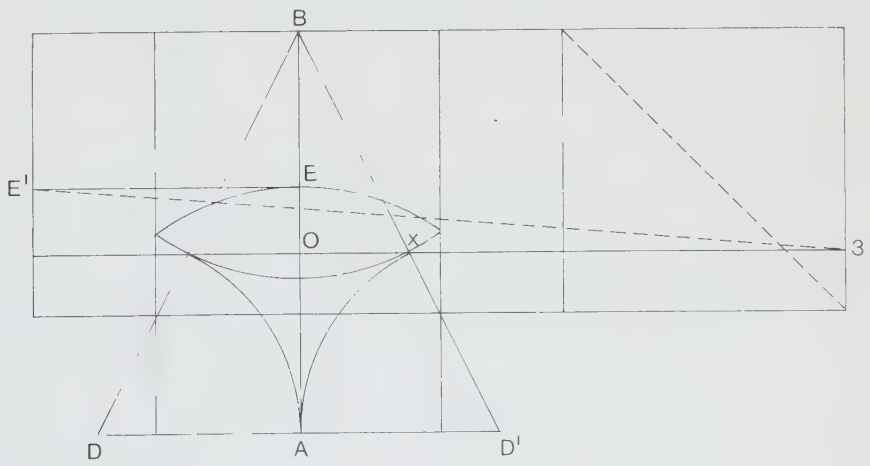


PLATE XVII A2

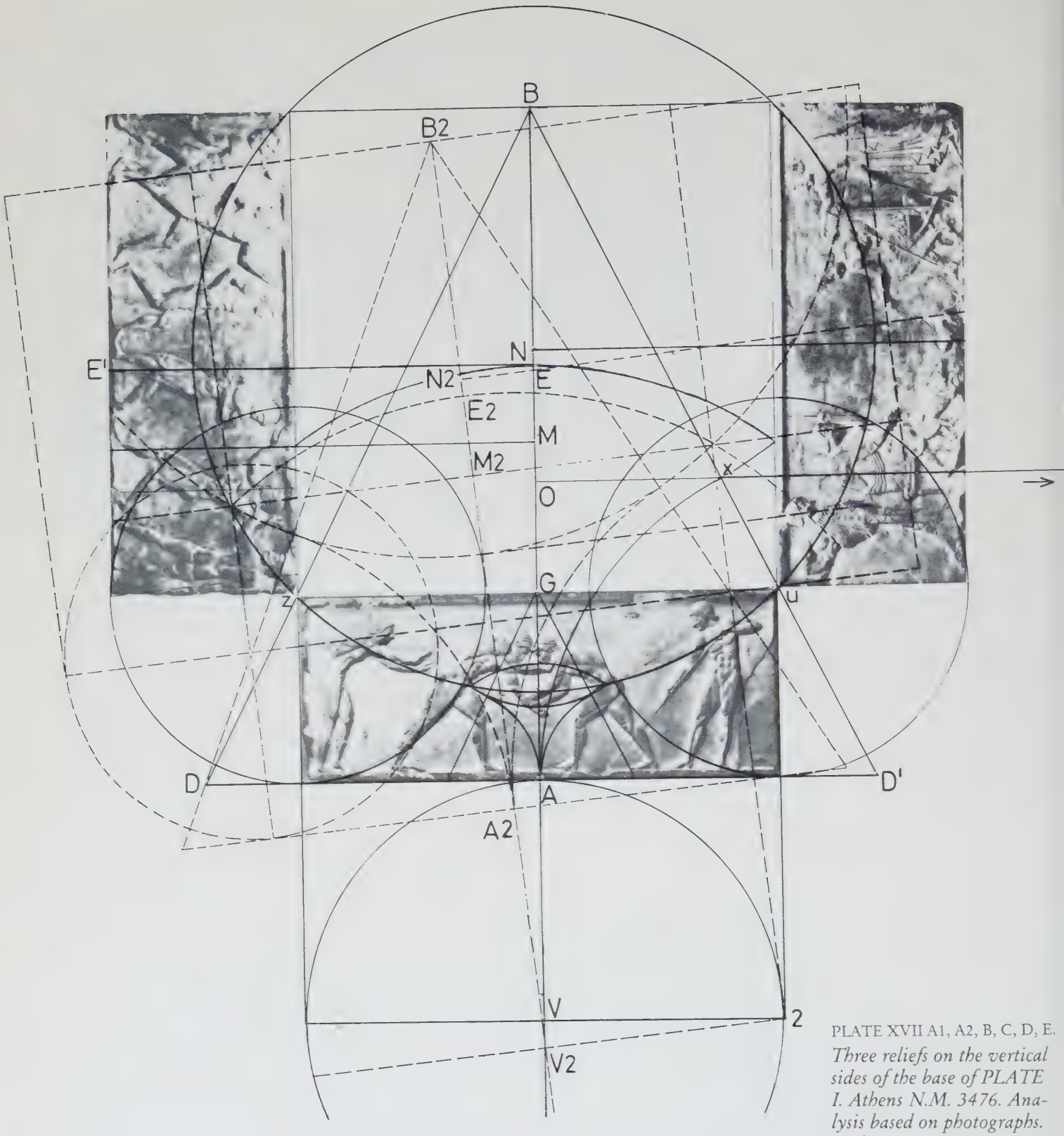
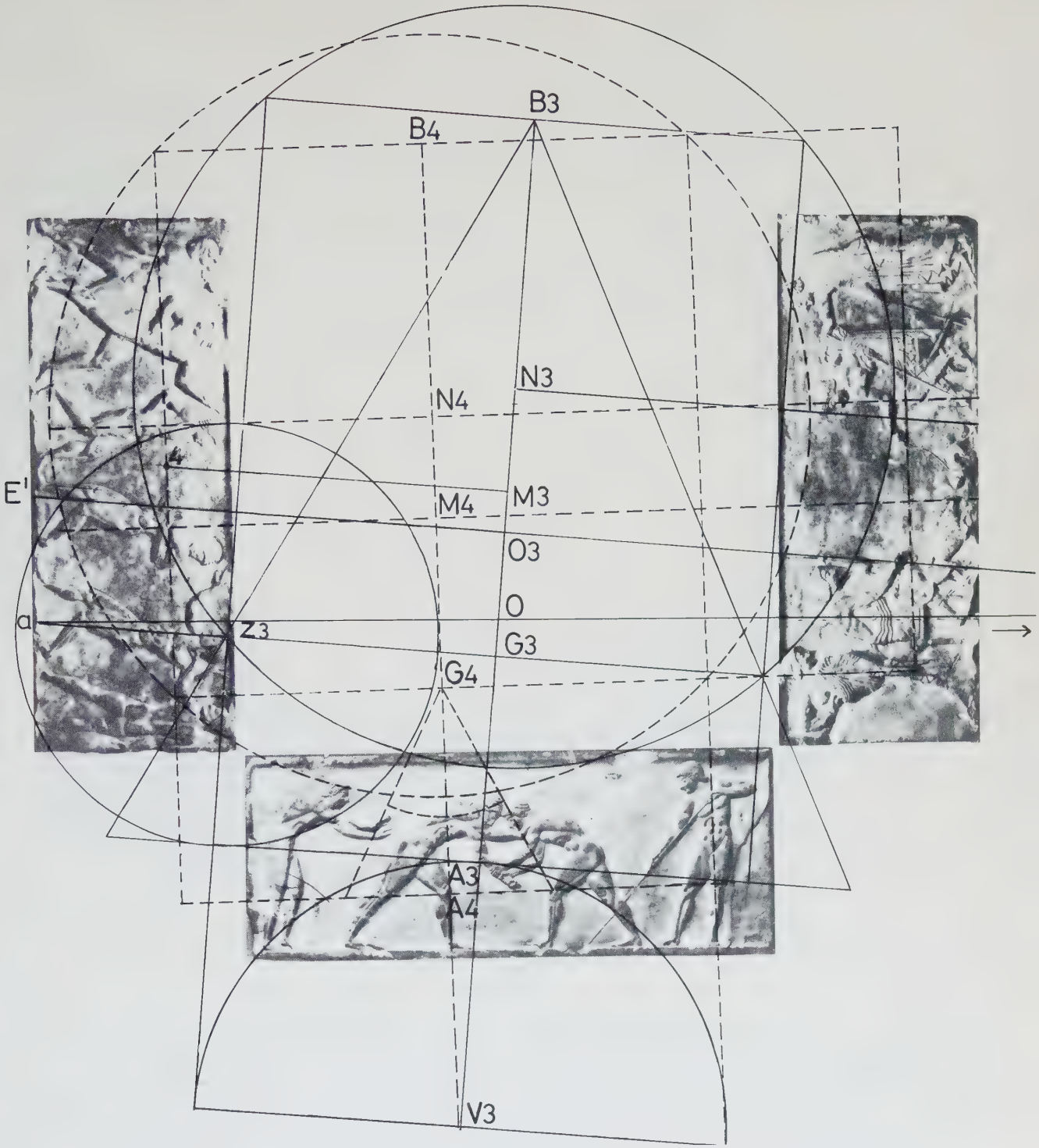
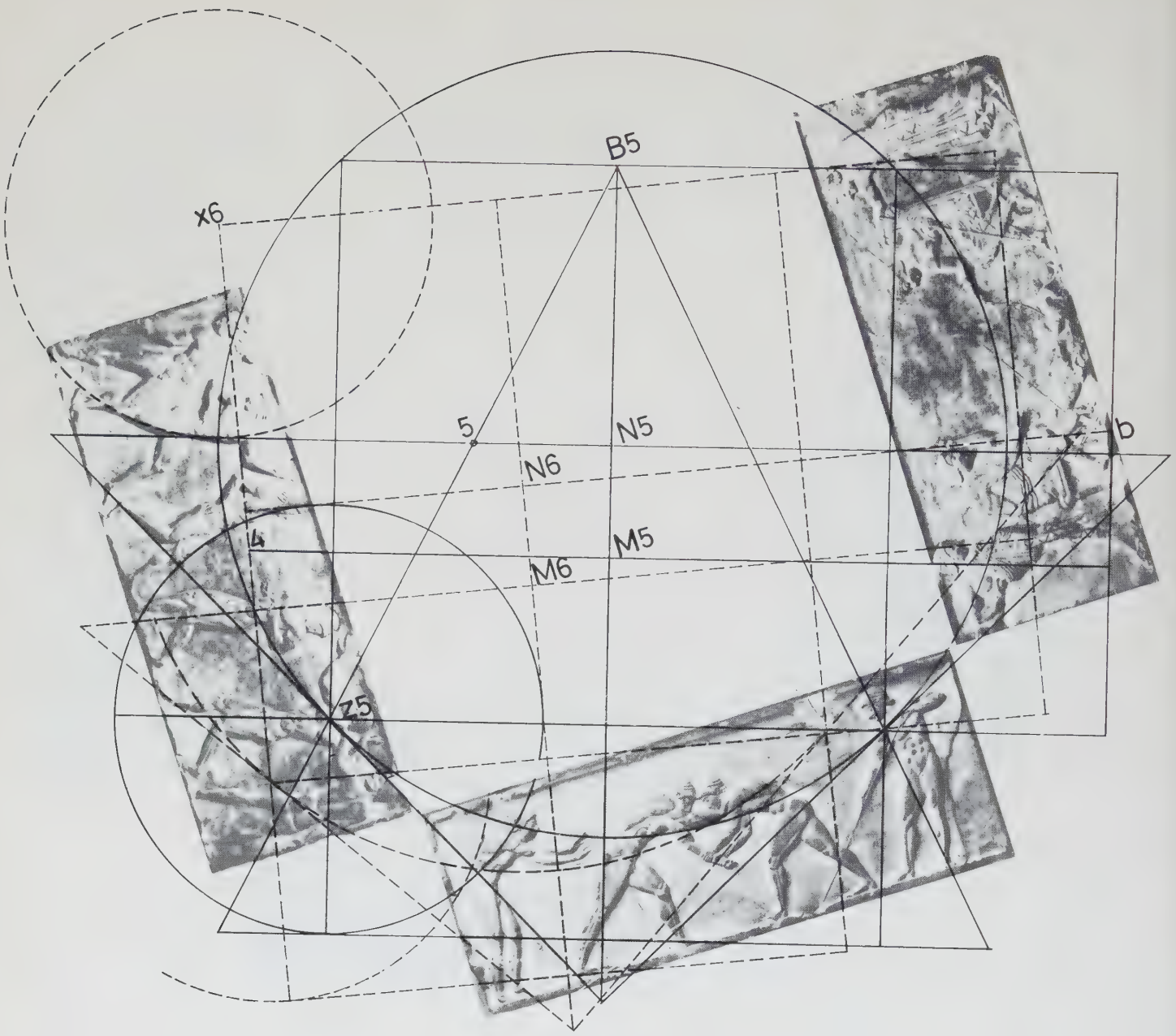
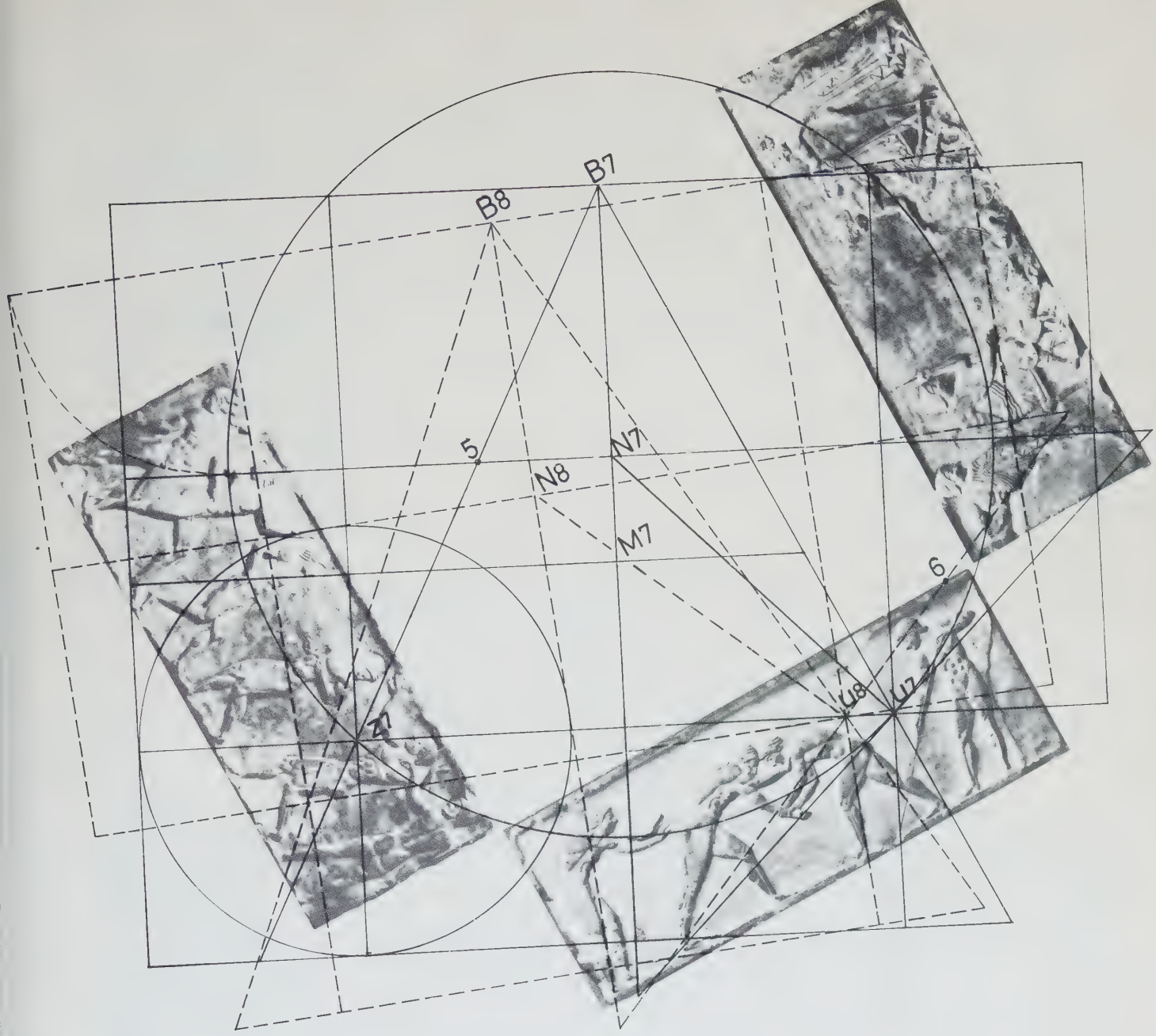


PLATE XVII A1, A2, B, C, D, E.
Three reliefs on the vertical sides of the base of PLATE I. Athens N.M. 3476. Analysis based on photographs. Scale 1:5.


B3
B4
N3
N4
E'
M4
M3
O3
O
a
Z3
G3
G4
A3
A4
V3

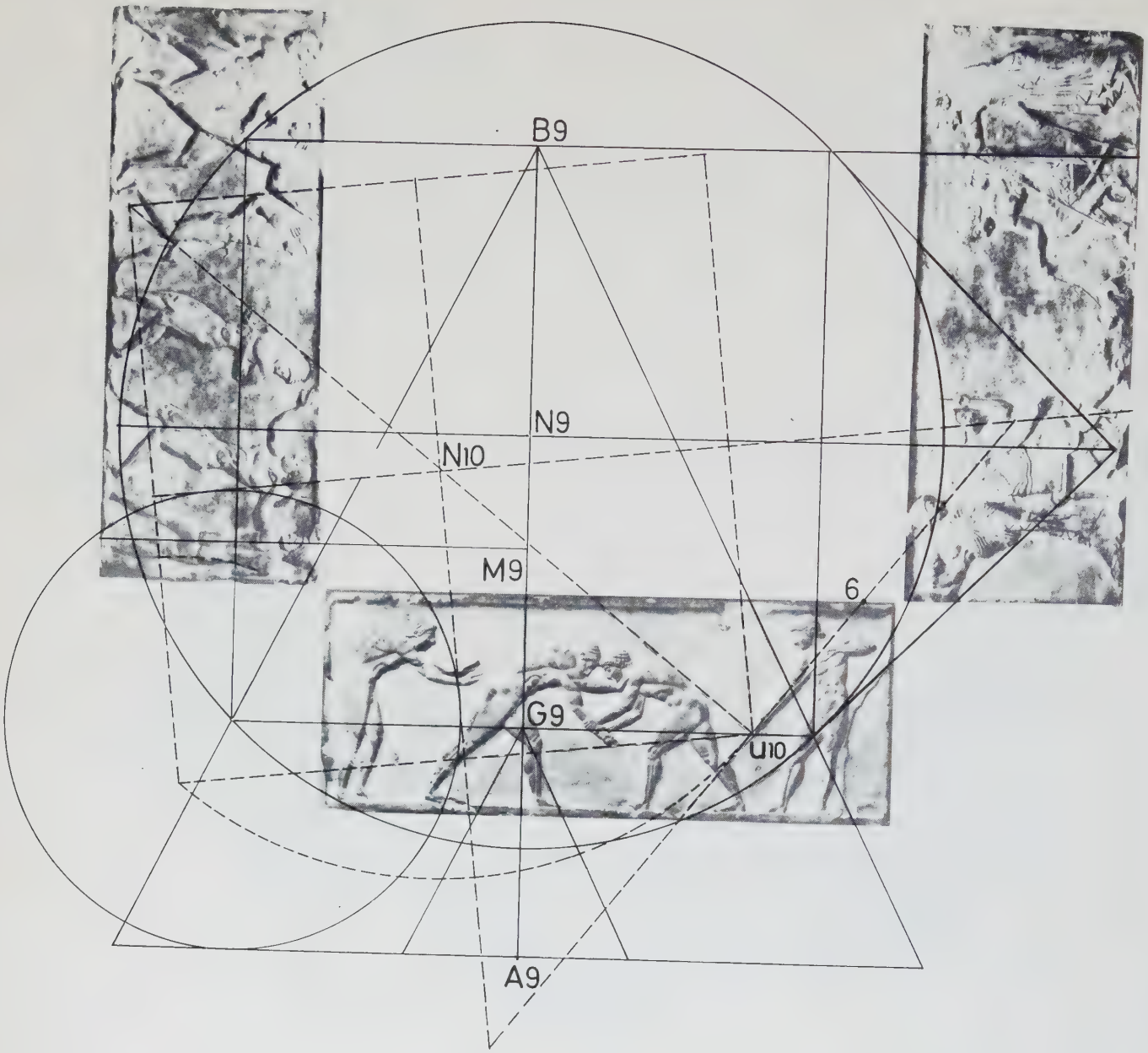


line has been produced until it cuts the lower side of the square on the base line of the right relief in point 3 (outside the plate, on PLATE A 2). The geometrical lines pivot to the right about point 3, until the O "horizontal" cuts the base line of the left relief (at E′ on PLATE A1) and the staff of the sitting man is determined. The large circle about N3 touches the head of the extreme left figure of the right relief, whose drapery depends on the dotted G4 horizontal (vertical here). This circle may be traced along the shoulder and upper arm of the sitting man with staff, and along the lower arm of the sitting man facing him. On the front relief, the geometrical pattern has moved up in such a way that the wrestlers are in contact with the base line of the relief in both positions, and the circle about Z – which enclosed the left thigh of the left wrestler on PLATE A – now encompasses the front of the body of the left second. The arc



about G4, its radius equal to the major of A4G4, reaches the lower edge of the great dorsal muscle of the left wrestler, and at the top this muscle touches the large circle about N4. The left relief shows the result of the geometrical lines having moved upwards: the dotted position of the geometrical lines shows the third figure from the left "at the centre of the scene", for the N4 "horizontal" passes centrally through this man; at the same time the extreme right figure is on the point of leaving the scene.

The next movement is around point 4, determined on PLATE B as the point of intersection between the left side of the top surface in its dotted position and the M3 horizontal. The extent of the movement is determined on PLATE C, as b, the point of intersection between the N6



"horizontal" and the base line of the right relief in position 5, lies on the base line of the right relief in its original position.

On PLATE C, a number of very clear marks are shown. First and foremost, we have *the staff on the extreme right in the right relief*, further the large circle about N5, which affects all three reliefs. On the front relief we note the lower edge of the left wrestler's left arm, and the tips of the fingers of the left hand of the man behind him. The front of this man's dorsal muscle is marked by the circle about z5, which at the other end reaches the front of the body of the athlete whom we, in connection with PLATE B, referred to as a "central figure" in the relief. Perhaps the most remarkable marking is the circle about the upper left corner of the top of the base, in dotted position. The radius of this circle about x6, like that of the circle about z, is

equal of the height of the base, and here *the front edge of the body and the thigh of the man with the extended lower arm are traced*, while his upper arm rests on the dotted line through x6. The geometrical lines (PLATE C) pivot about point 5, swinging farther to the right, until the right Φ hypotenuse from B7 is vertical. On PLATE XVII D we see that the seated man on the right of the right relief *has placed his staff so that it is aligned with the B8 "horizontal"*, while *the left second of the front relief stands at the centre of the scene*, extending his arms forwards, so that they touch the large circle about N7. On the left relief, the circle about z7 reaches the man with the upstretched arm, far back in the scene.

This is the limit of the right pivots, and balance needs to be restored. A tangent is constructed to the circle about N7 through u7 and a corresponding tangent through u8. This last dotted line intersects with the G horizontal at 6. On PLATE E the geometrical pattern swings back, pivoting about 6 until it is parallel with the position shown on PLATE A1. Now the staff advanced by the man on the right of the front relief is determined by the dotted tangent to the circle about N9, and his right hand reaches down to u10, the lower right corner of the dotted top of the base; the B9 horizontal is strongly marked in the right relief, where it appears clearly at the back of the seated man on the right. On the left relief, the man closest to the front relief marks the M9 "horizontal". We must also note the third man from the front on the left relief: his hands, chest and head touch the lower right diagonal through N10. This diagonal is tangential to the back of the right wrestler on the front relief. In this position of the geometrical pattern, the left wrestler crosses AB, and it would seem that he has the upper hand in the combat. However, tempting though it may be, we shall refrain from offering literary commentary. The part played by the dog and the cat we have not been able to discover. It is possible that their movement may be explained in terms of various positions of the horizontals through N and M? One thing, at least, seems clear: these three reliefs were planned on the same drawing as the base, on the basis of the method of geometrical pivots.

Fig. 37
Painting. 'Hector between two Scythians'. By Euthymides. München Staatl. Antiken Samml. 2308.

The experiments of Euthymedes and other masters

The Archaic artists mastered the method of geometrical pivots so completely that they were thereby inspired to a free and lively development of their art. This was of fundamental importance, for these men were possessed of a marked power of narrative, and of a need to create in art vivid scenes from all the legends and myths which formed part of their lives. Sooner or later they must find a new way of representing the human figure, which was then based on the Egyptian *profile–en face* style, incapable of meeting their requirements. Two vase-paintings by Euthymides, dating from c. 510 BC, clearly bear witness to this want, *Figs. 37, 38, 39.* Both show Hector at the centre, flanked by a figure on either side. The warrior, who is putting on his armour, is looking down thoughtfully. Both versions show him with one foot seen from the front, the other in profile; but while Hector puts his left foot forward in the one version, he advances his right in the other. It almost seems as though the artist were trying to find out which of these two positions makes for the more coherent figure, for both clearly have an oblique axis passing down through the right leg of the figure, traced by the folds of the chiton. We can hear Euthymides asking himself: which is better? Is the movement more coherent when the oblique axis passes through the frontal leg, or when it runs through a foot drawn in profile?

From our point of view it is regrettable that both these figures are painted on planes curving both ways, for this makes it difficult to control the course of the axes. It is clear, however, that both figures have two vertical axes in addition to the oblique axis, *Fig. 38.* We know that the

Fig. 38
Drawing showing Ethymides' experiment of combining the front view of Hector's right arm and his left leg with side views of his left arm and his right leg.

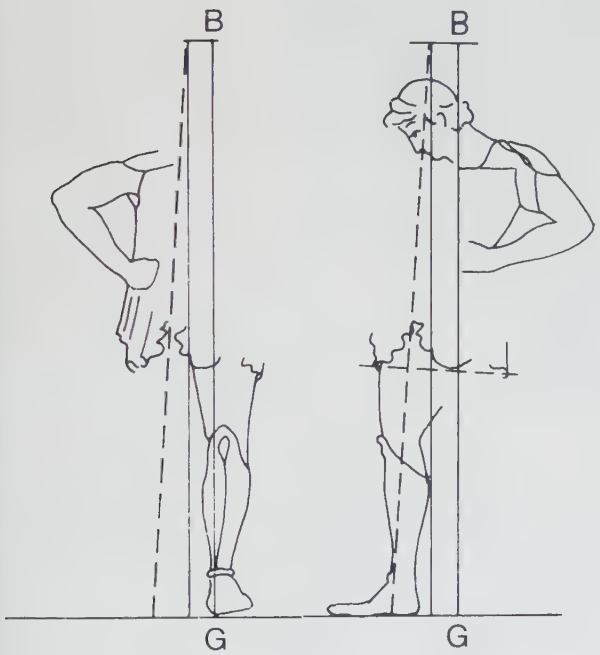


Fig. 39
Amphora by Euthymides. 'Hector putting on his Armour'. München Staatl. Antiken Samml. 2307. 510 B.C.

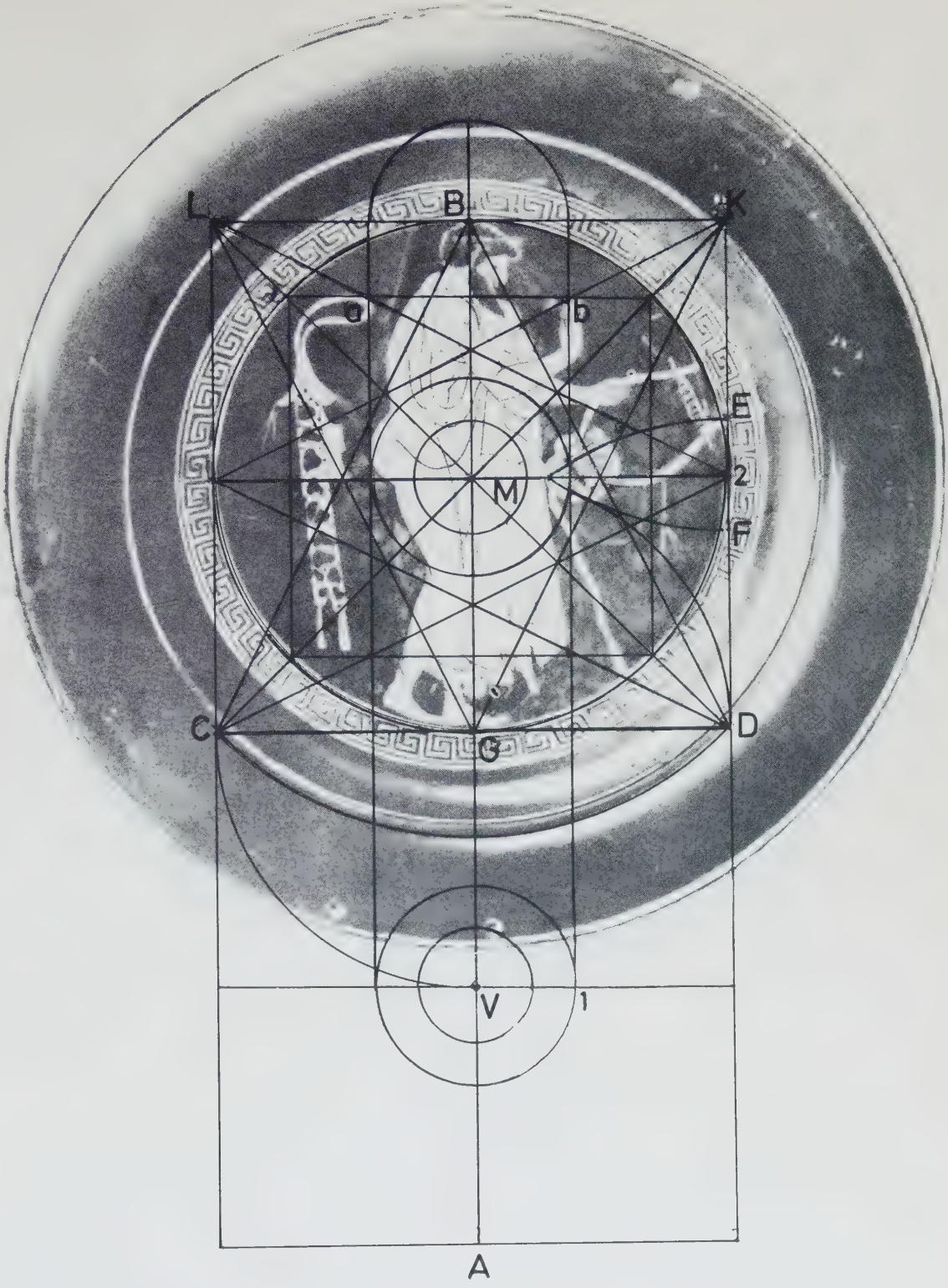
Egyptians made their flat pictures of human figures by combining a representation of the upper part of the body, viewed from the front, with head and legs shown from the side. It would seem that Euthymides was trying to combine the parts of the body in a new way. He can hardly be said to have been successful, either from the point of view of the Egyptian requirements as to absolute measurements being depicted, or from a more naturalistic point of view. That he was a great artist is, however, quite clear. The composition lacks neither power nor grandeur, *Fig. 39*, and it would be unfair to the artist to ignore Hector's close contact with his helmet, raised high behind his head, with his shield and with the two figures flanking him.

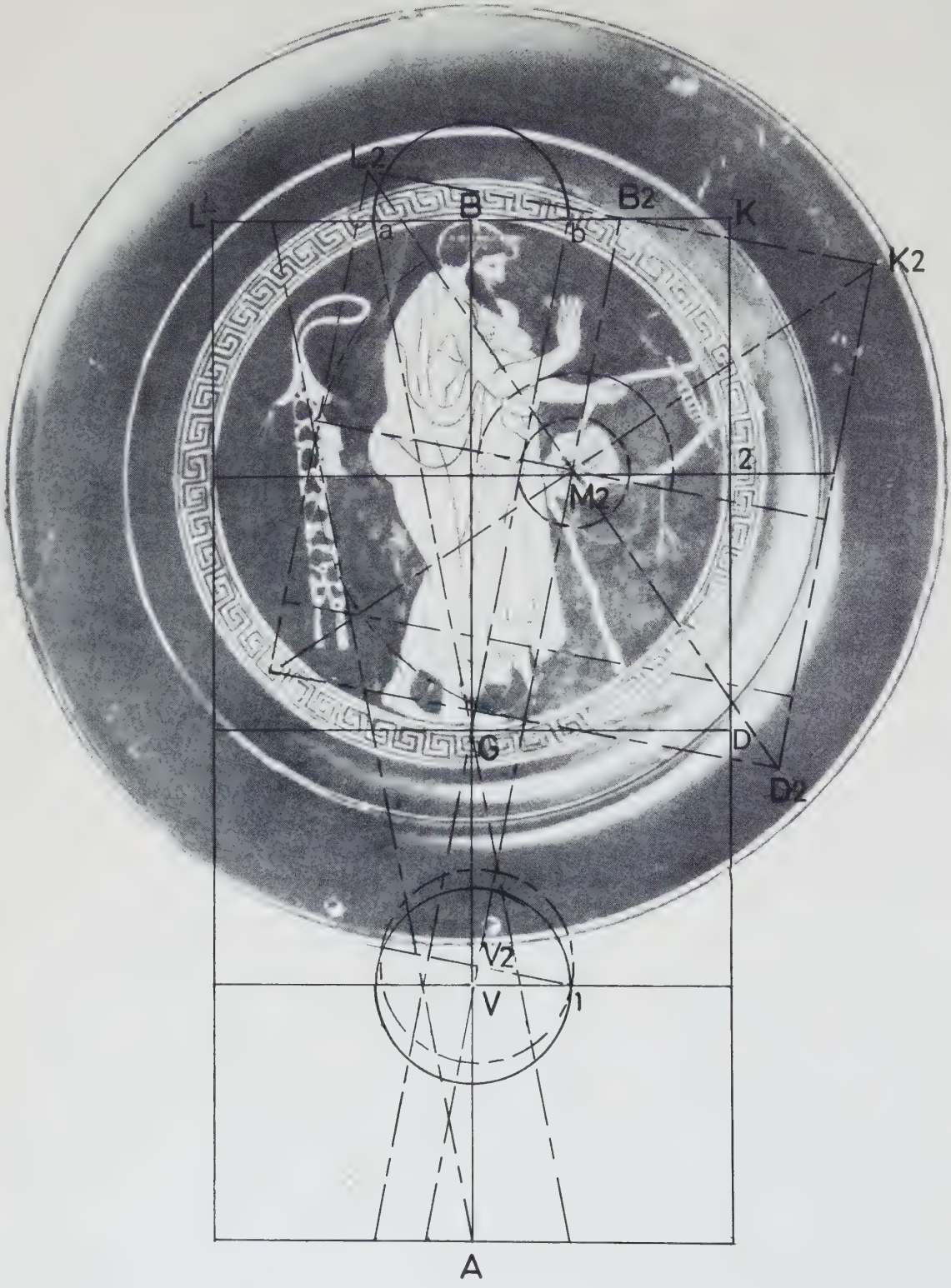
Fig. 40
Attic red-figure plate. 'Komast with Lyre'. Painted by Makron. Vth century. Nationalmuseet, Copenhagen.

Euthymides' attempts show that basic principles were not discarded – the basic principle in this case being *the combination* of parts of the figure designed to be viewed from the front, with parts shown from the side. It is clear, however, that any tampering with so strict and immutable a method as the Egyptian must be fraught with consequences. The picture of Hector immediately leads to the question: which leg carries his weight, the leg shown from the front? Surely that would have affected the height of his shoulders? In the following, we shall try to show how the supreme mastery of the geometrical pivot was to have a far-reaching influence on the new way of representing the human figure. It appears, however, that Euthymides was ahead of his time, for many decades were yet to pass before the new ideas were formulated. Our next two analyses show that the ancient way of thinking was still predominant, even though new features were beginning to make themselves felt.

PLATE XVIII A, B, C, D, E. Nationalmuseet, Copenhagen. Analysis based on the museum's photograph. Scale 1:1. *The Komast with Lyre* is an Attic red-figure plate, painted by Makron in the fifth century BC., *Fig. 40.* There is no sign of any uncertainty or unsureness in the figure of this man who walks along while playing his lyre. From the naturalistic point of view we can

PLATE XVIII A, B, C, D, E.
'Komast with Lyre'. By Makron. Nationalmuseet, Copenhagen. Analysis based on the museum's full scale photograph.



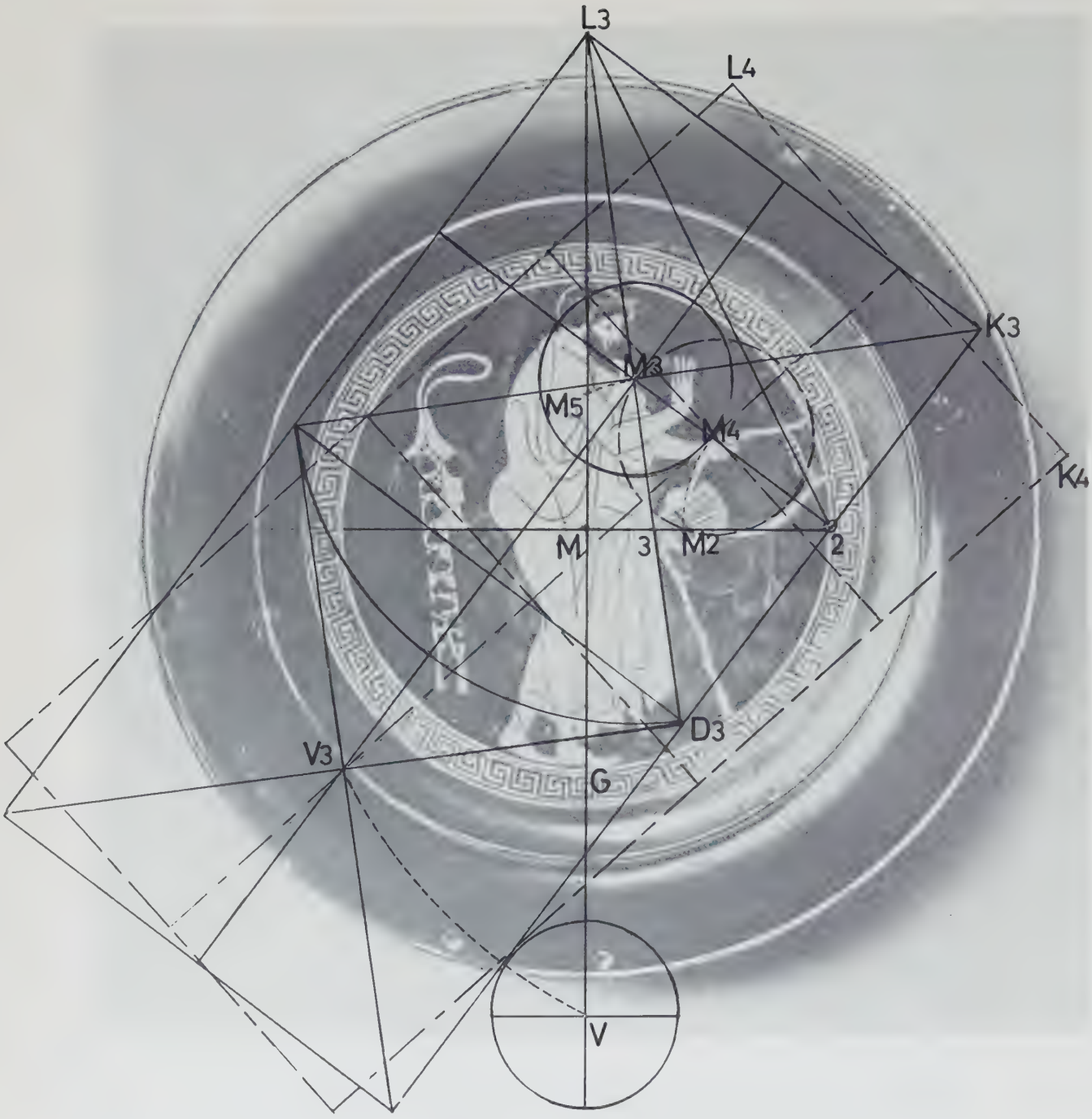

L2
L
B
B2
K
K2
a
b
M2
G
D
D2
V2
V
1
A

certainly find oddities, but they are not immediately noticeable – we are referring to the cover of the lyre, which hangs in the loose air, to the staff standing in front of the man, at an angle, but without any support, and to the drapery which falls in large folds across his chest, indicating an oblique line far beyond the front of the body. Basing his design on the geometrical way of thinking, the artist is able to depict a motif in movement in an entirely convincing way, while the human figure is still completely Archaic. (N. Breitenstein has called this lyre-player "a final vignette in Archaic vase painting".) The drawing of this motif provides a clear example of how the geometrical pivots may be connected with the figure: now we can speak of a definite technique employed in order to express a moving person.

The field is limited by the inner circle without border. We have constructed an inscribed as well as a circumscribed square to this circle. The height of the circumscribed square is GB, the height of the man, and the base of this square forms the base line of the scene. The base line of the block – CD – is repeated from G down to A, and we now have two squares, one above the other. The upper square, CDKL, marks a profile projection, the lower a horizontal projection. The centre of the lower square is V, which must be regarded as the horizontal projection of the vertical axis, GB, of the man in his original position. PLATE XVIII A shows that the figure depends on the division of the circular field in several respects. The hand plucking the strings of the lyre reaches up to point b, while the cover behind the man reaches point a. Both these points lie on the upper side of the inscribed square; they form the points of intersection between this side and the Φ hypotenuses from L and K to the mid-points of the vertical sides of the circumscribed square. M is the centre of the field. Three equal circles with a diameter equal to ab are drawn about centres M, B and V. The width of the lyre is equal to EF, the points of Φ division on DK. Circles with a diameter equal to EF are drawn about centres M and V.

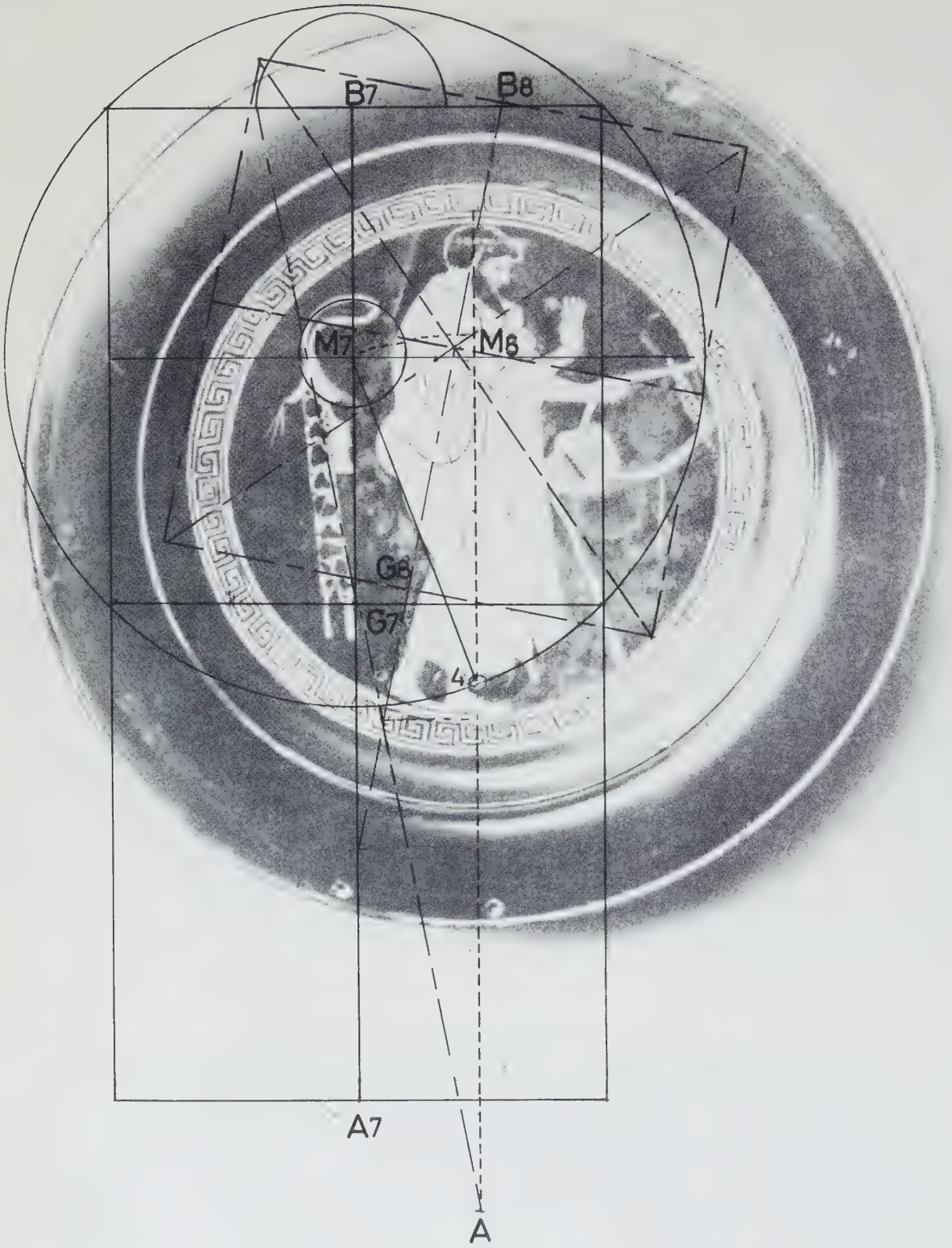
PLATE B. The artist employed the entire picture surface in order to create an impression of movement. By means of a pivoting movement about point 1 on the V-horizontal the interest is increased. This movement is determined in the horizontal projection; the geometrical lines swing to the right until AB is parallel to what we call a *space axis*. The angle of this space axis is determined by the broken line through G, a tangent of the circle about B. This first pivoting movement is calculated in the way employed in the preceding analysis, in the "orthodox" Egyptian manner, and it suggests that we may regard ab as representing the width of the man. We see that M in its new position lies at the centre of the crossbar of the lyre, and the direction of the bridge is determined by D2L2, the diagonal of the square, which passes through the folds of the drapery in front and traces the inside of the man's right upper arm. The position of the bridge at M2 suggests an identification of the man with his instrument. The horizontal diameter, in its first and its second position, is marked by the cover.

The next stage is this: in Plate C both these two geometrical positions, inseparably linked to each other, move on. The new pivot is point 2, the centre of DK. Both the geometrical positions swing to the left about this point, until L lies on the extension of AB (L3). The line from 2 to L3 traces the crossbar of the lyre, and the diagonal L3D3 describes the front edge of the cloak. Our attention is drawn to the upper part of the picture, where the right hand is lifted to pluck the strings, while the left, holding the lyre, touches M4. The arc about 2, which moves M2 up to M4, traces the curve of the lyre below the hand, while the larger circle about M4 follows the upper side of the lyre. The connection between the man and his lyre appears particularly striking on PLATE C: his left hand lies where the two M "horizontals" shown in this plate cross.



The next movement takes place about point 3, the point of intersection between the M horizontal (of PLATE XVIII A) and D3L3. This movement is *anti-clockwise*, and it is immediately apparent that the line which marked the front of the cloak now determines the direction of the staff. The movement is geometrically determined by M5 lying in GB of the original position (dotted). On PLATE XVIII D, the larger circle is drawn about M5, the smaller about M6, and we see that the hands depend on both these circles. Part of the circle of the picture field, in its broken line position, can be seen on the left, touching the cover. The staff rests at D5. A last

rotation turns the geometrical lines back into the original direction of the geometrical pattern of PLATE A. The pivot is point 4, the point of intersection between GB and the circumscribed circle of the square centre M5. On PLATE D a dotted line is shown, passing through 4, and parallel with G5B5. The geometrical lines of PLATE D swing to the left about 4 (PLATE XVIII E) until the same dotted line is aligned with GB. A7B7 then touches the bottom edge of the tassel of the cover, and the "small" circle about M7 can be traced in the loop behind the man. This again indicates an identification between the cover, the man and the lyre, and we note that the man has now moved forward from the position of the cover and is about to ad-


B7
B8
M7
M6
G6
G7
A7
A

vance further. The clear marking of the circumscribed circle of the square is also very striking. We must also note the line from M7 to point 4, which marks the actual stride ahead taken by the right leg. The "vertical" central line of the position G8B8, indicated in broken line, touches the right heel of the man, determines the angle at which the upper part of his body leans forward, and touches the back of his ear. The fact that the space axis shown in broken line, which passes to the left through G7, is identical with the left space axis trough A on PLATE B is strong evidence in favour of our arguments. As in the analysis of Dionysos on the Ocean, we could have charted the various phases of movement in the reverse direction, if required.

It should now be possible to understand how the artist exploited the various attributes of the principal figure in order to show the stages of movement, for there is a state of identification between the man and his attribute. Now, at the time of transition to Classical Greek art, these methods were clearly of great importance in what we may call an enrichment of the representation of the human figure. The ancient wisdom, the inheritance of old, was *not* discarded; on the contrary, its possibilities were now exploited to meet the demands of the new era. At this stage, no naturalistic tendency had yet made its appearance – *the method of the geometrical pivots preceded and paved the way for Greek naturalism.*

PLATE XIX A, B, C. Ny Carlsberg Glyptotek, Copenhagen, has a votive relief (I.N. 2308) showing a sick man on a stretcher. It is a provincial work from the time of Pheidias and is of special interest in our context as a late example of the Archaic style. The relief is stated to be 0.56 m high, 0.72 m wide, and the analysis is based on the museum photograph, scale 1:4. Checked on the original.

The scene, according to the Catalogue of Ancient Sculptures (1940), has been variously interpreted, *Fig. 41.* It obviously includes an Asclepios serpent, issuing from a hole in the trunk of a bare tree. The serpent must constitute an important aspect of the scene, and its connection with the sick man is essential. Martin P. Nilsson's interpretation, which is quoted in the catalogue, is of interest in the present context: he "imagines the relief as having been set up in thanks for the healing of a paralyzed man who suddenly found himself able to walk at the sight of the holy Asclepius serpent. Instead of two pictures representing first the fright, then the thankfulness, the frightening miracle and the thankfulness are combined in one scene. The two young men (who are throwing stones) represent the first phase, the grateful man on the stretcher – the result." We would also quote S. Eitrem, who wrote in the same catalogue: "The serpent in the tree is an epiphany, a divine revelation, which the man on the stretcher greets with gestures of adoration, the two young men with stone-throwing, which is to drive off not the serpent but the sickness. Thus the stone-throwing is of purgative (cathartic) significance. It relieves the sick of his torment, a well known feature of antique superstition, and therefore it is a rite performed before the sacred tree, from which the demon, the sacred serpent, can send both evil and good." Martin P. Nielsson's interpretation implies an *element of time* in the scene of this relief, closely related to that we found in the scene with King Arkesilas. We have seen how new positions of the geometrical pattern marked new stages of the action depicted – is the relief the result of similar methods? Our geometrical analysis begins, as usual, with an examination of the dimensions of the relief.

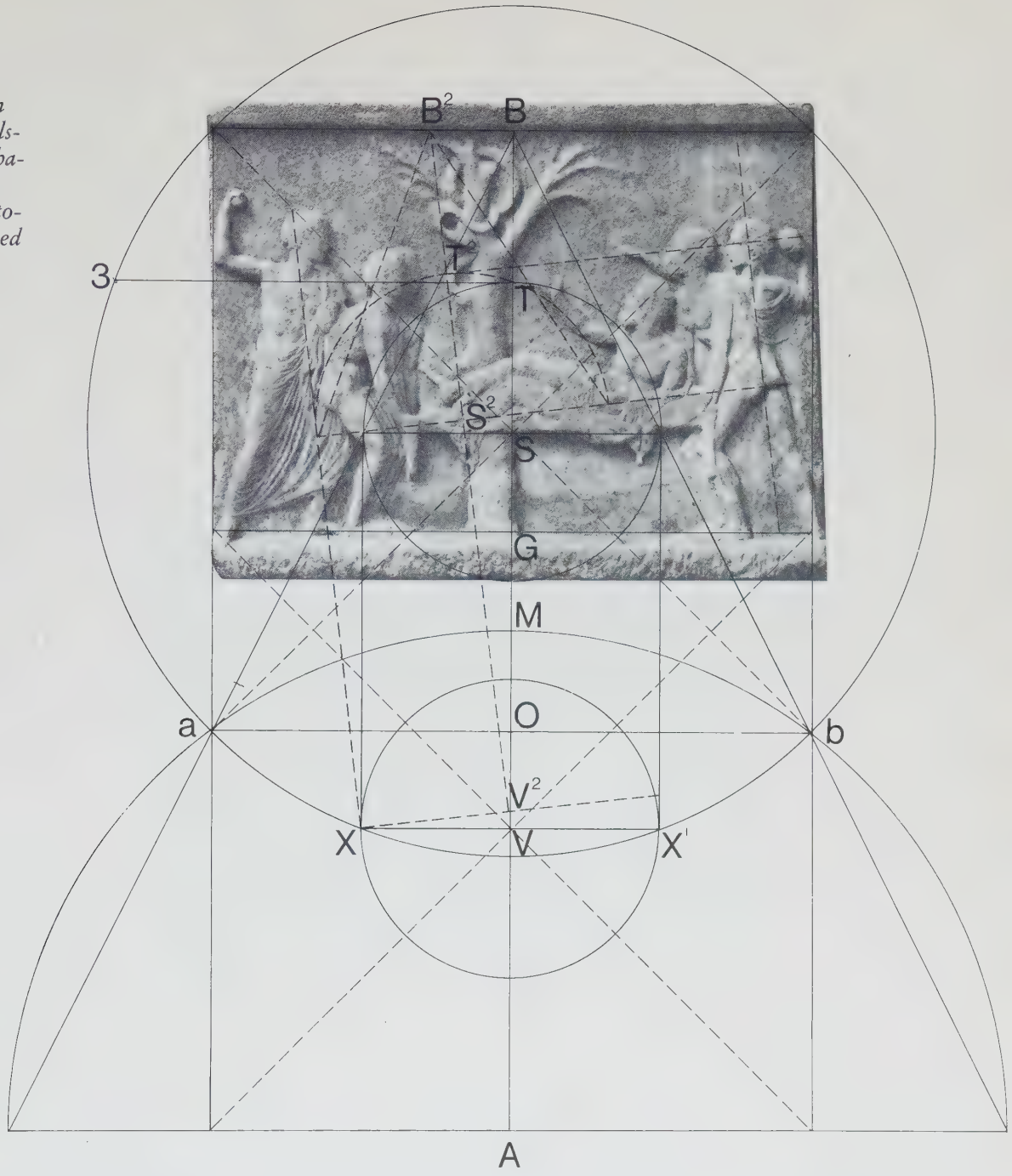
PLATE XIX A shows the basic dimension AB, which is equal to the sum of the height of the relief (without edge above and below) and its width. AM = MB. O is determined by the horizontal through the points of intersection, a and b, between the Φ hypotenuses from B and the arc about A through M. Now we see that *ab determines the width of the relief*, which is equal to AG. V is the centre of the square bisected by AG, and S that of the square bisected by OB. The

Fig. 41
Votive relief. 'Sick Man on a Stretcher'. Provincial work from the time of Pheidias. Ny Carlsberg Glyptotek, Copenhagen I.N. 2308.

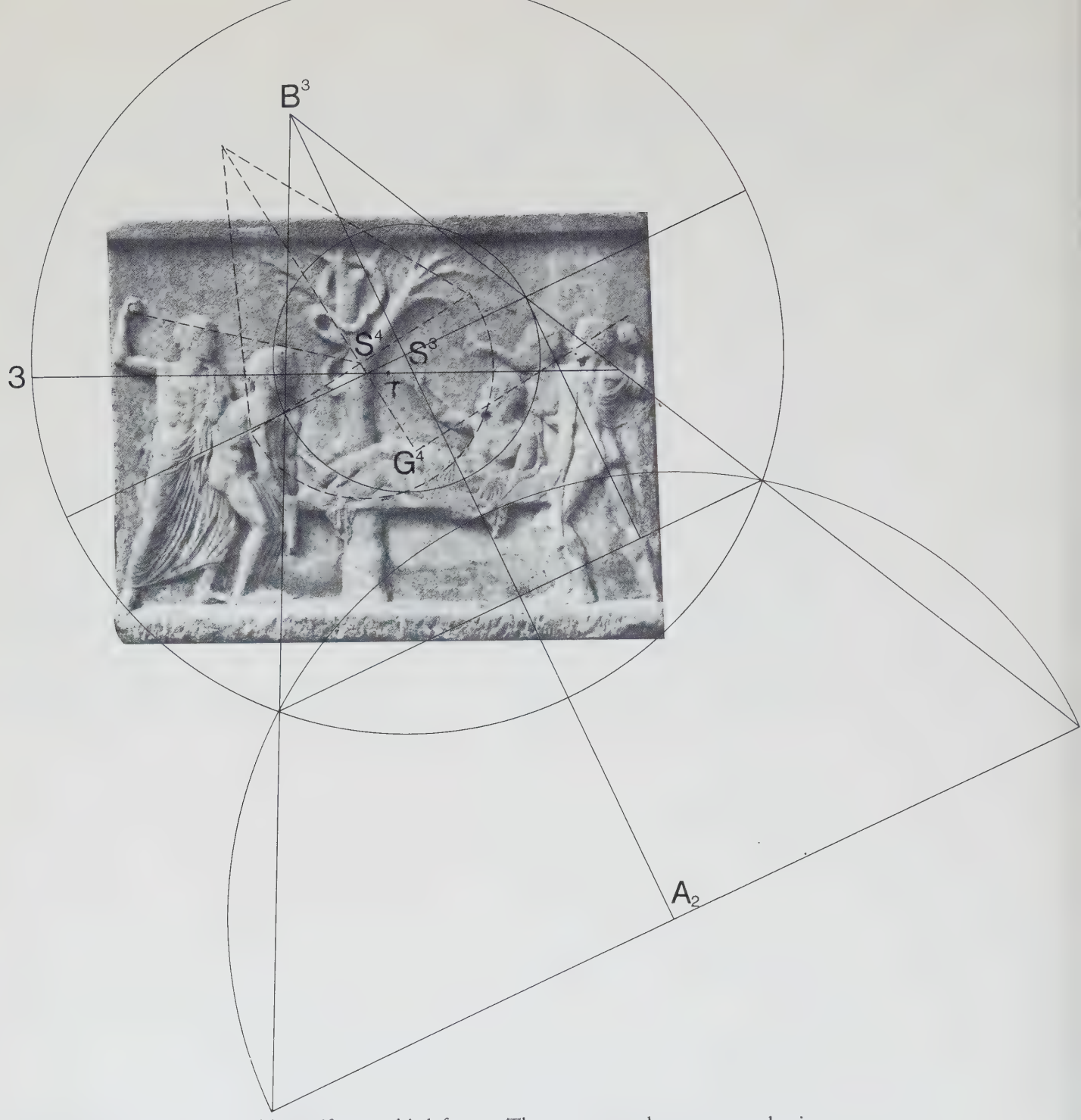
circumscribed circle of the latter square is struck, and it appears that the *length of the stretcher is equal to the distance between X and X'*, the points of intersection between the V horizontal and the large circle about S. The length of the stretcher XX′ is determined in horizontal projection, and V marks the vertical central axis of the relief, seen from above. We see that the stretcher is displaced to the right, and we must imagine it in its original position at the centre of the scene. A circle about S, the length of the stretcher forming its diameter, includes the outstretched hand of the sick man as well as the hole in the tree from which the serpent emerges – two essential parts of the scene.

The first pivoting movement of the geometrical lines is Egyptian, in the horizontal projection. The pivot is X, and the geometrical lines move towards the left, until the apex of the circle (T) lies on the left hypotenuse from B as T2. Now S has moved up to the edge of the stretcher to where the drapery is hanging down, marking V2B2 under the sick man's left foot. This line may be traced further up to where it is tangential to the serpent's hole, level T, and higher up, to the serpent. The circles about S and S2 mark the extended arm of the sick man as well as the back of the man on the left, who is lifting the stretcher. To the right we see the dotted horizontal through T2 passing through the stone in the right hand of the man. This stone is "outside" the picture, the dotted side line being the man's mid-line seen from behind. In relation to the dotted position, the left stone thrower has advanced "forward" into the picture.

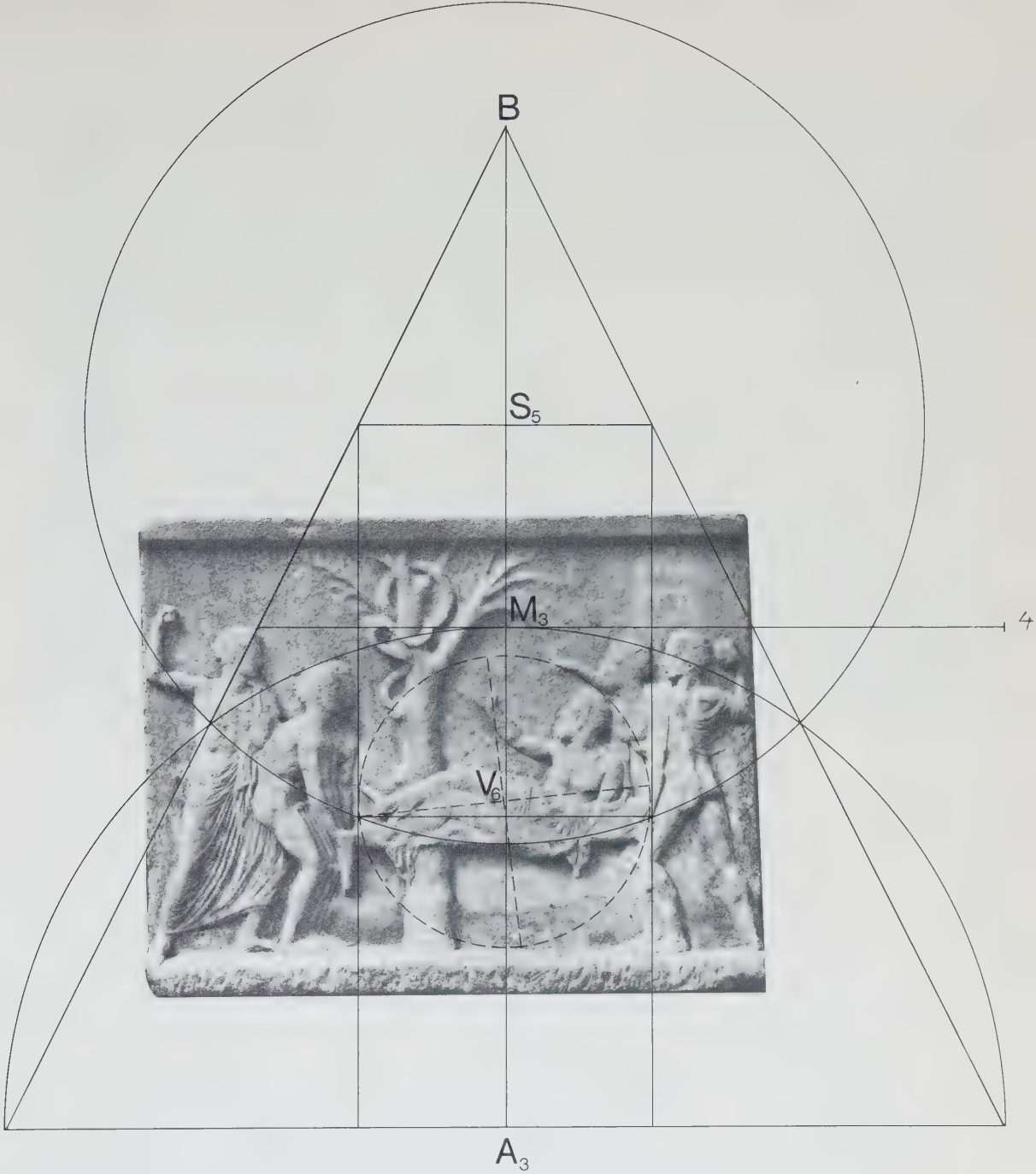
PLATE XIX A, B, C.
Votive relief. 'Sick Man on a Stretcher'. Ny Carlsberg Glyptotek, Copenhagen I.N. 2308. Analysis based on museum's photograph. Scale 1:4. Checked by the author.



The geometrical lines of the two positions A and B now pivot (*PLATE B*) together about point 3 (to the left on the large perimeter, and the level of the point is determined by T, apex of the smaller circle about the same centre S). The geometrical lines pivot until *the left Φ hypotenuse* from B3 is vertical. We can find a number of marks referring to this position of the geometrical pattern. This Φ hypotenuse cuts the left end of the stretcher, and the top of the tree, with the serpent winding about its branches, is at the centre of the scene, the circles about S3 and S4 encompassing parts of the sick man.



The sick man has half raised himself up on his left arm. The two stone throwers are also in contact with this pivoting movement. The man on the left raises his hand holding the stone, until it touches the left dotted diagonal through S4. We must note the firm drawing of the shoulder blade of the right stone thrower. But it is anatomically quite unconnected with the spine further down. The right foot is shown as seen from behind, which shows that the new style from the great centres of art, first and foremost Athens, was not quite unknown.



The last pivot is determined on PLATE C as the upper right corner of a square whose left side lies in A5B5. The geometrical lines pivot upwards to the right to the position parallel to that of PLATE A. The result is shown on PLATE C. Here the circle about V6 encloses the sick man, the serpent's hole in the tree and the stretcher and thus the scene is shown centrally also from above, embracing the sick man as well as the serpent's hole in the tree. The large circle about S5 is tangential to the lowest point of the stretcher. To quote Martin P. Nilsson once more, "the frightening miracle and the thankfulness are combined in one scene" — an idea which the Greek sculptors certainly mastered.

Fig. 42.
'The Contest between Apollo and Marsyas'.

"The Liberation"

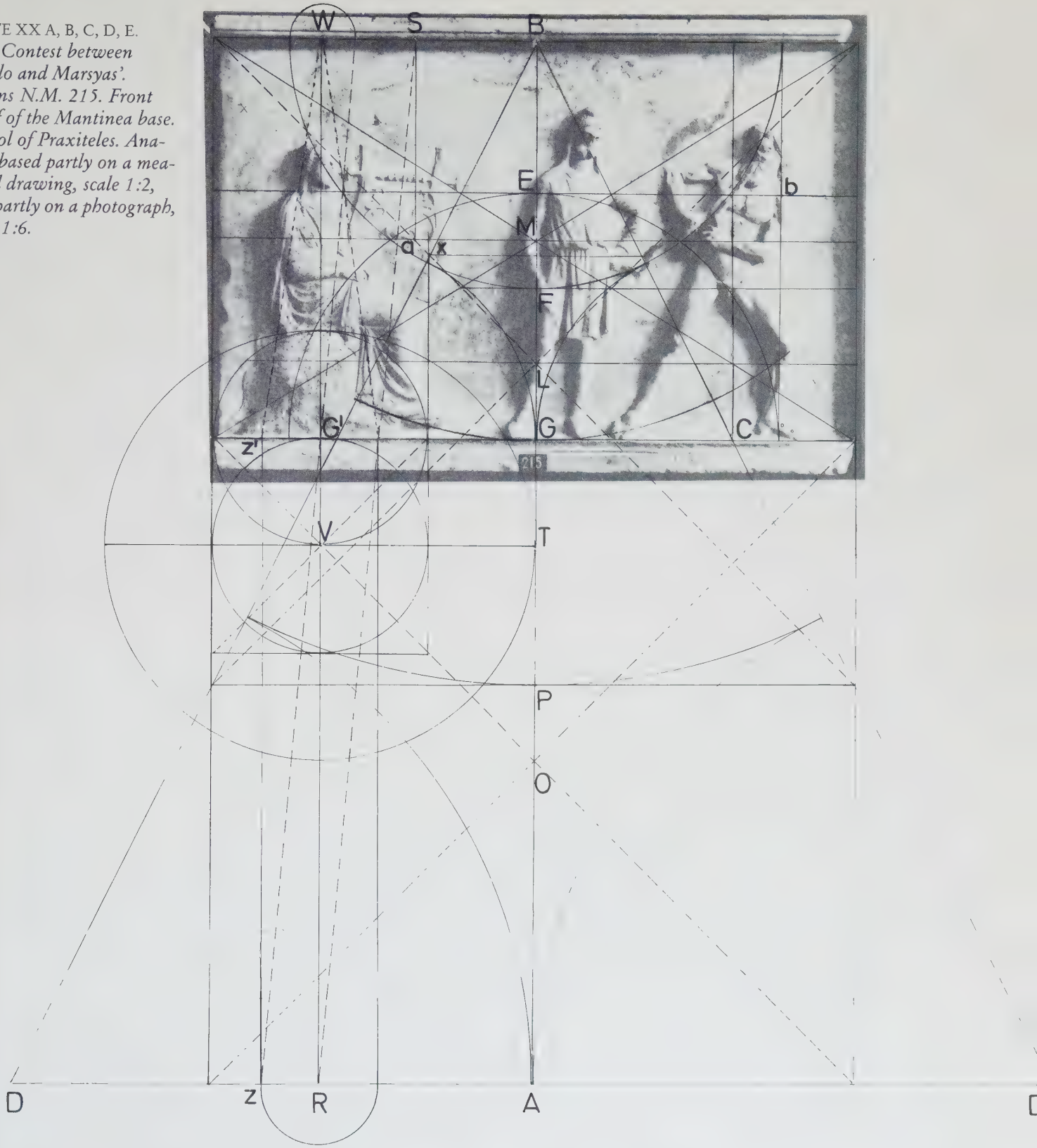
Displacement of the weight of the figure as a result of the pivoting movement. New "horizontal" axes arise

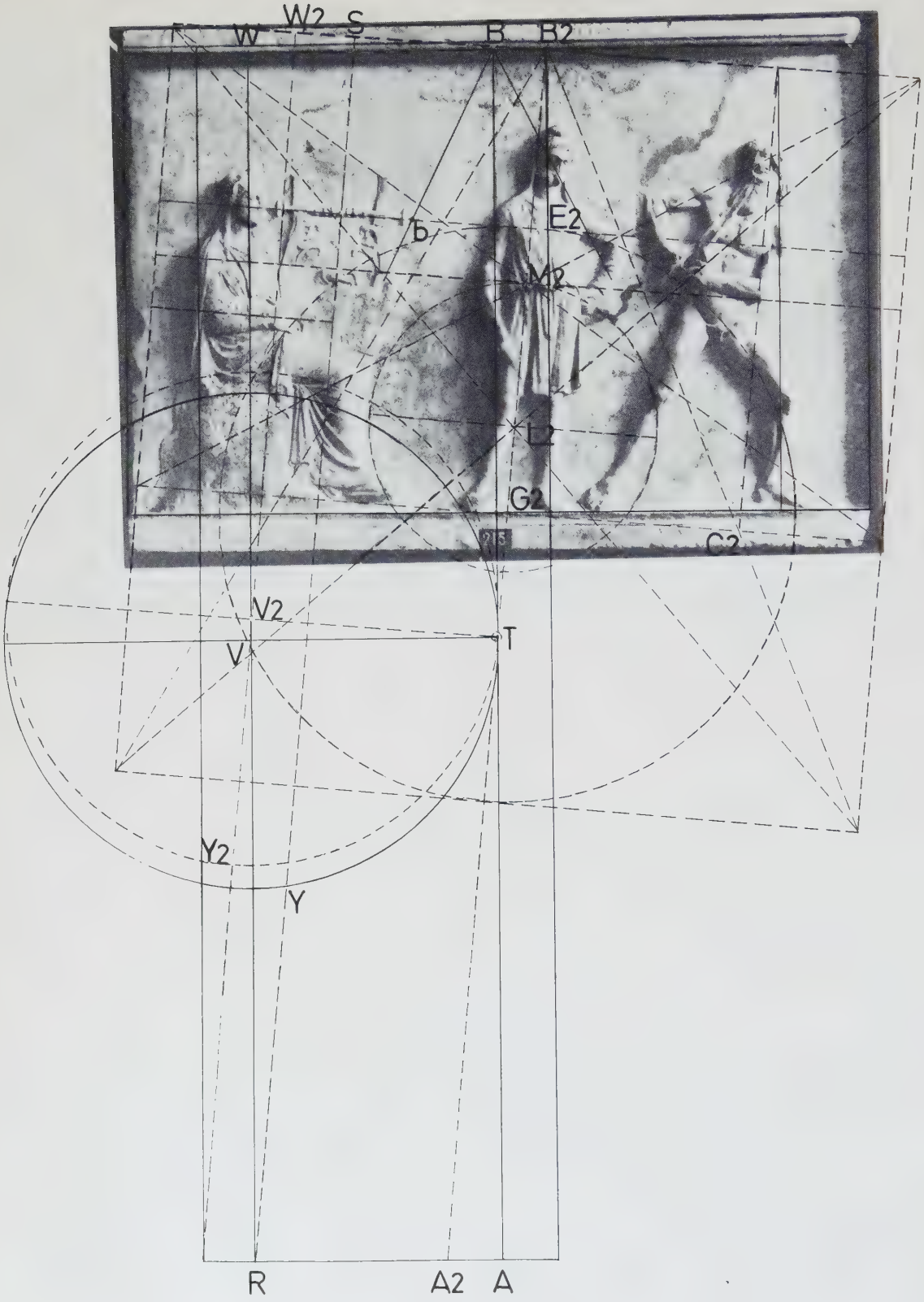
Unlike the Archaic reliefs investigated above (PLATE IV and PLATE V), which were hewn out of the sides of the statue base, the reliefs of the Mantinea base (school of Praxiteles), are hewn out of loose slabs 11.5 cm thick, placed outside the block in such a way that they cover its sides. According to the catalogue a fourth relief is lost. Perhaps it would be more logical to deal with the three reliefs together, as we did with the Archaic reliefs; several attempts, however, did not convince us of any connection. If such a connection could be found for the three reliefs side by side, which seems not impossible, one might perhaps arrive at the method underlying the long friezes of Greek art? An intriguing thought, but one which leads us too far from our present subject, the development of the method of representing the human figure.

The front relief of the Mantinea base illustrates the musical contest between Apollo, seated with his lyre on the left, and the satyr Marsyas, who had had the impudence to challenge the god. Every Greek knew this story, and they knew that Marsyas was defeated, even though the flute he had found was Athena's. A Scythian stands between the two principal figures, ready to slay the defeated Marsyas.

PLATE XX A, B, C, D, E. Marsyas, Athens N.M.215. Front relief of the Mantinea base, school of Praxiteles, ca. 320 B.C. Analysis based partly on a measured drawing scale 1:2, partly on a photograph scale 1:6. The length of the front relief is 1.36 m, and the inner height 0.835 m (measured by the author).

PLATE XX A, B, C, D, E.
'The Contest between Apollo and Marsyas'. Athens N.M. 215. Front relief of the Mantinea base. School of Praxiteles. Analysis based partly on a measured drawing, scale 1:2, and partly on a photograph, scale 1:6.




W2
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W
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A

The basic dimension, AB, is equal to the sum of the height GB and the width, laid down from G to A. As G here divides AB in Φ, both height and width are determined. The diagonals of the squares with mid-line AG (centre O) and PB (centre L) are shown in dotted line. GM = MB. GB is divided in Φ at E (above) and at F (below). The Φ arc centre B passing through F connects the two principal figures in a significant way: *it traces Marsyas' double flute and passes through Apollo's left hand at the point where it rests on the lyre.*

We determine Apollo's vertical axis, RW, so far without any comments. This line lies halfway between the left edge and point x on the lyre, the point of intersection between the Φ hypotenuse BD and the diagonal of the square centre L. The point of intersection between the M horizontal and the same diagonal is a, while S lies on the B horizontal, vertically above a. *A line from R to S passes through the centre of the lyre.* V is determined in the normal way, as the horizontal projection of Apollo's vertical axis. AB is tangential at T to a circle about V.

In the original position, the central figure may be assumed to have stood at AB, and Marsyas on the vertical line through C, the point of intersection between BD′ and the G horizontal. PLATE A shows an oblique axis parallel to RS, the axis of the lyre, and passing through G′, the point of intersection between BD and the G horizontal.

We consider this line to be the space axis of the figure, and can determine its profile axis in the normal manner. (The point of intersection between the space axis and the A horizontal is Z, and Z′, vertically above on the G horizontal, is connected with W). We may note that the profile axis slopes more strongly than the space axis — this is due to the format of the relief, whose width is greater than its height.

On PLATE B the geometrical pattern has moved to the right, pivoting about T, until AB is parallel to RS, the "axis of the lyre" (A2B2). *The crossbar of the lyre is in this position marked by the E2 horizontal.* Further, the circle about G2 through E2 marks *Marsyas' left thigh,* while the left side of this circle traces *the front of the drapery right across Apollo's thighs, cutting the axis of the lyre in his left hand.* The circle about L2 through M2 traces the *front of Apollo's legs.* This position of the geometrical lines is of the utmost importance to the drawing of the central figure: as AB moves to A2B2, *the weight is shifted to the right.* A vertical line is drawn down from B2, clearly marked by the leg bearing the main weight of the body. *This is Greek, a feature completely alien to the Egyptians.* As the weight is thus transferred from position 1 to position 2, these two positions are connected in a new way, and they must be regarded as part of one whole to a greater extent than previously. The displacement of Marsyas' weight is carried out in an entirely analogous manner. The C vertical (see PLATE A), which passes through the penis, now has a new position through C2; this is shown in broken line. A perpendicular is drawn from the point of intersection between this axis and the upper edge of the field, and down to the base line of the field. This is the "vertical of the weight".

Now *these two positions move* farther down towards the right, pivoting about Y2 (the lowest point of the perimeter of the circle about V2 through T). Y2W2 pivots until it passes through the point of intersection b (see PLATE B).

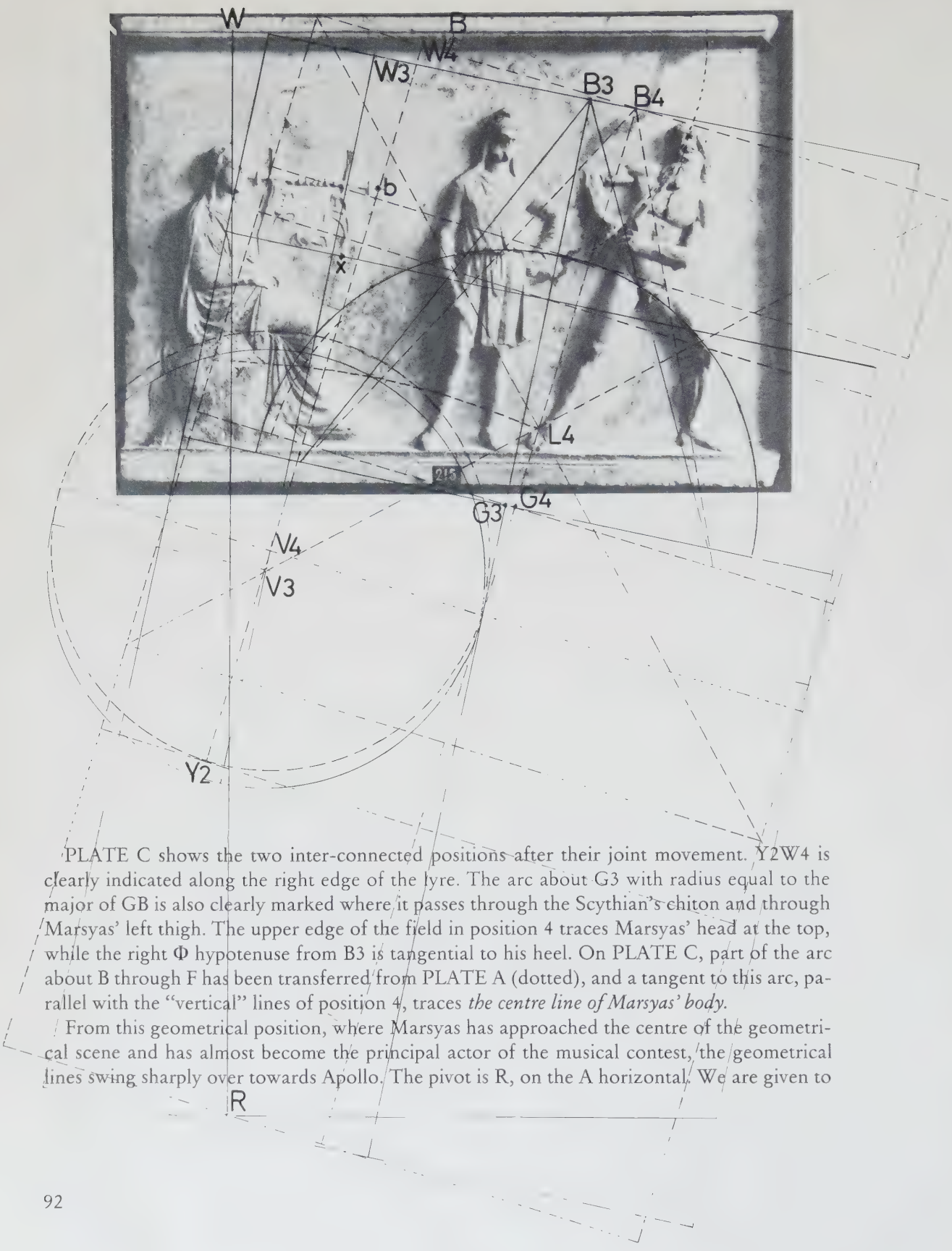
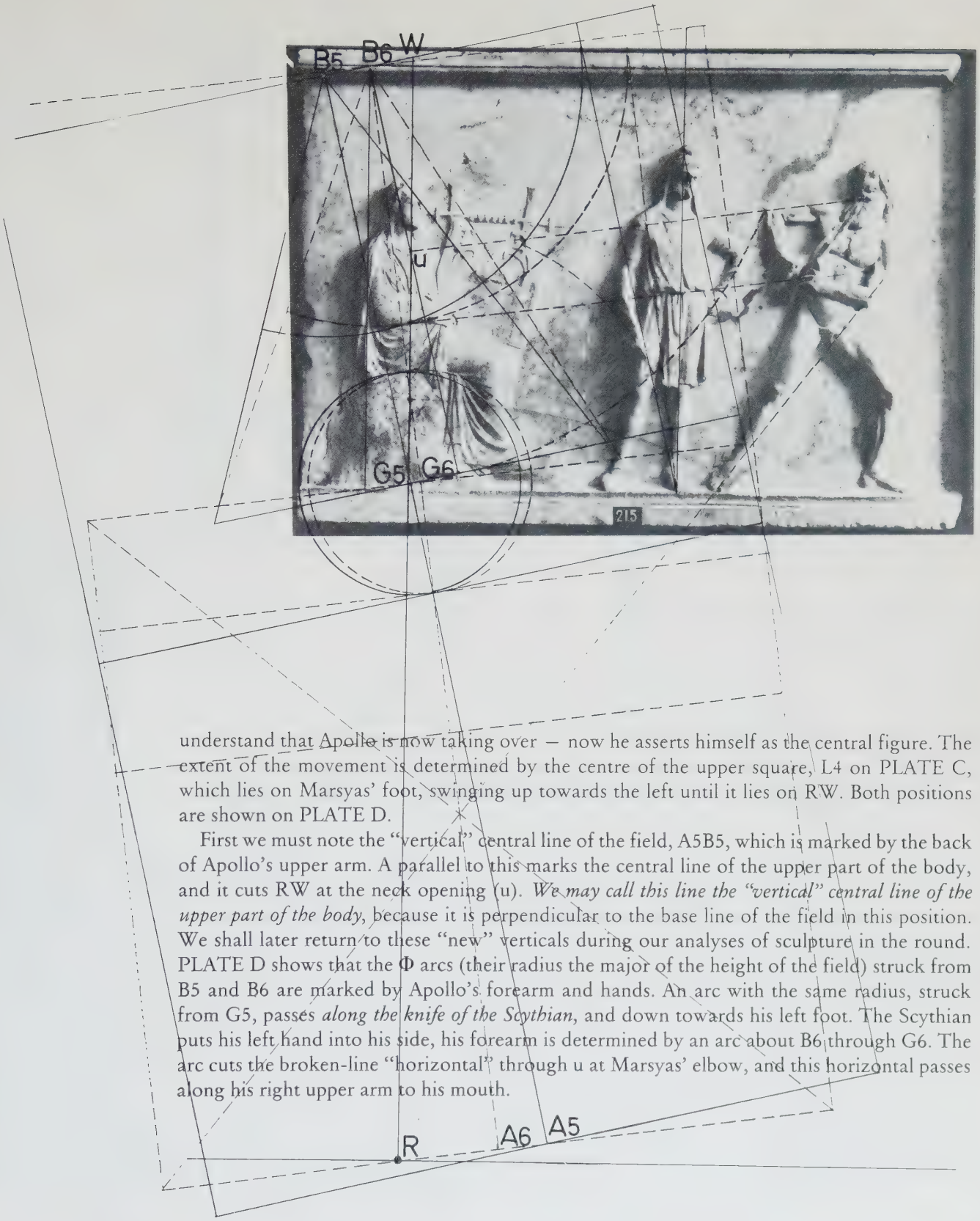


PLATE C shows the two inter-connected positions after their joint movement. Y2W4 is clearly indicated along the right edge of the lyre. The arc about G3 with radius equal to the major of GB is also clearly marked where it passes through the Scythian's chiton and through Marsyas' left thigh. The upper edge of the field in position 4 traces Marsyas' head at the top, while the right Φ hypotenuse from B3 is tangential to his heel. On PLATE C, part of the arc about B through F has been transferred from PLATE A (dotted), and a tangent to this arc, parallel with the "vertical" lines of position 4, traces *the centre line of Marsyas' body.*

From this geometrical position, where Marsyas has approached the centre of the geometrical scene and has almost become the principal actor of the musical contest, the geometrical lines swing sharply over towards Apollo. The pivot is R, on the A horizontal. We are given to



understand that Apollo is now taking over – now he asserts himself as the central figure. The extent of the movement is determined by the centre of the upper square, L4 on PLATE C, which lies on Marsyas' foot, swinging up towards the left until it lies on RW. Both positions are shown on PLATE D.

First we must note the "vertical" central line of the field, A5B5, which is marked by the back of Apollo's upper arm. A parallel to this marks the central line of the upper part of the body, and it cuts RW at the neck opening (u). *We may call this line the "vertical" central line of the upper part of the body*, because it is perpendicular to the base line of the field in this position. We shall later return to these "new" verticals during our analyses of sculpture in the round. PLATE D shows that the Φ arcs (their radius the major of the height of the field) struck from B5 and B6 are marked by Apollo's forearm and hands. An arc with the same radius, struck from G5, passes *along the knife of the Scythian*, and down towards his left foot. The Scythian puts his left hand into his side, his forearm is determined by an arc about B6 through G6. The arc cuts the broken-line "horizontal" through u at Marsyas' elbow, and this horizontal passes along his right upper arm to his mouth.

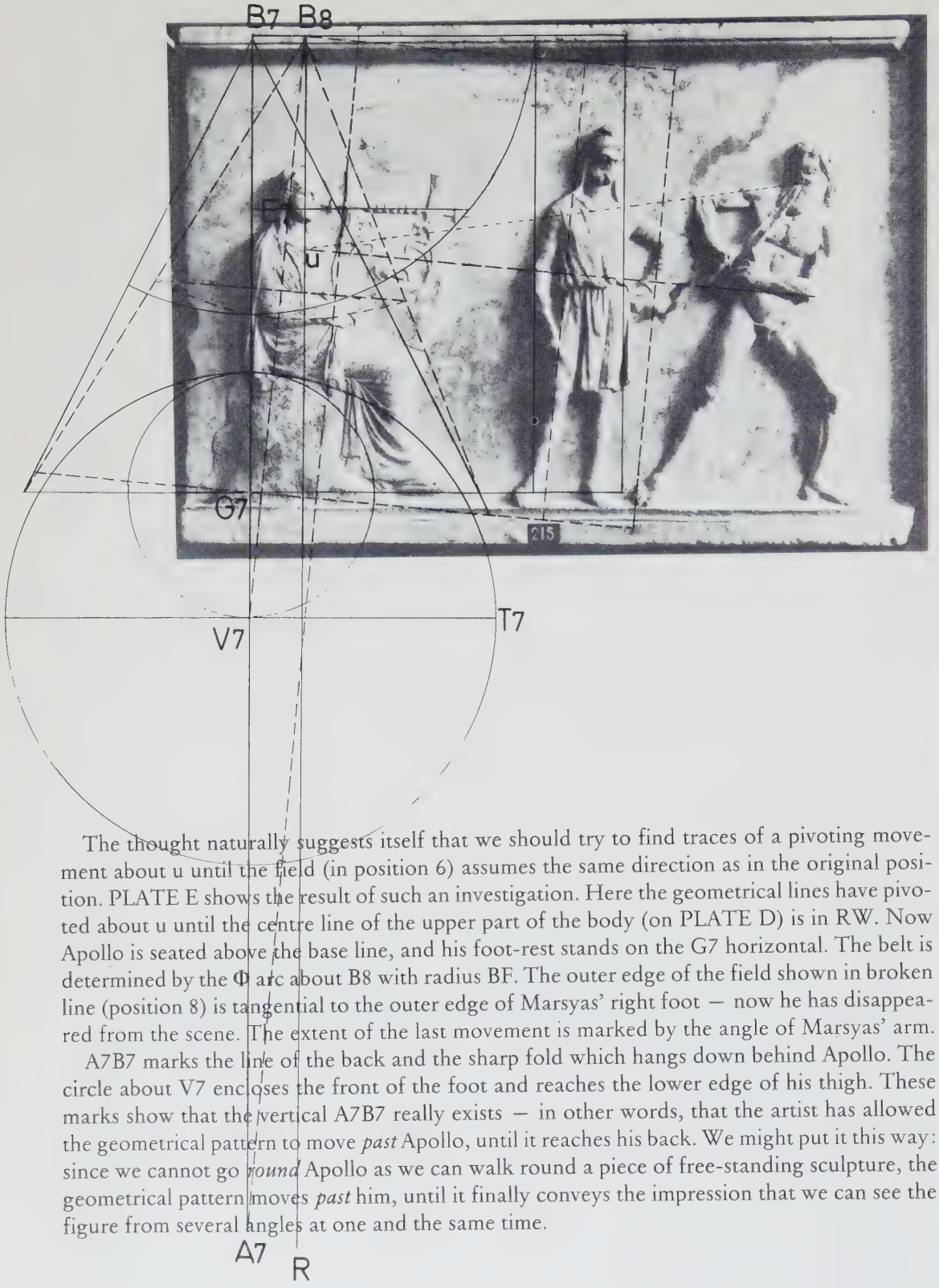


Fig. 43.
Diotima from Mantineia (Athens N.M.226), C. 410 B, C.

The thought naturally suggests itself that we should try to find traces of a pivoting movement about u until the field (in position 6) assumes the same direction as in the original position. PLATE E shows the result of such an investigation. Here the geometrical lines have pivoted about u until the centre line of the upper part of the body (on PLATE D) is in RW. Now Apollo is seated above the base line, and his foot-rest stands on the G7 horizontal. The belt is determined by the Φ arc about B8 with radius BF. The outer edge of the field shown in broken line (position 8) is tangential to the outer edge of Marsyas' right foot — now he has disappeared from the scene. The extent of the last movement is marked by the angle of Marsyas' arm.

A7B7 marks the line of the back and the sharp fold which hangs down behind Apollo. The circle about V7 encloses the front of the foot and reaches the lower edge of his thigh. These marks show that the vertical A7B7 really exists — in other words, that the artist has allowed the geometrical pattern to move *past* Apollo, until it reaches his back. We might put it this way: since we cannot go *round* Apollo as we can walk round a piece of free-standing sculpture, the geometrical pattern moves *past* him, until it finally conveys the impression that we can see the figure from several angles at one and the same time.

Fig. 44.
'Poseidon' from Artemision. About 460 B.C. Athens N.M. Br. 15161.

The somewhat more recent stele, *Diotima from Mantineia* (Athens N.M. 226), *Fig. 43*, c. 410 B.C., shows this quality to a pronounced degree.

Our lengthy analyses of the flat pictures are intended to do more than simply explain these particular works of art; they have been placed before our further discussion of sculpture in the round in the hope that they will enable the reader to understand more readily our view on how the Greeks gradually developed the distinctive characteristics of their style. We have now arrived at the point where it seems natural to continue our investigations concerning *sculptures in the round.* The over-life size bronze Poseidon from Artemision (Athens N.M. Br.15161), *Fig. 42*, is the work of a very great artist. Poseidon's weight is clearly borne by the advanced left leg. The god thrusts forward his weapon, confident of his own strength, and magnificent in his pondus and repose. Like the small bronzes of a similar type, this figure might almost be said to be trespassing on the preserves of the relief, whose base line is analogous to the depth (here: length) of the base. There are a number of related figures all dating from the middle of the fifth century, and thus it seems obvious that one such figure must be investigated in the light of the knowledge we have gained about the Greek method of representing movement on the picture surface. We have chosen the following small bronze to demonstrate the method common to all such figures, Poseidon being too large; moreover, his base is missing.

The following measurements have been taken:
(the word "plinth" has been substituted for "base")

Length of the figure from the right heel to the top of the head 11.7 cm
Depth of the plinth 6.6 cm
Width of the plinth 2.5 cm
True length of the weapon 3.3 cm
Vertical height to the top of the head 11.1 cm
Vertical height to the end of the thunderbolt 11.5 cm
Vertical height to the point of the thunderbolt 10.4 cm
Vertical height to the right elbow 8.9 cm
Vertical height to the bent knee 3.2 cm
Vertical height to upper edge of left hand, shoulder level 9.2 cm
Height of head from the chin up 2.4 cm
Height from the right nipple to the top 8.2 cm
Height from the left nipple to the top 7.8 cm
Distance between the two nipples 1.8 cm, the right lying 0.15 cm from the edge, the left 0.55 cm
The horizontal distance from the back of the plinth to:
The tip of the left hand 0.1 cm
The rear heel 0.7 cm
The little toe of the rear foot 0.4 cm
The front heel 4.3 cm
The little toe of the front foot 6.5 cm
The inner edge of the rear foot 1.4 cm
The point of the weapon 2.7 cm

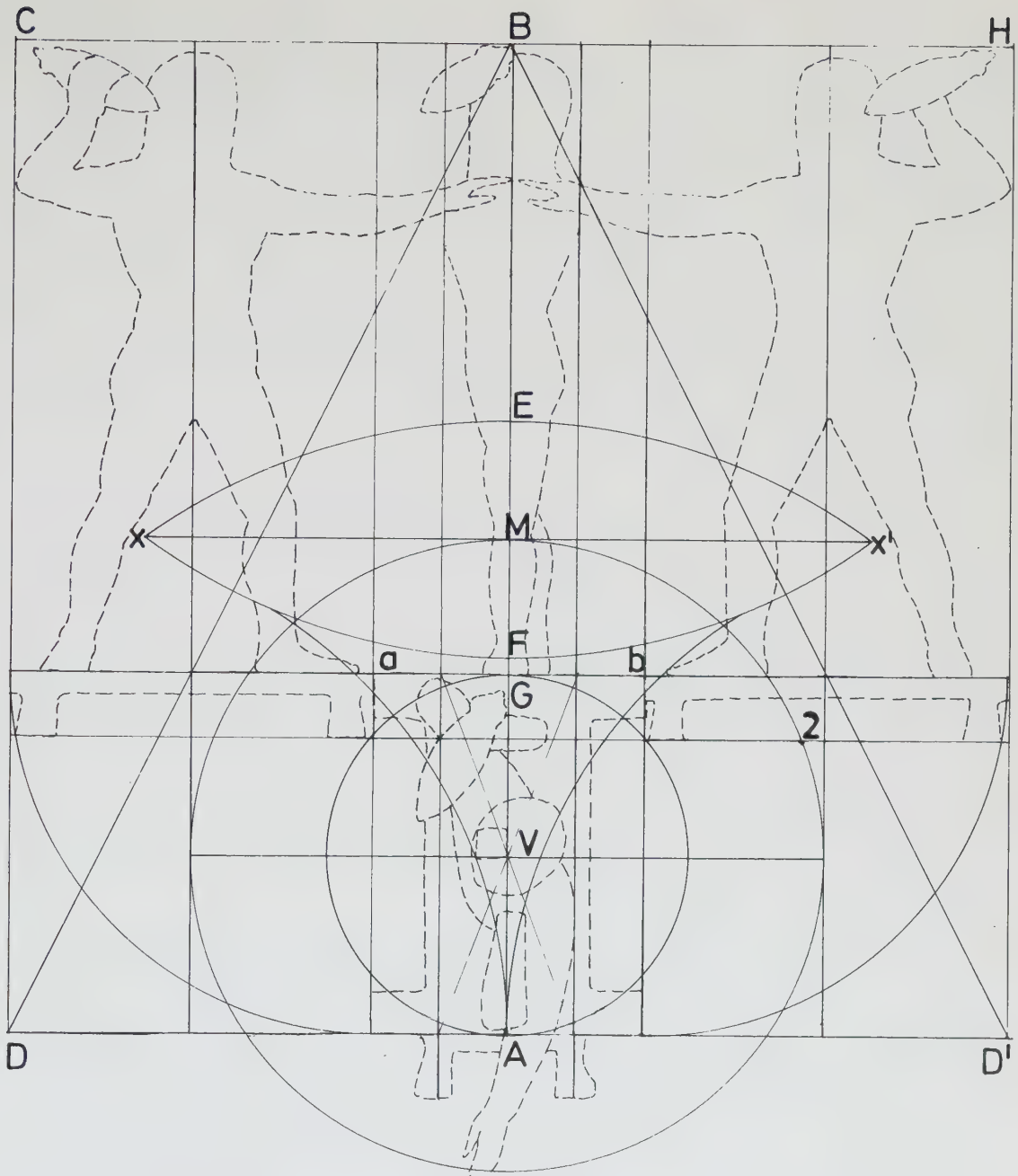
Fig. 45
Bronze statuette. 'Zeus Holding the Thunderbolt'. About 458 B.C. Athens N.M. Br. 16545.

PLATE XXII A, B, C, D, E. *Zeus Holding the Thunderbolt* (N.M. Athens 16546). Bronze. 1:1. *Fig. 45.*

The top of the weapon lies on AG, the elbow on the rear edge of the plinth, 4 cm beyond the side edge. The left hand extends 2.7 cm beyond the plinth in front, the bent knee lies 1.5 cm within the front of the plinth. A vertical plane through the longitudinal axis of the weapon cuts the rear of the plinth 1.2 cm from the right corner. The greatest width of the head is 1.6 cm, that of the face 1.2 cm, the width of the body at the front, measured over the muscles of the hips, is 2.3 cm.

The bronze is entirely intact; nothing is missing from either the figure or the plinth — even the weapon is complete. This is of great importance, for we know from two-dimensional art that a weapon or other attribute of a figure often provides the key enabling us to understand the composition of a work of art. In this investigation we shall not go farther than attempting to explain *the position of the weapon in space.* Accurate measurements show that the weapon is *not* parallel either to the horizontal plane or to any vertical plane parallel to the front or the side of the plinth. *This is not Egyptian,* for if we project the weapon on to a drawing of the block with all its sides placed flush with one another, it would appear foreshortened from above, from the front, and from both sides. To the Egyptians, it will be remembered, the *true* measurements were of the utmost importance. We shall now employ the knowledge we have gained from the preceding analyses to investigate the figure. We start with AB, the sum of the base and the height of the figure.

PLATE XXII A, B, C, D, E. *Bronze statuette. 'Zeus Holding the Thunderbolt' Athens N.M. 16546. About 450 B.C. Analysis based on author's measured drawing. Scale 1:1.*



The three drawings (PLATE A), which show the figure from the front, from above, and from both sides, are sketches, and must not be regarded as measured drawings. They are included here because they are necessary to an understanding of the analysis. The *weapon*, however, *is shown as accurately as possible on all the three sketches.* It seems unlikely that a Greek sculptor should ever have made a naturalistic drawing like this; a constructional drawing met his requirements, for he must surely have had a clear idea of how he intended the figure to look — at least, in broad outline. This applies not only to the artist who designed the original figure, but also to all those who later produced various versions of it, in different sizes. Their

proportions may vary, the movement of the figure may differ; one thing, however, they all had in common — *the method.* Our basic dimension is AB, the sum of the depth (length) of the plinth and the height of the figure, measured from the right heel to the top of the head. AB is divided in Φ at E (above) and at F (below), while M is the mid-point. The Φ arc about A through E intersects with that about B through F at x and x′, and it appears that *the depth of the plinth, AG, is equal to xM.* Thus, GB, the height of the figure in the original position, is also determined. The rear edge, seen from the side, aligns with the verticals through D and D′ (as in the analysis of Aristodikos, PLATE XIII). In the present case, however, the front edge of the plinth stops a little short of the sides, at a distance from the side equal to the height of the plinth, which is therefore shown flush with its horizontal projection. It may be assumed that the reason why the artist calculated his measurements in this way was that he was thereby able to keep the profiles apart, so that the tips of the fingers of the outstretched hand might meet at AB. However, the artist had there arrived at a different solution: He has extended the plinth, so that the base is longer than the plinth proper.

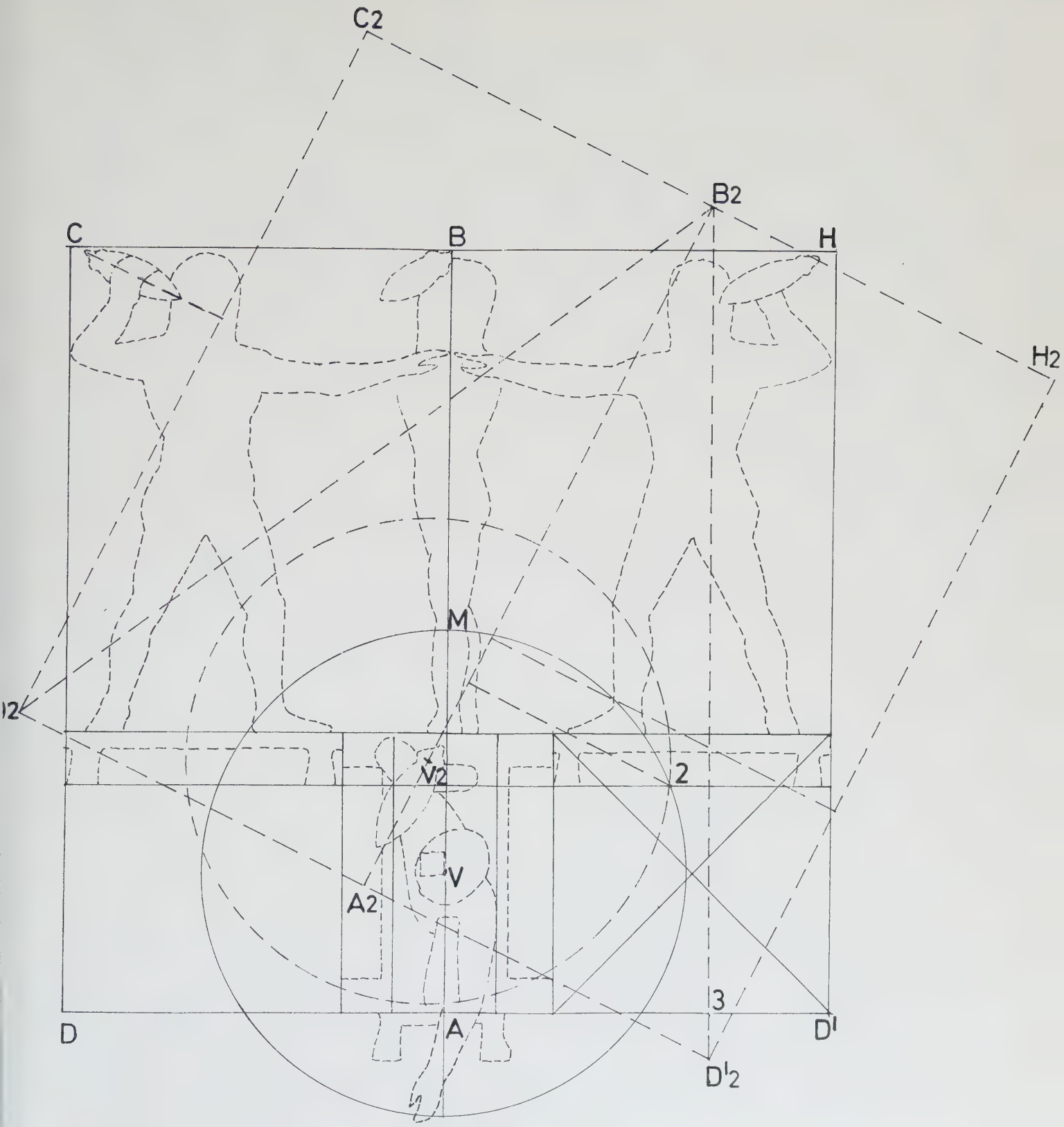
The further drawing of the figure shows us the results of this displacement of the profiles: seen from the side, the figure reaches AB with the fingertips of the outstretched arm. Seen from above, in other words, the man with his arm extended has at his disposal an area defined by a circle about V, the centre of the plinth, and with a radius of VM. We must think of him as rotating around his vertical axis which, seen from above, is identical with point V (we know this way of thinking from Egyptian reliefs as far back as 3000 BC).

We have, up to now, described a hypothetical original position. Now, however, the figure begins to move out of this position, and we shall attempt to chart the stages of this movement in the same way as that on which our analyses of flat pictures were based. The geometrical pattern moves (PLATE B) — for the figure itself is unable to do so. The first pivot is point 2, and the first movement is to the right. (2 is the point of intersection on the right, where the horizontal at the lower edge of the plinth cuts the larger circle about centre V.) The geometrical lines swing until BD′ is vertical as B2D′2, and V2 lies in the hand which holds the weapon. The circle about V, which in the original position enclosed the outstretched hand, now encompasses the foot which is placed at the front of the plinth.

These two closely interconnected positions now swing farther to the right, about 3 as their pivot (see PLATE B). (This is the point of intersection between DD′ and B2D′2.) The limit of this pivoting movement is given by V2 lying on A B (PLATE C) (as V4). *The point of the weapon now lies at A4, and the vertical central plane of the weapon's length is determined.* This line, A4B4, cuts the rear edge of the plinth a little to the right of the centre, and this accords well with the measurements taken. Zeus leans so far back and no further. — From this position Zeus thrusts his weapon forward with his raised hand (PLATE D). This movement is determined by a rotation about point 4, (the centre of the lower square on the right (on Plate C)), a movement which stops when H5 lies on AB at B. Seen from above, the vertical plane through his right shoulder and upper arm appears to be parallel to A5B5, while A6B6 is of importance to the turning of the upper front part of the body.

The next pivot is 5 (the point of intersection between AB and A5B5). Around this point the geometrical lines swing back towards the right until A7B7 is aligned with AB. We now see that the height of the arms is determined by C7H7, the line on which the hand rests, seen from the front (PLATE E).

How did the sculptor who carved the wooden figure over which the bronze was to be cast transfer the calculations of the drawing to his block? He must surely have marked the position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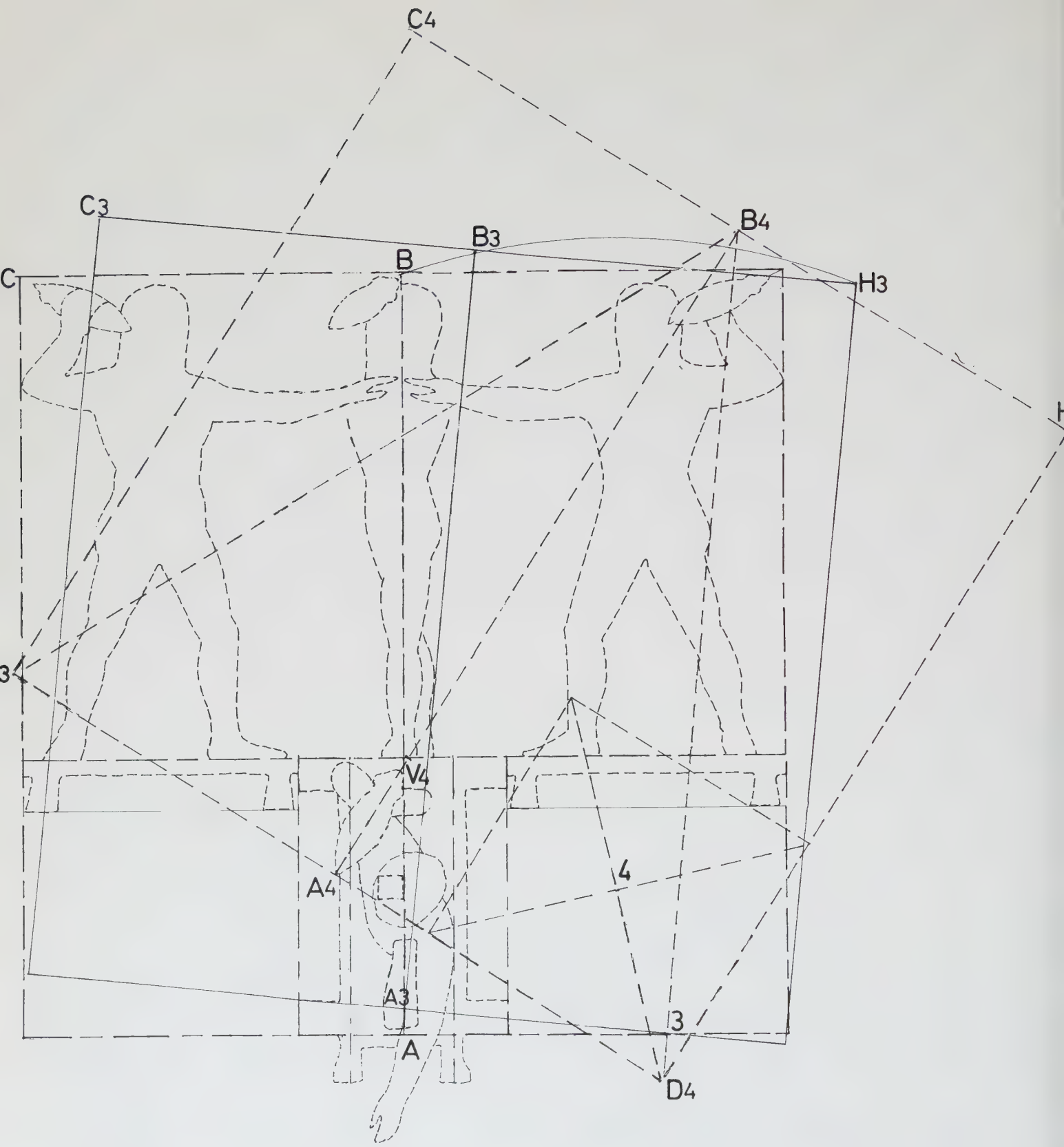
C4
C3
B4
B3
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3
V4
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4
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A
D4

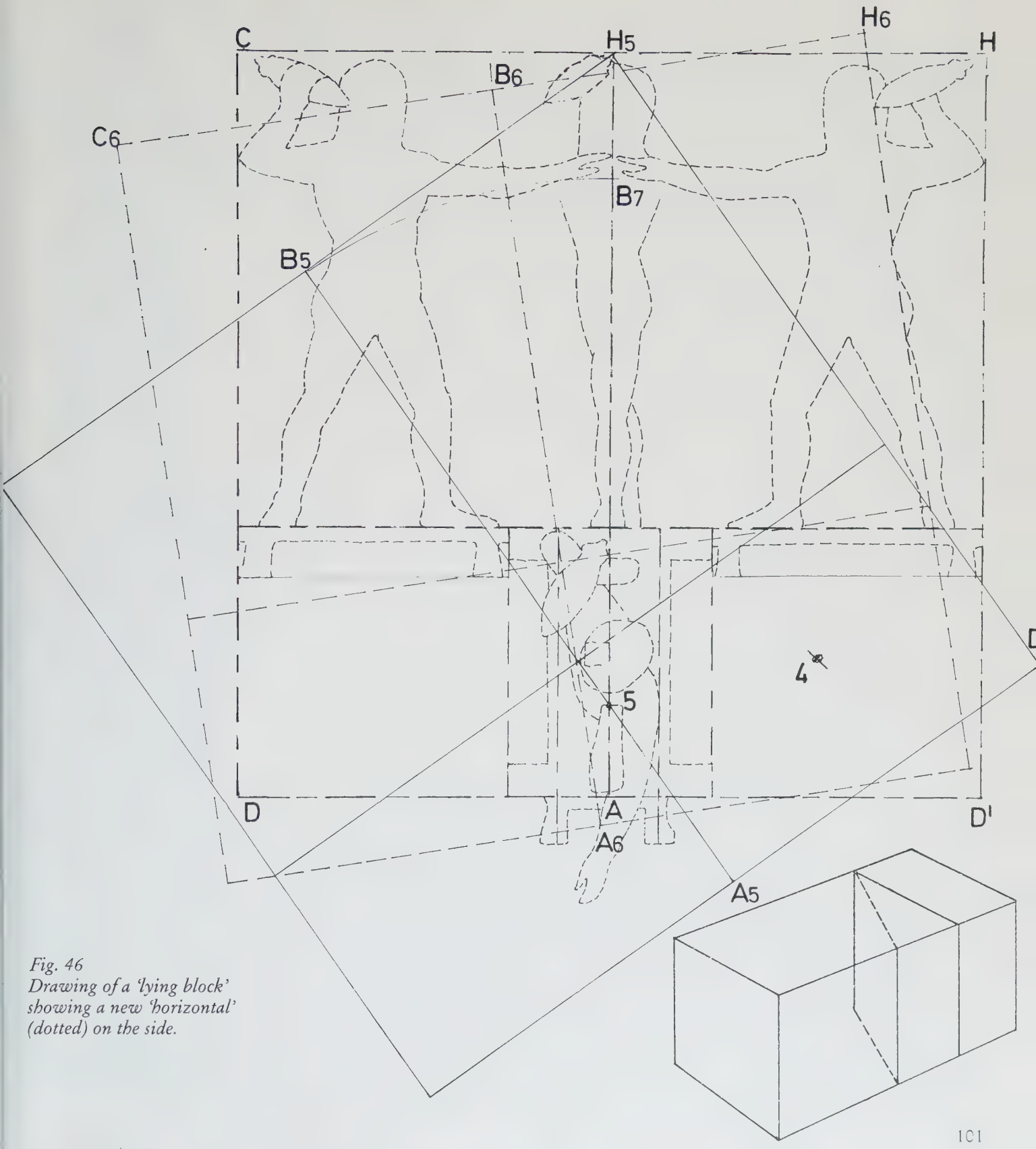
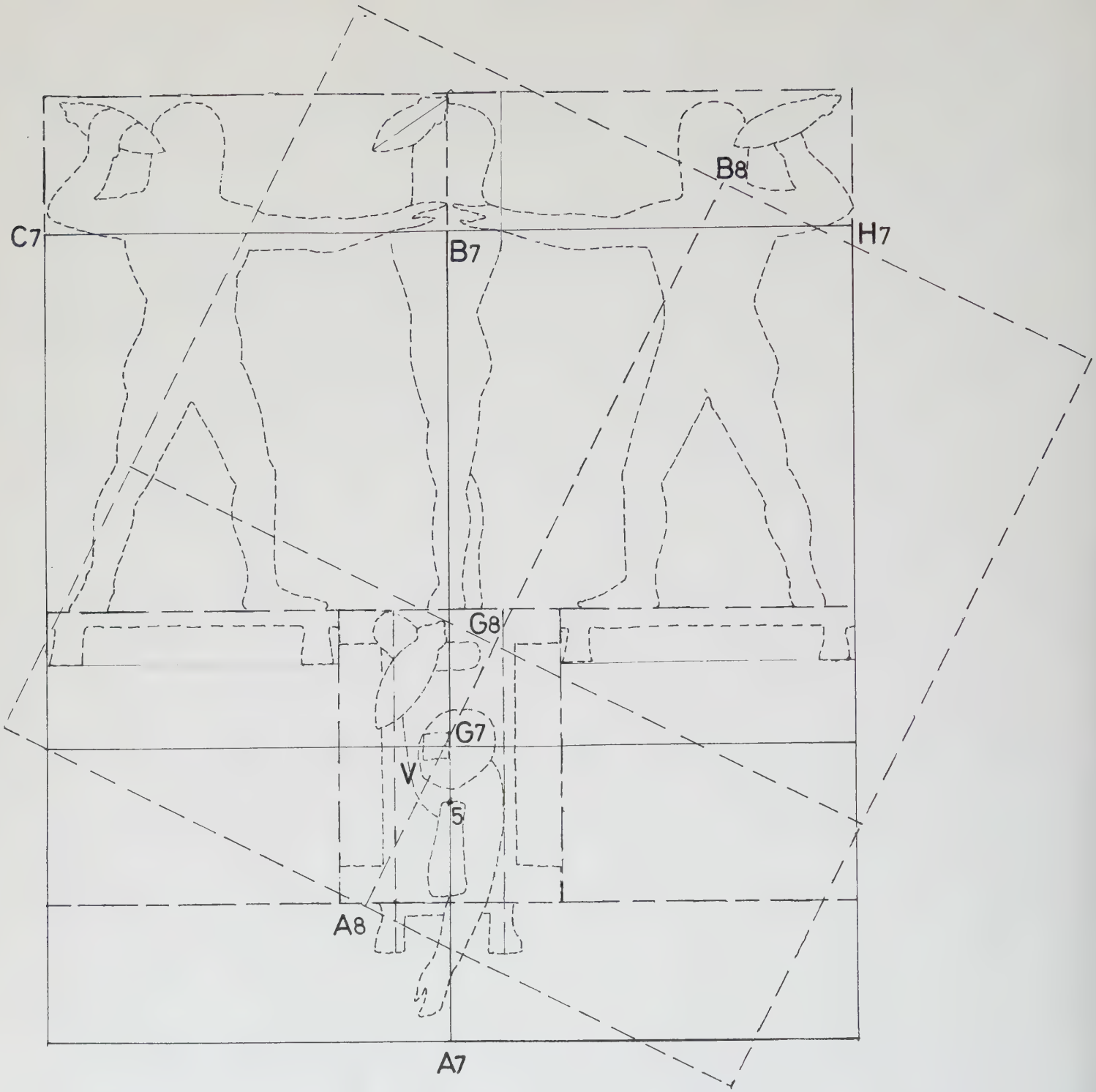


Fig. 46
Drawing of a 'lying block' showing a new 'horizontal' (dotted) on the side.



of the weapon in horizontal projection on the top surface of his rectangular block. This projection is determined on PLATE C, from which the direction of the vertical central plane of the weapon appears, as well as the position of its point. He must have tilted it vertically in the direction indicated by this central plane, the distance from the point to the rear edge of the block remaining unchanged. PLATE B enables to find the inclination of the weapon, seen from the

Fig. 47
Sculpture of 'Young Man in Himation'. Provincial, Attic work. School of Agoracritos. Athens N.M. 199 Total height 1.94 m.

side, and we may assume that the sculptor marked the "horizontal" axis of the thunderbolt on the side of his block at the angle shown on the plate (dotted), which cuts the front of the block in a horizontal line (Fig. 47). This plane is completely analogous to the Egyptians' new, vertical plane, which we shall demonstrate by means of a cardboard box (Fig. 48). *The line of intersection between the oblique* – but *nevertheless vertical – plane of the top surface* (PLATE C), and *the oblique, "horizontal" plane of the side* (PLATE B) *is the axis of the weapon.* This is the *Greek innovation*, a natural consequence of the way of thinking which was their inheritance. It seems so very simple, and yet no one except the Greeks took this step – it must have been inspired by their tremendous need to find ways and means of expressing their new attitude to life.

PLATE XXII A, B, C, D. Statue of Young Man in Himation (Athens N.M. 199). Attic work, school of Agorakritos. About 420 BC. Analysis based on the author's measurements, scale 1:5. Height of figure – 0.90 m, height of the marble base – 1.04 m, height of upper rectangular part of base – 0.305 m, width of this part – 0.45 m, depth of same – 0.305 m, width below – 0.37 m. The width of the base, which is part of the statue, equals the depth of the marble block – 0.305 m. Thus V is the centre of a square whose sides are equal to AG.

The *Young Man in Himation* shows a displacement of weight on to one leg, Fig. 47. We shall investigate the method employed – was it the same as that responsible for the slave of the relief?

The depth of the base is equal to 1/4 of GB. *Thus the height of the figure, GB, and the depth of the plinth, AG, are determined.* The width of the top of the plinth is ½ GB, and it now appears that the height of the wide top of the base is equal to its depth. The centre of the base, seen from above, is V, and around this point we now describe a circle with radius equal to GF (the minor of GB). The lowest point of the circle is C, and the full height of the stele is determined as $2CA' = CG$.

In its original position, we must imagine the figure standing in repose about AB; seen from above its centre is V. The geometrical lines then pivot about point 1, the right end of the horizontal diameter of the circle about V. The extent of this pivoting movement is determined, as *A2B2 is parallel to the dotted axis sloping up to the right, Z'*B. On PLATE B we see how the weight is moved to the right in the same way as that observed in the case of the scythian between Apollo and Marsyas (PLATE XX B). A perpendicular is dropped from B2, and this marks the point of the drapery which hangs heavily down from the hip. It is indicated by an incision in the stone at the lower edge of the drapery, perpendicular to the G2 horizontal. Further we see that the circle about V, in this position, touches the young man's right knee, while R2 marks the raised heel of his right foot, in profile. The arc about B2, with radius equal to the major of GB, apparently traces the lower edge of the drapery, where this lies across the outer edge of his right thigh, seen from in front. Moreover, this arc appears to be connected with the execution of the profiles. (We must, however, make the reservation that the drawing of the profiles must be regarded as no more than sketches based on a few secure measurements, *not* as complete measured drawings. The preparation of a measured drawing would have required a scaffold, and this I was not allowed to use in the galleries (1969). The points which I was able to determine securely include the position of the feet on the top of the base, and also the drapery where it touches this surface.)

The geometrical lines continue to pivot, this time about R2 at the man's right heel. All the lines of PLATE B swing farther up towards the left, *until GB (as G4B4) is parallel to GT, the space axis.* PLATE C makes it quite clear how much we miss the right, upstretched hand of the figure and part of the arm. We must assume that the hand, possibly holding the man's attribute, would have indicated the highest point of the entire geometrical pattern in this position. But in spite of their absence, we feel the presence of the two interconnected positions in this pivoting movement so strongly that a further development of the analysis appears justified. The Φ divisions of G3B3 and of G4B4 are so clearly marked by their arcs that they certainly support this view. *The arc about B3, its radius equal to the major, traces the upper edge of the drapery across the abdomen,* continuing on the left side, where the lower edge of the drapery is determined. The equivalent arc from G4 reaches the shoulder.

We must also note how the feet are placed in relation to the horizontal projection in this position. A final movement, shown on PLATE D, occurs about point 3, the lower left-hand corner of the top of the plinth in position 3, marked by the toes of his right foot. The geometrical lines swing down to the right until the position shown in broken line assumes the same direction as the original position. Now the arc about R5 through G5 is particularly clearly marked by the drapery which hangs obliquely across the man's right thigh. The large circle about V5 is marked across the drapery, which reaches down to the base. Now the figure *really* has moved, for G6B6 lies on the line which we call the "vertical of the weight". A parallel to G5B5, to the left of the circle about V5 and tangential to it, surely indicates the importance of this position for the backward inclination of the upper part of the body. PLATE D shows the *completed* movement. The young man has moved forward and to the right on the base. We have repeated the large arc about B3 (from PLATE C) here, and we gain a true impression of the supreme artistic technique of which this statue is an example — and *one that is typical of the Classical Greek statue.*

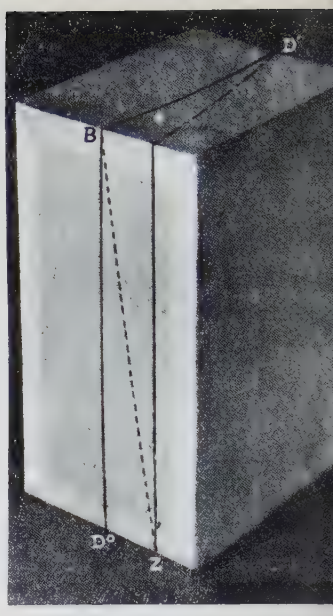
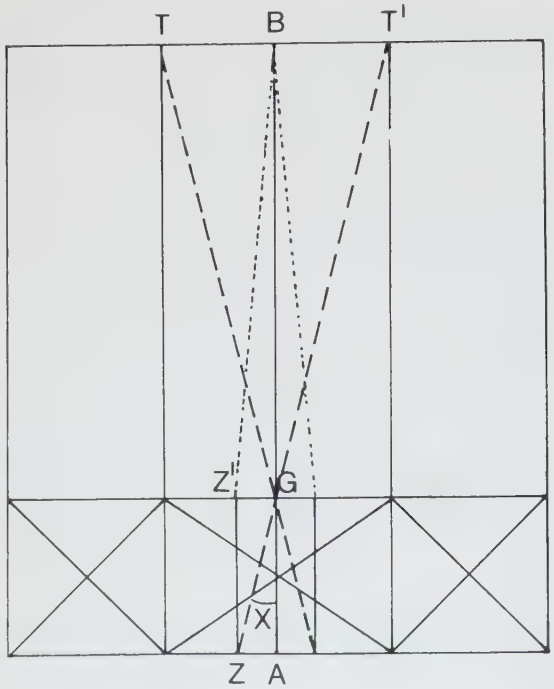


Fig. 48a
Cardboard box cut by two vertical planes.



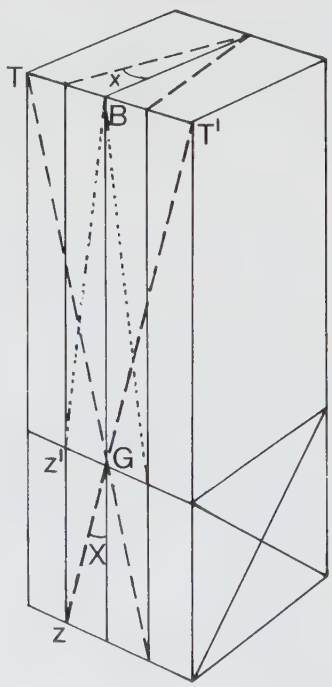
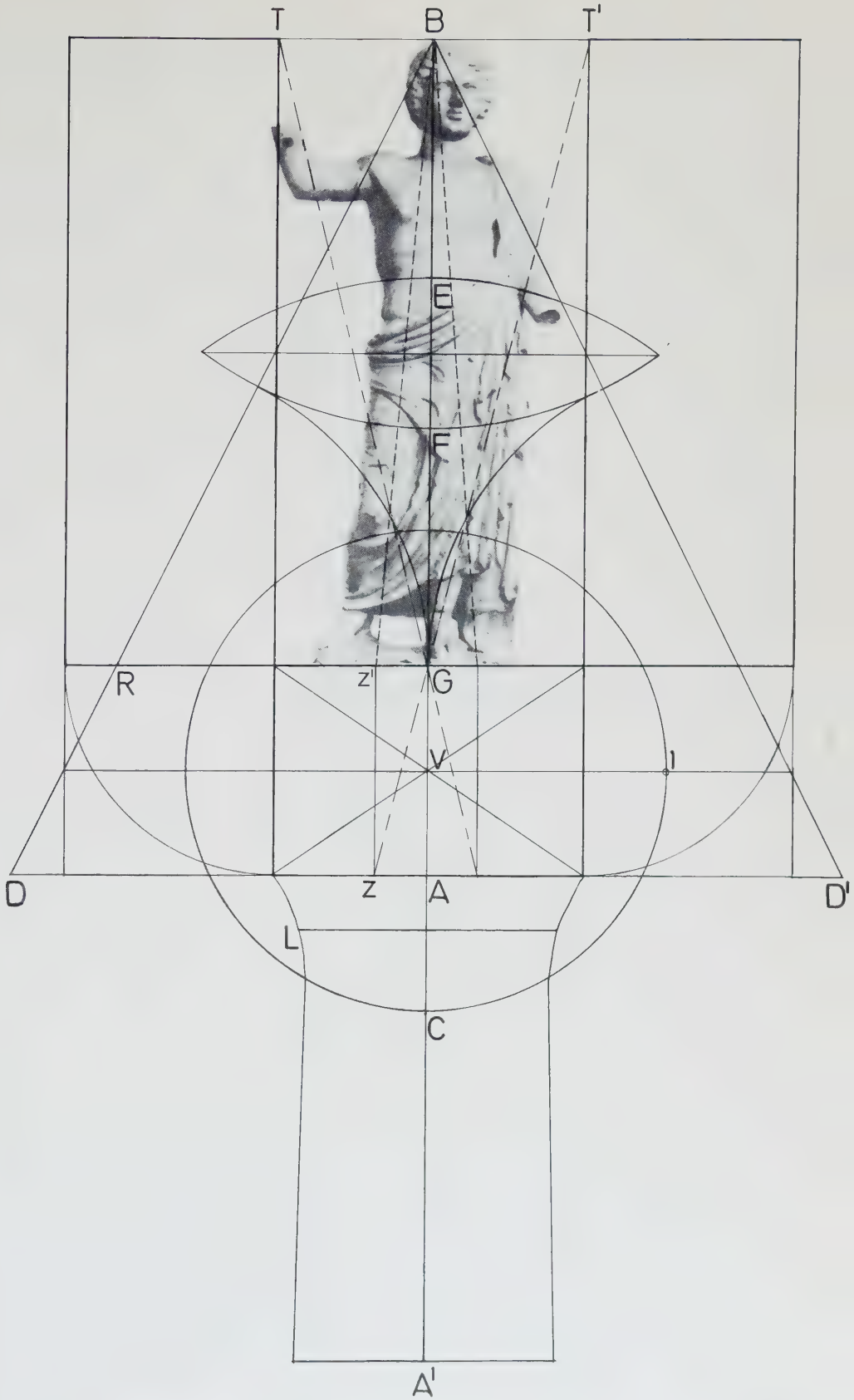
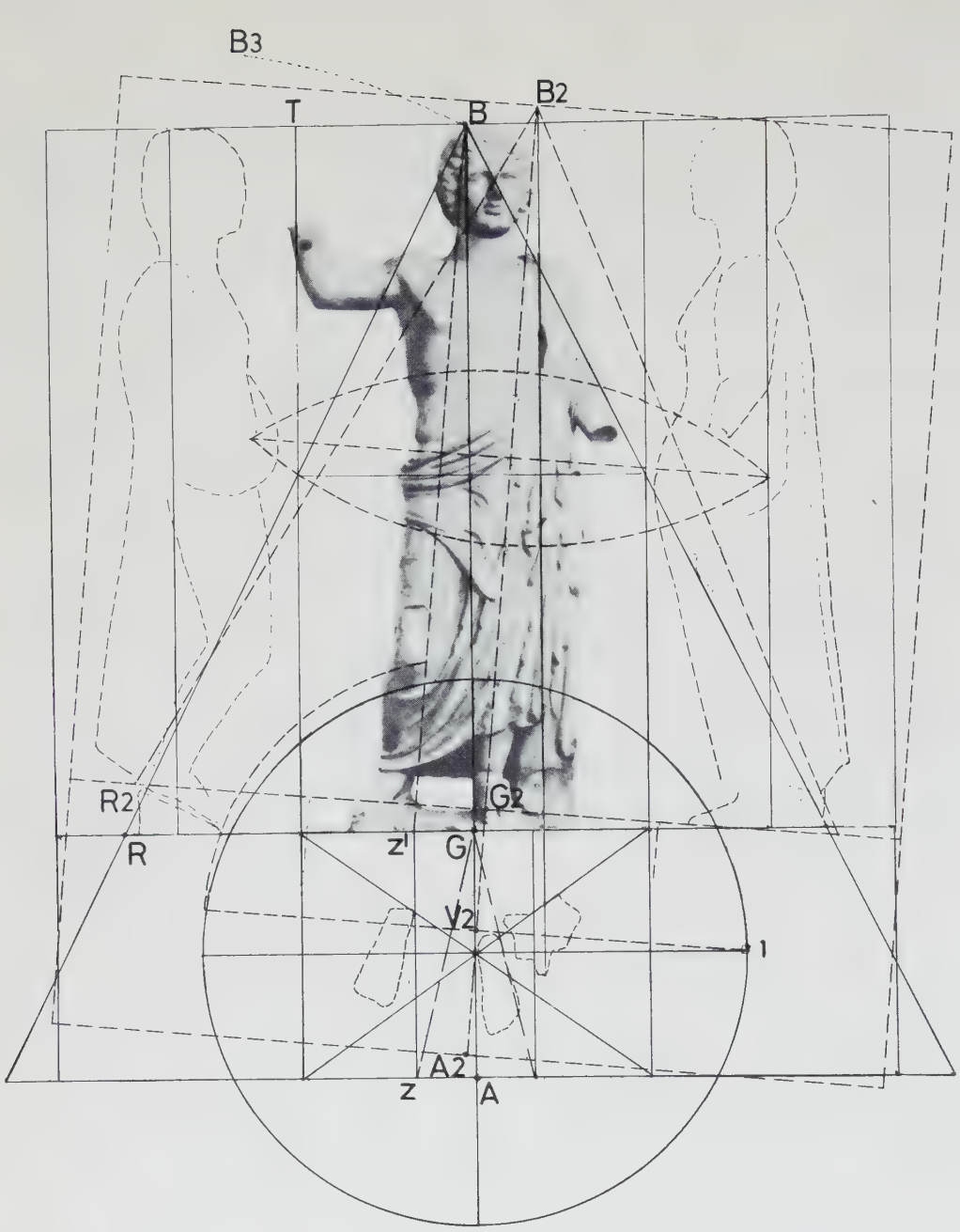


Fig. 48b, c
Block with space axes (broken line) and profile axes (dotted).

PLATE XXII A, B, C, D.
Statue. 'Young Man in Himation'. Athens N.M. 199. Attic work. School of Agorakritos. About 420 B.C. Analysis based partly on the author's measured drawing, scale 1:1, and partly on a photograph, scale 1:5.

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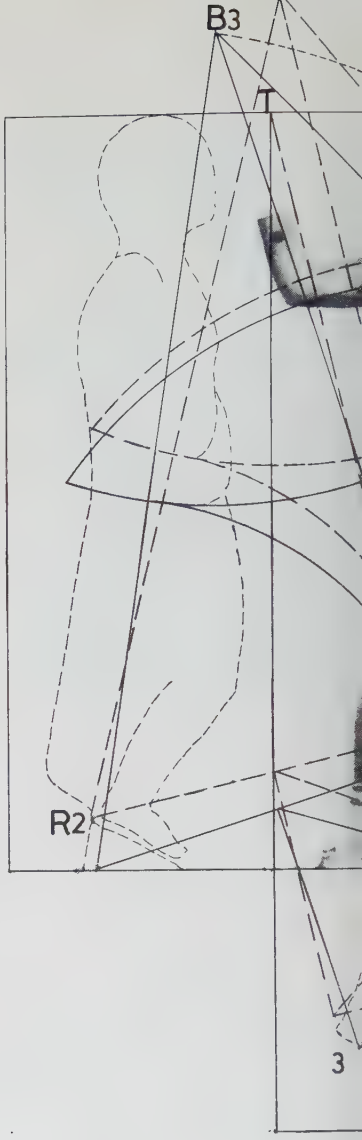
G2
R
z'
G
V2
1
A2
z
A


B3
T
R2
3

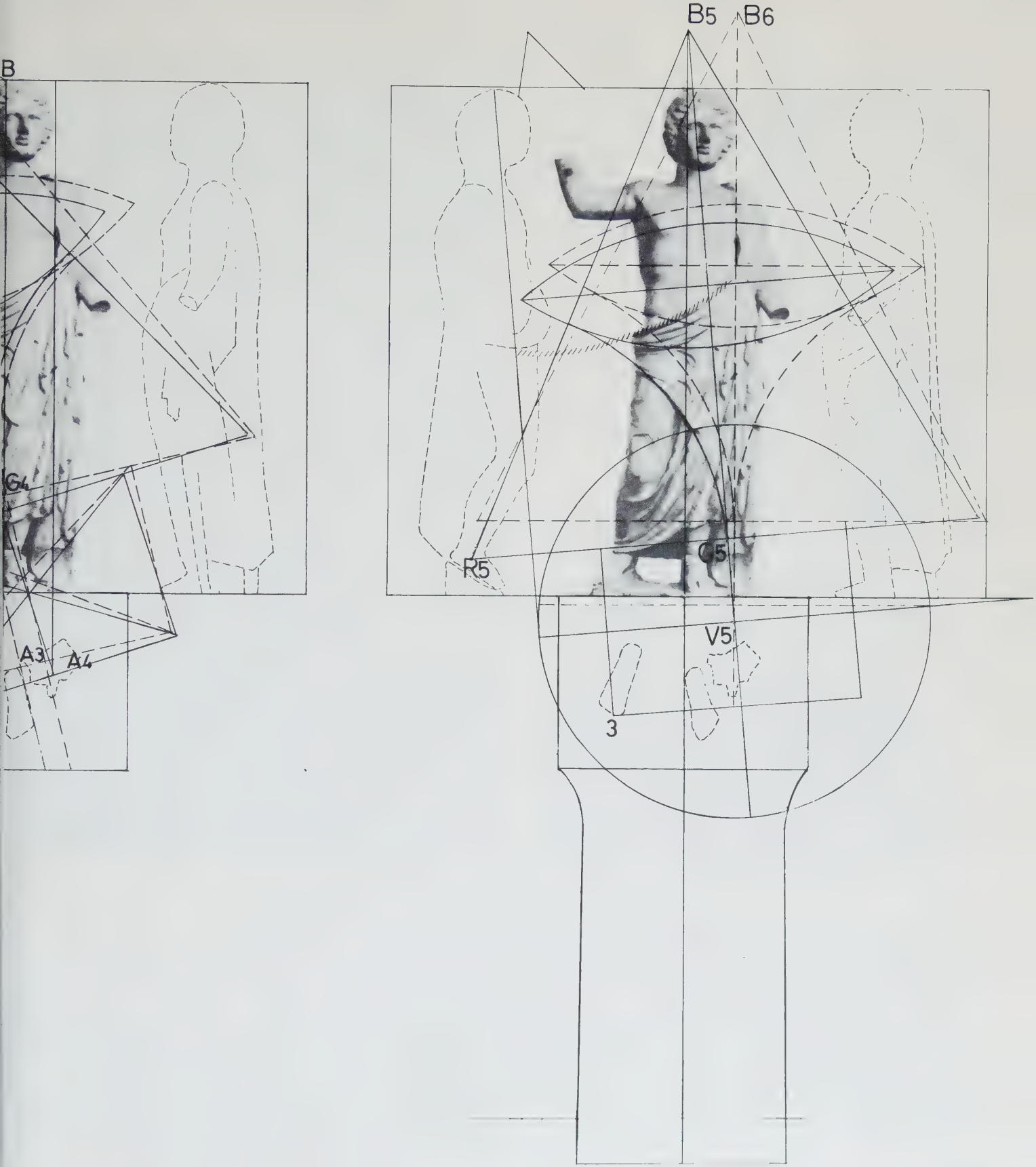

B
G4
A3
A4
B5
B6
R5
G5
V5
3

Fig. 49
'The Lenormant Athena'. Probably from the 1st century A.D. Athens N.M. 128. Total height 0.42 m.

Fig. 50
'Athena Varvakeion'. Copy of 'The Parthenon Athena' by Pheidias, 1/12 the original size. IInd – IIIrd century A.D. Athens N.M. 129. Total height 1.105 m.

The next geometric analysis of this chapter concerns *Athena Varvakeion*, a copy of Pheidias' colossal statue of Athena in the Parthenon. This statue should be studied together with the so-called *Lenormant Athena* (Athens N.M.128), which is known by the name of the scholar who identified it as a statue of Athena. When we compare these two figures, it is obvious that Lenormant's identification is correct (*Fig. 49* and *Fig. 50*). The catalogue of the National Museum of Athens describes this statue as "a modest copy of the Pheidian Athena of the Parthenon". It is not a replica in the strict sense of the word; this Athena and Athena Varvakeion were both inspired by the same model, Pheidias' Parthenon statue. They have certain features in common, of great interest to us in the present context. Both figures show Athena in the same position, resting on her right leg, while her left arm holds the shield; the latter is in both ver-

sions supported by a small stand. Both figures, moreover, hold the right arm in the same position – a *Nike* stands on the right hand of *Athena Varvakeion*, but the *Lenormant Athena* lacks this *Nike* as well as the helmet. A complete analysis of the Lenormant Athena is thus impossible, as the helmet forms part of the height of the figure, which, in turn, is a component of the basic dimension, AB. The unfinished figure measures no more than 42 cm including the plinth, and thus it would have made an excellent object for analysis. Its main features are roughhewn, and the principal planes appear so clearly that it is immediately obvious that this little Athena must be the result of conscious planning. The plinth and the figure are treated as component parts of one unit. Typical, for instance, is the fact that the front of the plinth slopes fairly steeply, thus forming an oblique line running up towards the right, a line surely connected with the long oblique axis of the figure, which passes through the left leg and down into the relief on the front of the plinth. These two lines may perhaps have been perpendicular to each other on the plan drawing on which the design was based? The sculpture appears monumental, in spite of its small scale – this is certainly no "minor" art.

Athena Varvakeion, on the other hand, is in a good enough state of preservation to make an analysis possible; again we shall attempt to trace the method on which our previous analysis of sculpture in the round is based. Pheidias' original was installed in the Parthenon in 447 BC, and this copy (to a scale of 1:12) was made at some time during the second or third century AD, while the original was still in existence, as stated in the catalogue of the museum "a valuable work as an archaeological proof". A copy such as this would have been very difficult, if not impossible, to make unless the sculptor was acquainted with Pheidias' way of thinking, and could follow the same method as the master had employed. We may assume that he first analysed the figure, in roughly the same way as that we shall employ below, but his knowledge of Pheidias' methods must have been far greater and more complete than ours. Considered purely as a work of art, this copy is flat and academic; but it nevertheless provides us with an extremely useful object of study.

This particular analysis we submit with certain reservations. The upper part of the figure has not been measured in detail, a task made impossible by the fact that the use of scaffolding was not permitted in the museum. Nor has the *Nike* (without head) on Athena's right hand been measured in detail, and the same applies to the serpent which fills the space below the shield. But the statue has some very interesting features which we were able to investigate thoroughly, such as the displacement of weight; this is why we have decided to publish the analysis in spite of its shortcomings. Moreover, the plinth and the columns are very interesting. But possibly most important from our point of view is the placing and positioning of the shield. How can this have been determined purely on the basis of the plan drawing? The shield is not parallel to any of the vertical or horizontal planes determined by the original position. Is it possible that methods similar to those explained in the last analysis may underlie this Athena?

PLATE XXIII A, B, C shows the *Athena Varvakeion* (Athens N.M.129) of the third century AD. The copy is to a scale of 1:12 of Pheidias' *Athena in the Parthenon*, 447 BC. The analysis is to a scale of 1:2.

First we shall investigate the dimensions of the statue (PLATE A) based on AB, the sum of the height of the figure, GB, and the depth of the base, AG. AB is divided in Φ at E (above) and at F (below), AM = MB. An arc is described (in dotted line) about F, tangential to the arcs about D and D′ through A. The lowest point of this circle determines G. *Thus the height of the figure, GB, and the greatest depth of the base, AG, are geometrically determined.* The points of intersection between the arcs about D and D′ through A, and the Φ hypotenuses – x and x′ –

The following measurements were taken:
Height of figure (with helmet) 0.940 m
Greatest depth of base, measured along right side 0.320 m
Depth of left side of base 0.275 m
Greatest width of base (ST) 0.400 m
Width at back of base 0.370 m
Height of base 0.110 m
Greatest thickness of shield 0.035 m
Greatest diameter of shield 0.400 m
Greatest diameter of shield without rim 0.340 m
Length of stand 0.180 m
Distance from stand to back of base 0.060 m
Height of stand, at the point where the shield rests on it 0.038 m
Height of column 0.440 m
Diameter of column below uppermost ring 0.040 m
Diameter of column at base level 0.048 m
Width of front of abacus 0.053 m
Width of back of abacus 0.048 m
Length of outer edge of abacus 0.058 m
Length of inner edge of abacus 0.053 m
Height of abacus 0.013 m
Level of Gorgoneion on Athena's breast 0.600 m
Level of belt buckle 0.510 m
Level of waist-line nearest the column 0.460 m
Level of left knee 0.215 m
Height above top of base of uppermost point of shield 0.415 m
Horizontal distance to right edge of base of uppermost point of shield 0.110 m.

determine the O horizontal. OB is divided in Φ at I (above) and K (below), and ON = NB. GP is equal to OI, and the *height of the column is determined.* z and z′ determine *the greatest width of the base and also the greatest diameter of the shield.*

The investigation shows that the sculptor responsible for Athena Varvakeion, like he of the "Young Man in Himation", operated with space axes (shown in broken line), and corresponding profile axes (dotted line), in the true, Egyptian manner, Fig 48. The angle of inclination of *the space axes* is determined by the fact that these axes pass through G, and through y and y′, the points of intersection between the Φ hypotenuses and the arcs about x and x′ through O. The corresponding *profile axes* are determined by C and C′, the points of intersection between the space axes and the A horizontal. They correspond to H and H′ on the G horizontal, the former seen from above, the latter from the front. We also see that an arc about A through C and C′ determines *the height of the base* (A′).

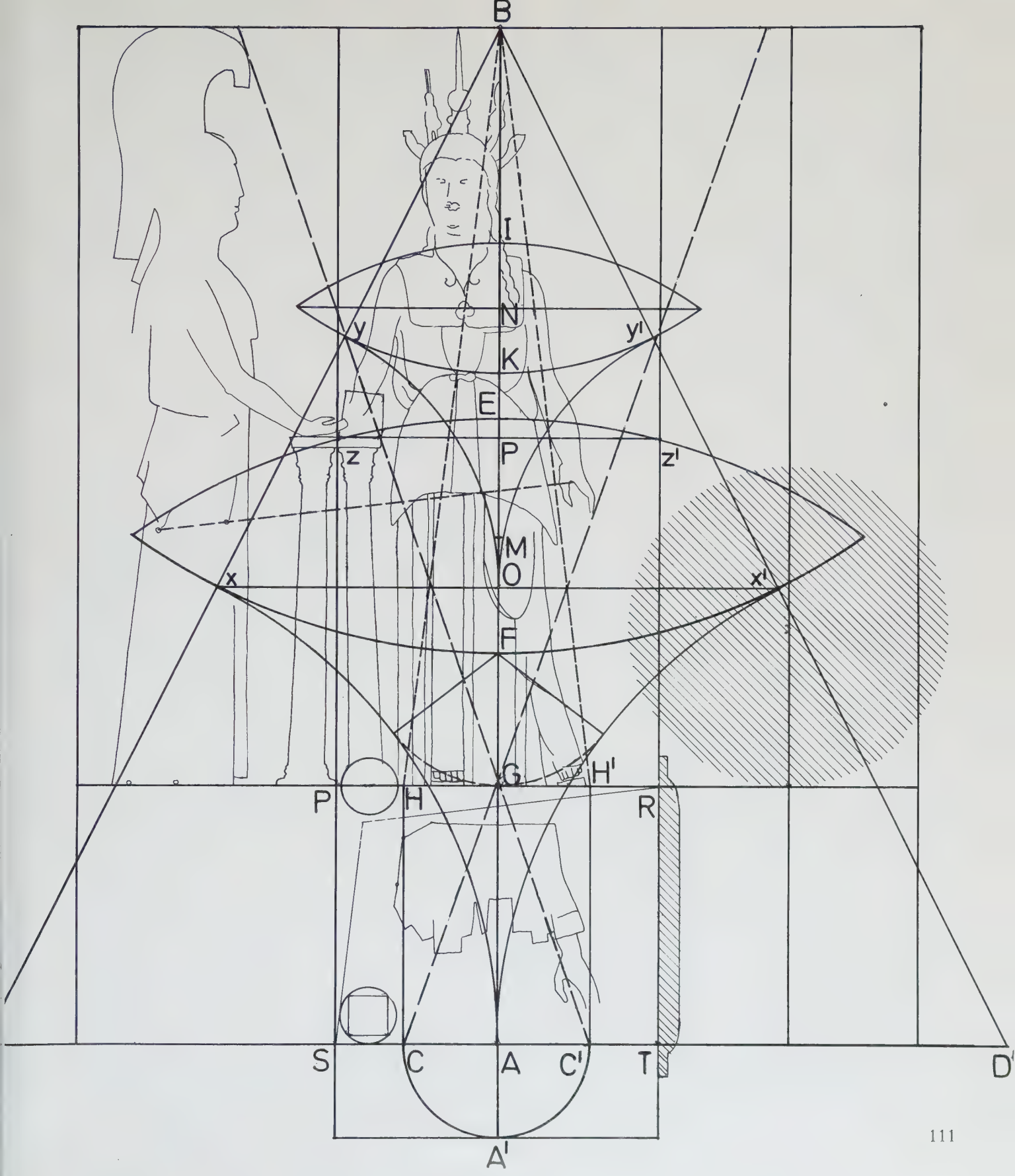
This statue provides evidence of the great importance of the method of placing the different planes of the hypothetical block flush in the picture surface, and also of the determination of space axes and profile axes. The form of the base, seen from above, shows this very clearly. It is far from being rectangular: the front and the right side are at right angles with each other, but the left side forms an acute angle with the front and an obtuse angle with the rear edge, the rear edge slopes up towards the right, meeting the right side on the G horizontal (R). (Classical Greek art has many such irregular bases: for instance, a base found together with the *Charioteer* in Delphi, which was displayed in the same gallery as this statue in the Delphi Museum.) Our analysis revealed that the *left side of the base* of *Athena Varvakeion* is *parallel to the left profile axis, while the rear is perpendicular to the right profile axis.*

It further appears that this irregularity of the base is transmitted to the column and to its capital. Seen from the front, the column slopes to the left; seen from the side, it slopes to the front. On PLATE A, the quadrilateral abacus at the top of the capital is shown in horizontal projection on top of the column. Its front edge is aligned with zz′, the left corner coinciding with z. The length of the front is equal to the side of the inscribed square of the circular cross-section of the column at the level of the A horizontal — and thus the front of the abacus is determined. The column stands at the left front corner of the base, tangential to the front and side edges; but as the side slopes inwards, the centre of the circle is also displaced slightly towards the middle of the statue. This results in a peculiar sensation of movement, or of irregularity, to keep to the term we used above. The left side of the abacus is parallel to the left side edge of the base, and the right edge is parallel to AB — thus the abacus acquires its irregular form. This determination agrees completely with the measurements taken.

We assume that *the shield*, in its original position, was vertical, aligned with the right edge of the base and half-way between the front and the back. This position is drawn as seen from above and from the side. But Athena's shield, just like Zeus' thunderbolt, is not parallel to any of the planes of the rectangular block. The question arises whether there is a connection between the design of the figure and the different geometrical positions indicated by the shield, similar to that established in the case of Zeus with the Thunderbolt, so that the shield is "vertical" in one of these positions. In other words — do the true dimensions of the shield appear in the B and C positions of the geometrical pattern, so that they may be transferred from the plan drawing to the block?

We shall begin by assuming that the geometrical lines first pivot about point 2, the point of intersection between the horizontal centre line of the base seen from above, and the rear edge seen from the left (PLATE B). We are now working with the dimensions of the original rec-

PLATE XXIII A, B, C.
Athena Varvakeion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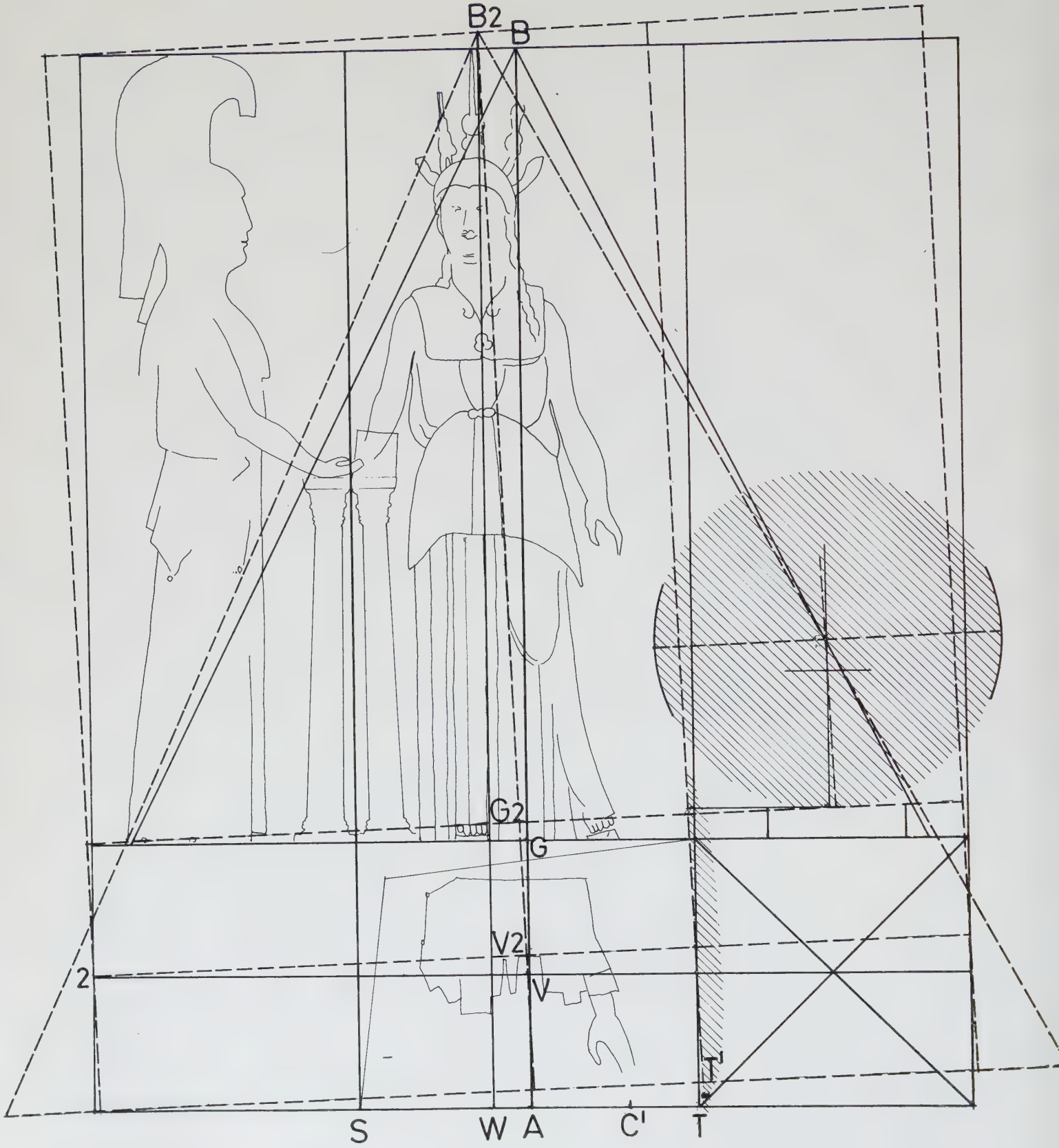
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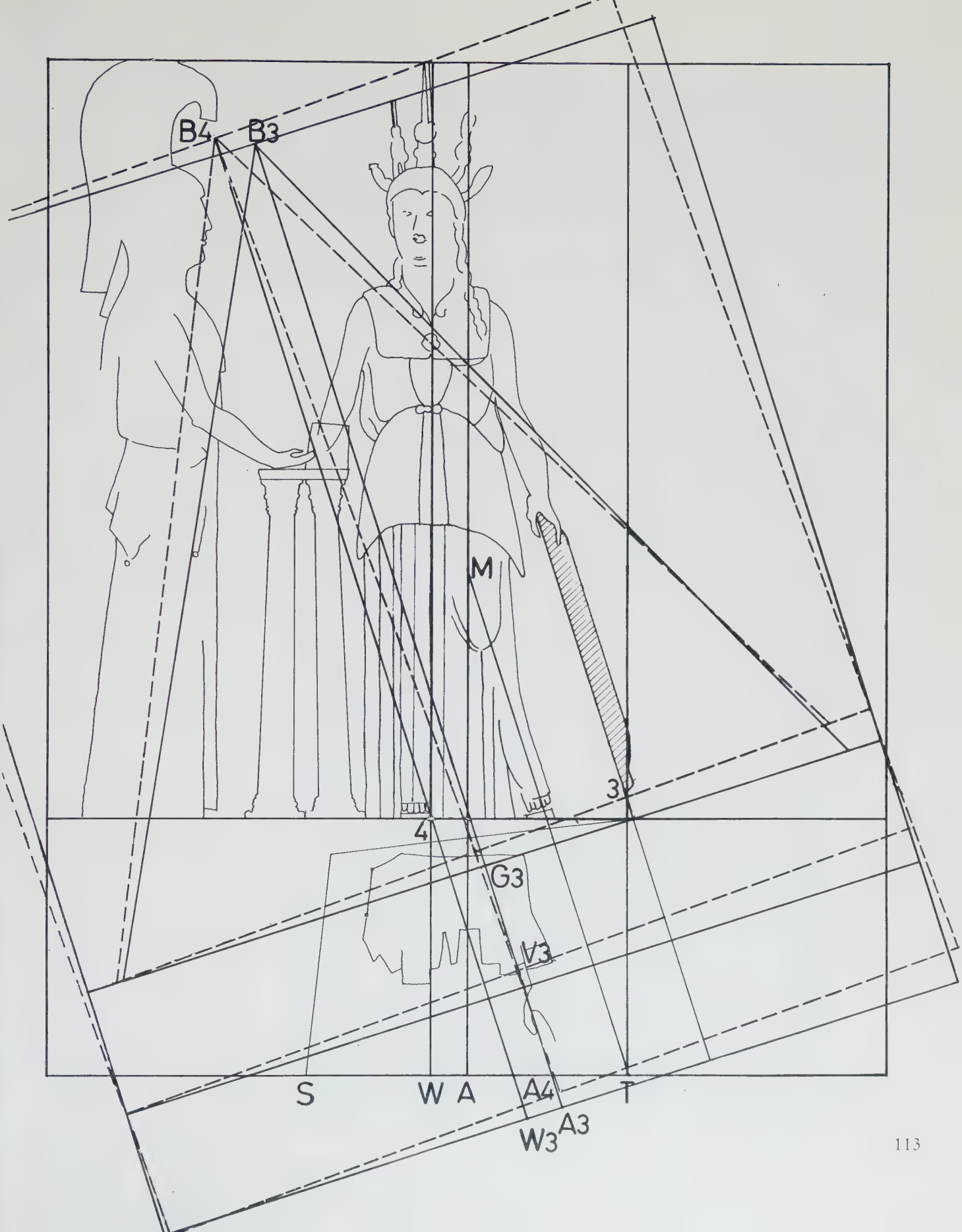
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W3
A3

tangular block, before the sloping sides of the base were hewn. The horizontal centre line swings upwards, resulting in a new "horizontal" position. This pivoting movement is of decisive importance to the shield and to the figure, as we shall soon see. The extent of the movement is geometrically determined by B2 lying on the vertical which is dropped half-way between S and C′ (W). In fact, the weight *moves* to this vertical, which traces the large fold along the inside of the leg bearing the weight of the figure; it is also marked by the centre of the Gorgoneion on Athena's breast. The new "horizontal" through point 2 is marked by the deepest fold of the drapery at the front of the figure, seen from above. In its new position, the G horizontal passes over the feet, seen from the front, but its most important effect appears on the far right: *in this position the shield is lifted into place on its small stand.* The length of this stand is determined on PLATE A, where it is equal to GF. But we see that the stand is closer to the back of the base than to its front – this may be due to the fact that it is the *centre* of the shield, seen from the side, which determines the new position. A perpendicular dropped from this new centre to the dotted "horizontal" determines the centre of the stand.

Now the slight turn of the shield has been determined; this appears most clearly in horizontal projection on our drawing. It is obvious that further movement of the geometric pattern has been executed, but we stop here. The investigations made here should give us an idea of how Greek monumental art was created. We have every reason to believe that the methods discussed on the preceding pages could be retraced in, for instance, the Parthenon sculptures, the masterpieces of Pheidias. On this temple several sculptors worked, Pheidias was not the only one. They were all artists, familiar with the geometric methods whose main features we have tried to elucidate. In addition to the artists, a great number of stonemasons must have been working on the temple. Their craftmanship made it possible for them to carry into effect the intentions, the plans and ideas of the creative artist. We might call to mind our analyses of the Theseion, where the relation between the foot and geometry was discussed. The great sculptures of the temple gables may be regarded as sculptures in the round as well as reliefs, and their connection with the size of the temple building itself is evident. It should be clear that an attempt at an analysis of these gables is impossible. In any case it would be a gigantic task. We must be content with photographs as messengers of this art at its highest level.

Fig. 51a
Olympia. Zeus kidnapping Ganymede.

Fig. 51b, c
Bronze statues of two hoplites, c. 440 B.C

Chapter III
Greek Vases.

Proportions of the vases based on the Φ division of the longest horizontal perimeters.

In a publication from 1944, entitled "*Potter and Painter in Ancient Athens*", J. D. Beazley concludes by saying that up to then the study of the Greek vase had concentrated on the paintings; it was, however, high time that their shapes were also studied. In this connection he mentions the Americans Caskey and Hambidge, who had done valuable pioneer work in the study of the shape of the vases, and also the Swiss, Bloesch: "It will not be enough to note the general proportions and the features of the shape: the eye must become accustomed to perceive minute refinements of curve and line. Then it will be possible not only to write the history of Attic vases from the point of view of the *potters*, but, in the long run, to shed fresh light on the *painters*, with whom they collaborated."

Bloesch does not in any way work with geometry in his study of the development of the shape of vases. Caskey and Hambidge's important contribution, on the other hand, lies in their systematic examination of the geometry of Greek vases. In their publications of 1920 and 1922, these two scholars show that the shape of the vase is determined on a perfectly geometrical basis, depending on the so-called "root-rectangles", which in turn are based on the Φ proportion. However, their work was confined to vertical sections of vases, and they did not even touch on the decoration of the surface. Seeing that the pictorial decoration of Greek vases assumed an importance which makes it impossible to disregard it, it seems unlikely that their theories would lead us to the methods in fact employed by Greeks.

Volume and surface.

In the light of our knowledge of Greek art we expect to find an intimate contact between the proportions of the vase and the design of its decoration. Like a rectangular block, a vase is *a three-dimensional object*; if we are to be consistent, we must assume that the line which we term the basic dimension, AB, must relate to the volume of the object as well as to the surface confining this volume.

Analyses of about seventeen *lekythoi* as well as of a number of vases of other types showed that *the basic dimension, AB, is equal to the greatest horizontal perimeter of the vase*; in the case of *lekythoi*, however, we must make the reservation that AB is equal to the length of the perimeter at the edge between the shoulder and body of the vase, regardless of whether the vase is wider below this point or not.

The method of proportioning revealed by our investigation is obviously extremely practical. Once the basic dimension is determined, contact is established between the dimensions of the

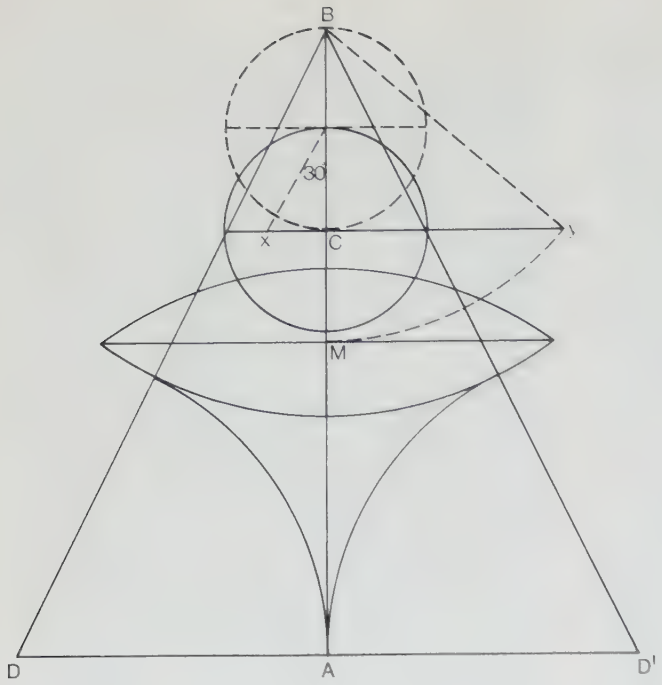


Fig. 52
Construction of the length of the perimeter of a circle, published 1651 by Kochansky.

vertical section and those of the circles representing the vase as seen from above. Random tests show that the Egyptians were also acquainted with this method, a fact not in the least surprising, and clearly proved by three vases from El Anrah, excavated by Flinders Petrie (Oslo Etn. M. 10535, 10531 and 10540).

When planning the design of a vase, it is natural that one should *first decide its greatest diameter.* We know that the Greeks were acquainted with the ratio between diameter and perimeter, and thus were able to calculate the length of the latter *arithmetically.* But our investigations would seem to show that they also knew a *geometrical* method of arriving at the perimeter, a method still in use today (cf. O.P. Arvesen: "Beskrivende geometri", Universitetsforlaget, Oslo 1966). On Fig. 52 the circle about C is repeated about C', here dotted, and an angle of 30° from C' determines x. xy is equal to 3 radii, or ½ perimeter. The arc yM = MA. This method, published in 1685 by the Jesuit *Kochansky,* also occurs in a camouflaged version in the Renaissance vase drawings published in Venice in 1559 by Sebastiano Serlio. It is not impossible that Sebastiano Serlio should have found it in a Greek manuscript; it may, on the other hand, have been handed down by tradition. O. P. Arvesen says that many constructions similar to this one are known, but none of them is as exact as Kochansky's. The construction gives a value of π with four correct decimals: π = 3.14153 . . ., a divergence from the correct value of no more than 0.00006 . . . No difference from the theoretically correct value would be discernible with a radius of less than 60 cm. Of course the longest perimeter may also have been measured.

PLATE XXIV. Ethnographical Museum, Oslo 10540, from El Anrah. We assume that the greatest width of the vase is fixed first, so that its perimeter may be determined, arithmetically or geometrically = AB. The geometrical determination is shown dotted on the plate. As DD' is equal to AB, both these represent the greatest perimeter of the vase. In this vase the greatest width is at level M, the midpoint of AB. AB is divided in Φ in the usual way, E being the upper

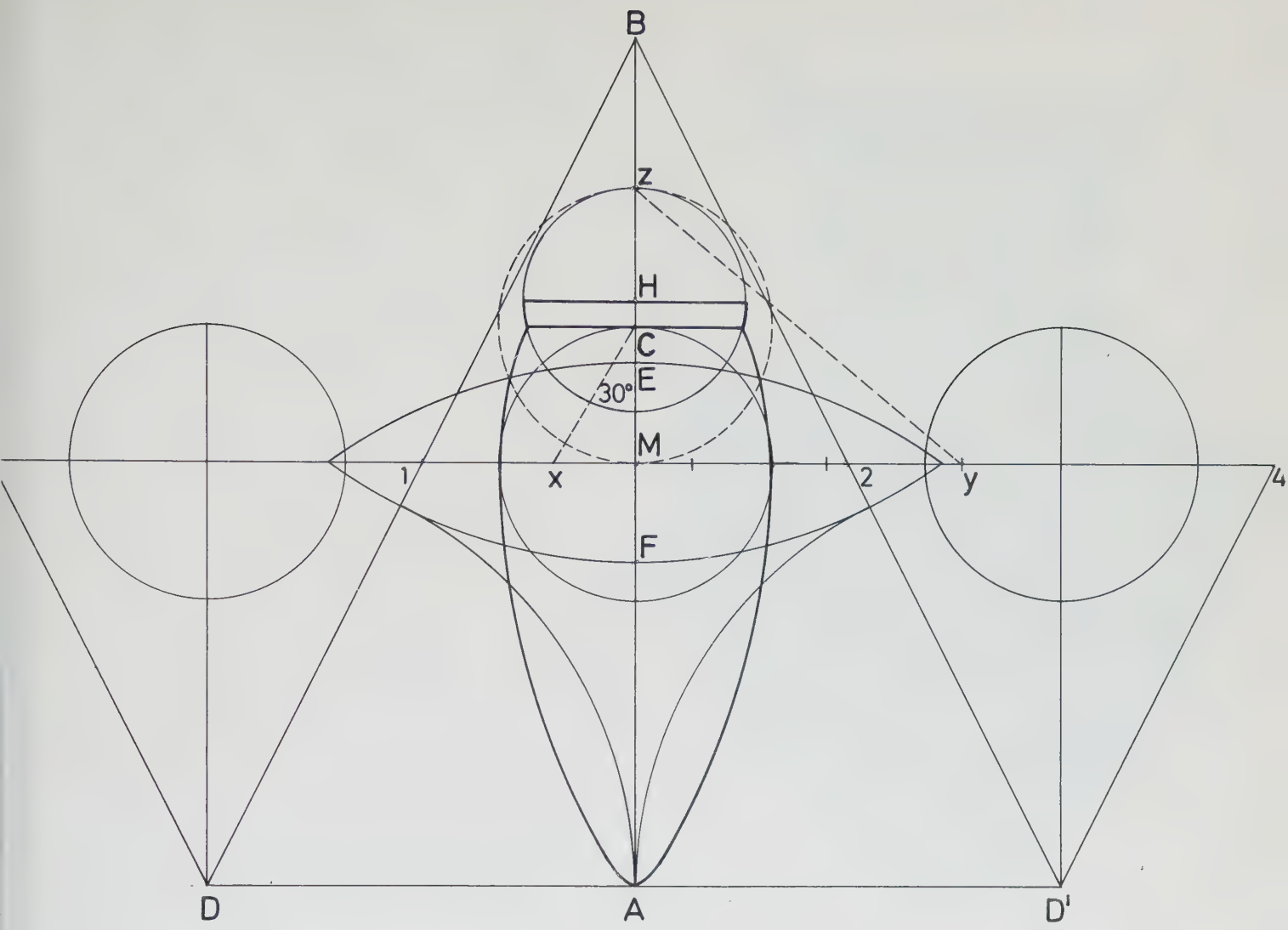


PLATE XXIV
Egyptian vase. Etn.M. Oslo 10540. Excavated by Flinders Petrie. From El Anrah. Analysis based on the author's measured drawing. Scale 1:1.

point of division and F the lower. AH, the height of the vase, is determined, as FH = HB, and the width at level H is equal to 2 HZ. The outline of the vase is described by arcs about points 1, 2, 3, and 4. The two former describe the outline of the vase up to level C, while the arcs from 3 and 4 trace the outline down to the base. Thus the shape of the vase — we may call it boat-shaped — has been geometrically determined. It would seem that the basic principles underlying the design of the shape of vases were fully developed when Greek Geometric vases were first produced.

The great specialist on vases, Gisela M. Richter, who had worked as a potter herself to learn all details of the procedure, states (The Craft of Athenian Pottery, Yale 1933) that a potter can make his product correspond exactly with his carefully planned design: "Modern potters of standing work in the same way today. They first make a drawing of a vase, full size or to scale, and then proceed to follow this drawing in every detail, using rulers and calipers for their guidance. Of course it needs a great deal of skill and patience to be able to work so exactly; but Greek potters, we can be quite sure, had a good supply of both these qualities . . ." She states that the larger vases are best thrown in sections: "While throwing the respective pieces use

Fig. 53
Large Geometric vase.
Etn.M. Oslo 40153.
Height 69.6 cm.

AH 69.6, diameter H 32.5
AR 64.0, diameter R 26.7
AT 53.5, diameter T 25.0
AP 47.5, diameter P 27.0
AS 43.5, diameter S 37.4
AO 41.5, diameter O 41.0
AC 32.5 – 33.3, diameter C 46.5
AK 28.0 – 29.5, diameter K 45.4
AL 21.0 – 22.0, diameter L 40.4
AV 7.0, diameter V 24.0
AU 1.4, diameter U 13.8
, diameter A 14.0

must be made of measuring sticks and calipers to obtain the right heights and diameters. The measurements should be those of the soft clay, which will of course be slightly larger than those of the final shape. This shrinkage will naturally be proportional: so that the relation of every part to the whole will be same in the fired vase as in the thrown product."

PLATE XXV A, B. We start our series of analyses of Greek vases with a large Geometric vase, now in the Ethnographical Museum in Oslo (No. 40153). *Fig. 53.* The analysis was carried out to half scale. The following measurements have been taken (in cm):

The greatest perimeter of the vase (at height C) is marked off from A to B (off the plate). AB is divided in Φ at F (below), and *the height (C) of the greatest diameter of the vase is determined by the circle about C, which passes through F.* This circle depends from F. AM = MB, and AM is divided in Φ at E and K. AN = NM, and a circle about C with radius equal to MN determines H, the height of the vase. An identical circle struck about N determines the dia-

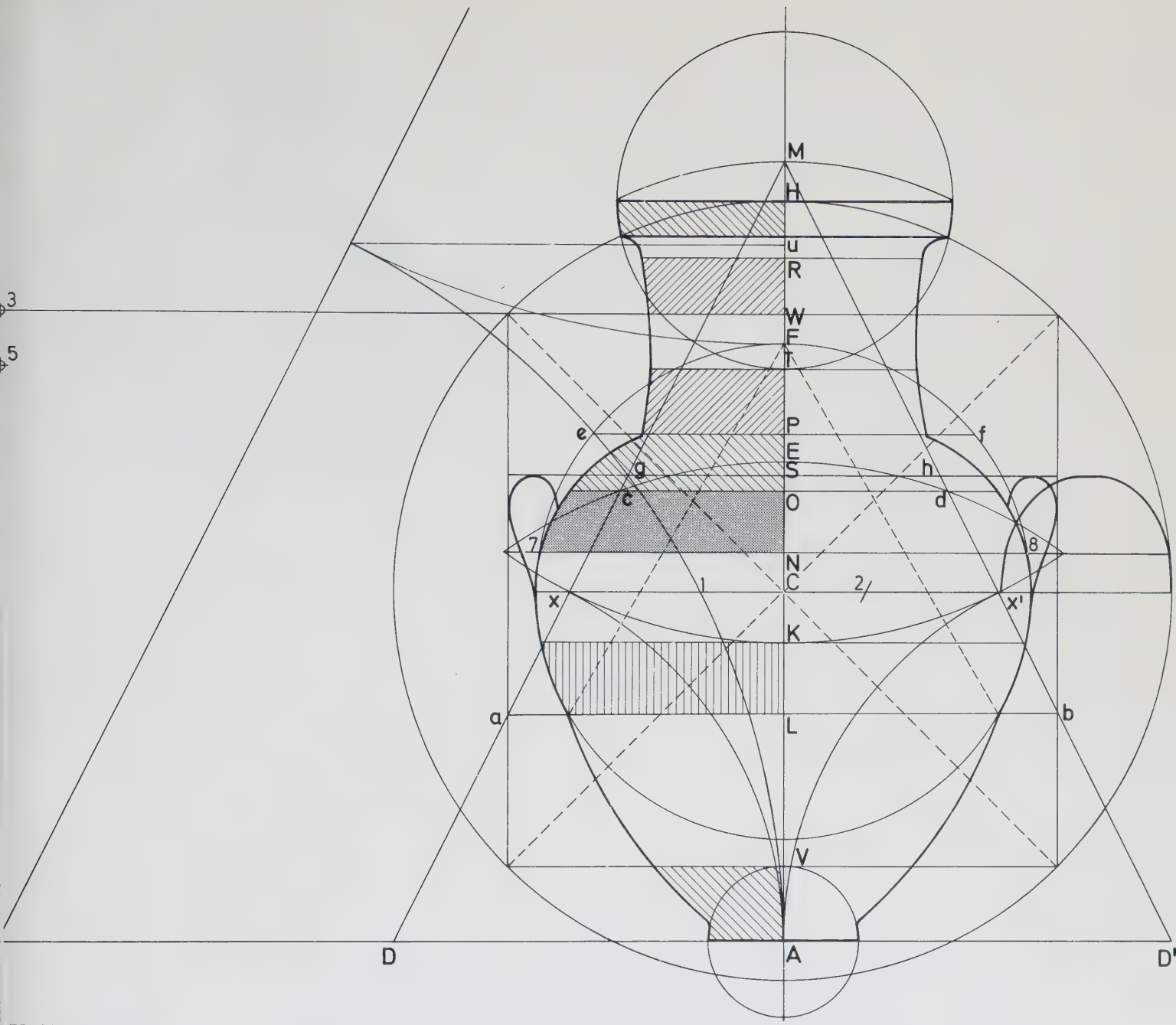


PLATE XXV
Large Geometric vase. Etn.M. Oslo 40153. Analysis based on the author's measured drawing. Scale 1:1.

meter at level H, and the horizontal projection of the greatest width of the mouth of the vase can now be drawn. The horizontal tangent below this circle, at T, is marked on the surface of the vase by the lower edge of the light band across the neck. The upper edge of this band is marked by the upper horizontal through W of the inscribed square of the circle about C through H, while its lower horizontal marks the dividing line between the dark and the light zone, just above the foot, at V. AV is equal to the radius of the foot. The horizontals through L (a – b), K, (lower point of Φ division of AM), O (c – d), and P (e – f) are marked, as is also R, which lies half-way between H and W, the top of the square.

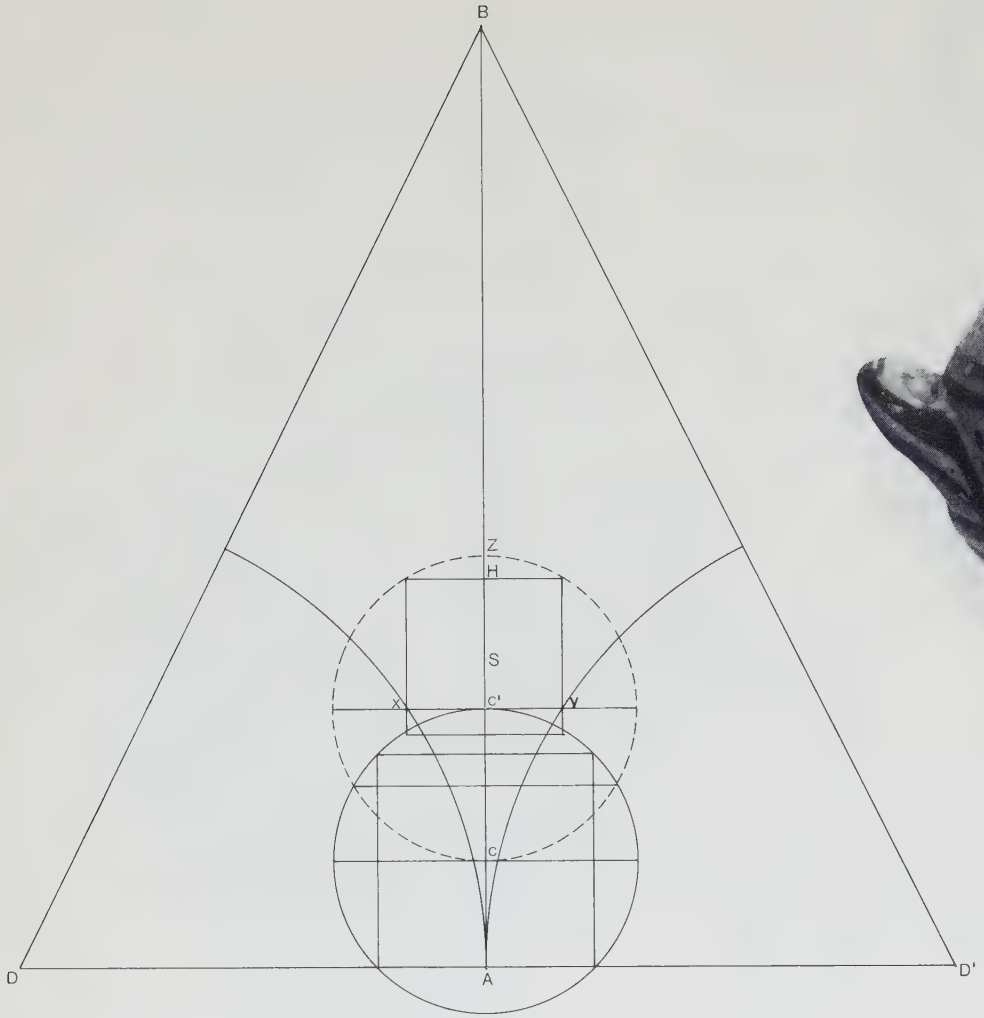


Fig. 54
Small Geometric vase. Etn.M. Oslo 6914. Height 9.4 cm.

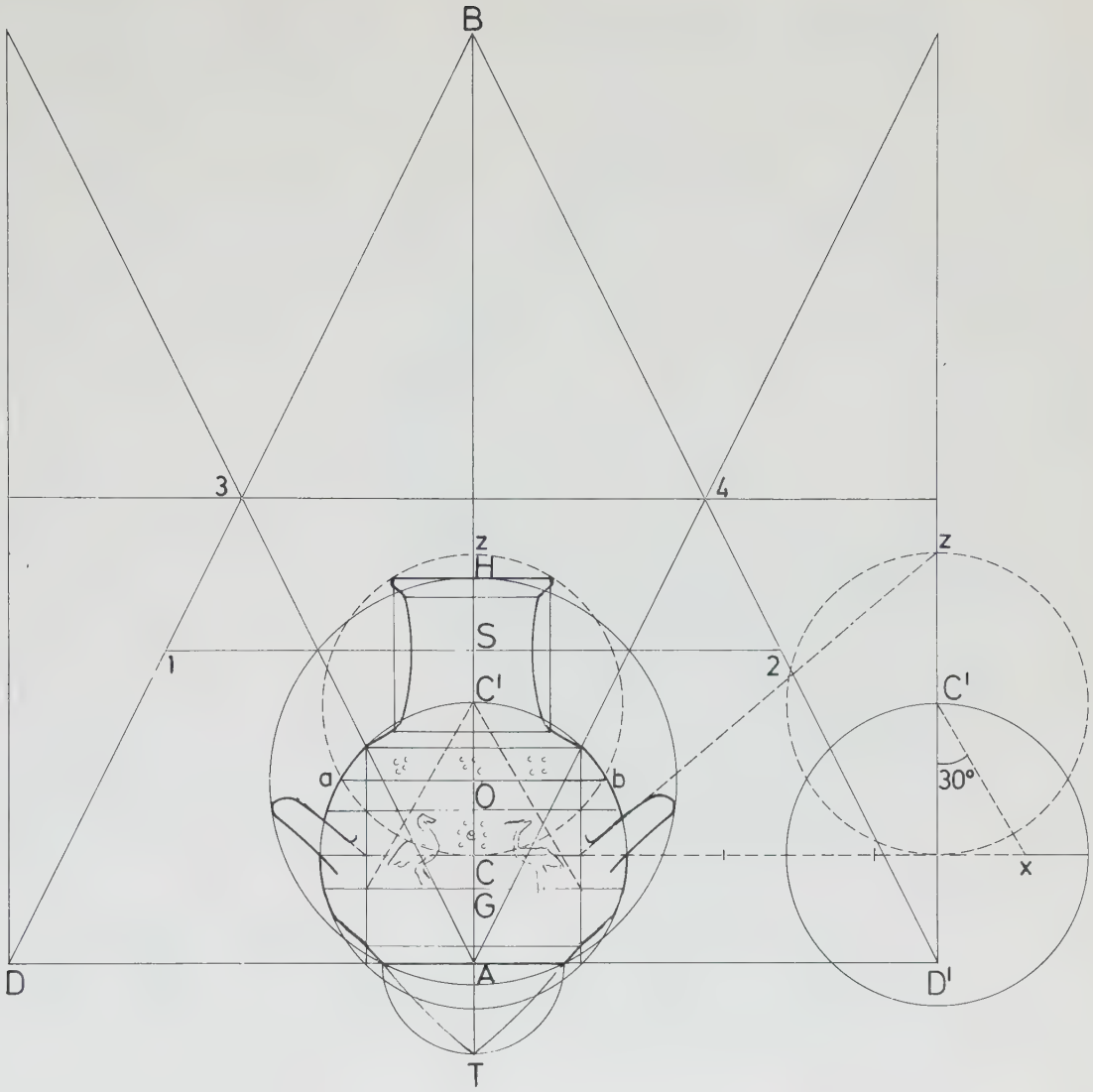
PLATE XXVIII A,B.
Small Geometric vase. Etn.M. Oslo 6914. Analysis based on the author's measured drawing. Scale 1:1.

The geometrical pattern determines not only these dimensions, but also the outline of the vase. This follows the circle about C from level C to level L. Between level C and level P an arc is struck about 1, between level P and level T an arc is struck about 3, between level T and level U an arc is struck about 5, points 3 and 5 lying at a distance from AM equal to AM on the horizontals through W and T.

Arcs about 7 and 8 on the N horizontal trace the outline of the vase from level L down to the foot, whose height is determined by their points of intersection with the circle of the foot.

The vertical sides of the square centre C through W are tangential to the outer edge of the two handles, seen from the front; the greatest width of the handles (seen from the side) is equal to the distance from point x left on the C horizontal to the large circle about C. The S horizontal, determined by g and h, is tangential to the top of the handles.

PLATE XXVI A, B. Subsequent analyses of other vases, like that of *a small Geometric vase,* height 9.4 cm, diameter at C 7.3 cm, also in the collection of the Ethnographical Museum in Oslo (No. 6914), would seem to show that the construction of the perimeter was known. It is, in fact, of decisive importance for the proportions of this vase and for the simple decoration of its surface. The analysis is based on a measured drawing, where all measurements were taken from the vase, full scale, and the outline was checked by a templet. *Fig. 54.*



We shall concentrate on the main proportions based on AB, which is equal to the greatest perimeter of the vase. The circle about C is described, and a square is inscribed in this circle. Its lower horizontal side is aligned with the A horizontal, and *thus level C, the height of the greatest width of the vase is determined.* Vertical lines are constructed through the intersection points of the arcs about D and D' through A and the C' horizontal, x and y. These verticals cut the upper part the dotted circle about C' and the *height of the vase, AH, is determined.* The chord through H defines the *greatest width of the mouth.* At the same time the chord is the upper side of a square whose lower side is marked by the *dividing line between the painted black neck of the vase and the reddish ware of the body.* Thus the vase is determined by two squares, the square of the body, and the square of the neck.

On PLATE B the intersection points of the circle about C and the dotted circle about C' a and b, are marked, and ab determines the width of the vase at that level (O), while the lines is marked by the painted ring on the body, as shown on *Fig. 54.* A circle is constructed about O

through H, and it cuts the A horizontal, determining the width of the vase at level A. The circle also encloses the handles. We have shown the construction of the perimeter about the vertical through D′. The line from Z′ down to the left, marking half the perimeter, is tangential to the inner side of the handle, and on the vase the two dotted lines from C′, marking two 30° angles, cut the vertical sides of the body's square, defining the base line of the picture panel, G. The outline of the vase is completely dependent on the geometrical pattern. The neck is traced by arcs about 1 and 2, through the closer of the two lower corners of the square of the neck, while arcs about the same points, enclose the vase at level A and trace the outline until the vertical sides of the body's square are cut. From this level the outline is drawn by arcs about 3 and 4 until they meet the circle about C. The outline then follows the circle about C to the upper corners of the square of the body. The outline between these corners and the corresponding lower corners of the neck appears to be traced by arcs about 3 and 4.

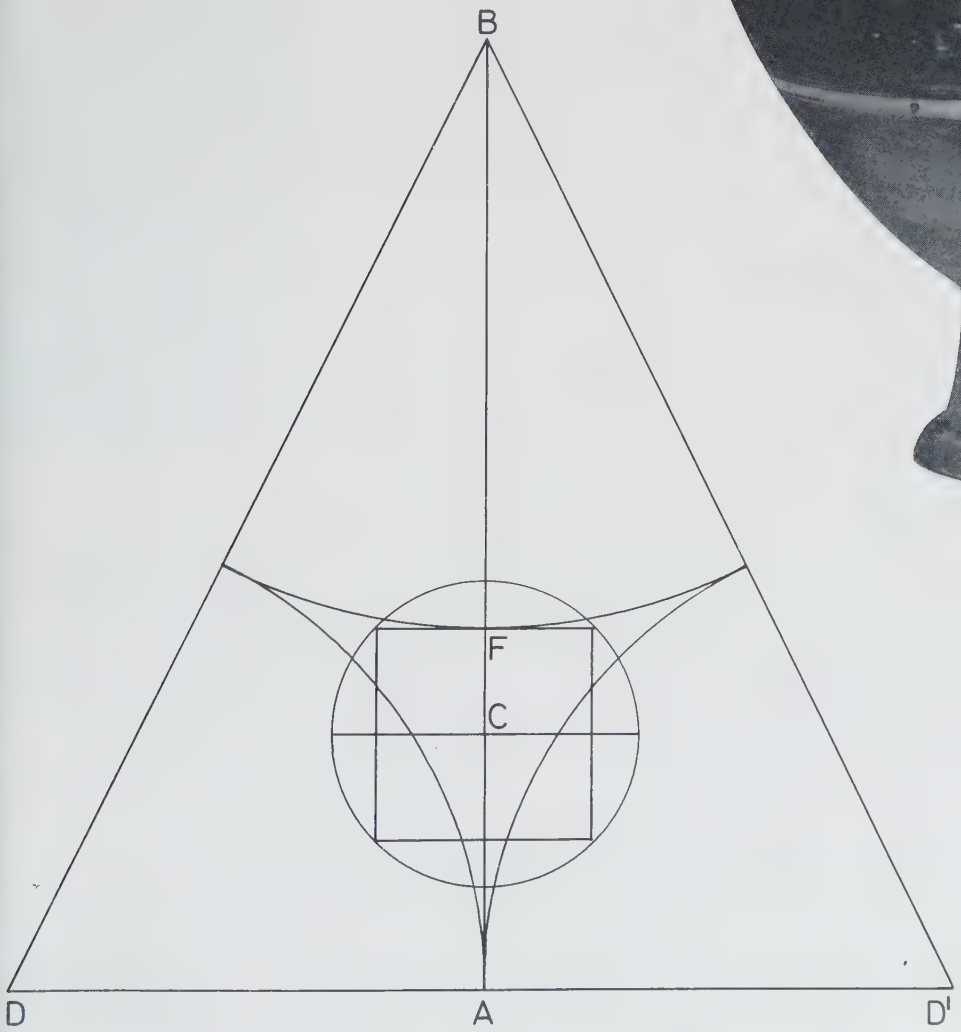
The various types of Greek vases are all a result of different geometrical combinations being employed, all based on AB. We may note, for instance, that the miniature hydria just examined has one feature in common with the large *Cycladic amphora in Stockholm (Stockholm N.M.1)*, a work of the highest quality, PLATE XXVII, Fig. 55; in both cases, AC is determined in terms of the inscribed square of the circle about C. In the case of the hydria, the lower side of the square marked the base line of the vase — for the Stockholm amphora, *the upper side of the square* is decisive, for it depends from F, the lower point of Φ division of AB. Thus level C is determined.

The peculiar shape of this amphora, apparently without any practical function, is a result of this construction. On the large Geometric vase in Oslo (PLATE XXVII) it is the *circle* about C which depends from F. The combination of possibilities is legion, and therefore *we have never found identical vases*, only more or less similar specimens of different types. An examination of a series of one type would be interesting — we shall attempt such an examination of a number of lekythoi.

The simplicity of construction is almost incredible; by means of models we were able to investigate whether the geometrical lines are equally important to the decoration of vases also at a later date, when vase painting was mainly concerned with depicting living creatures, human figures and animals. Is Greek *vase painting* of periods later than the Geometric also based on the geometry of the vase?

Fig. 55
Large Cycladic amphora. Stockholm N.M. 1. Measurements given by the museum: height 58.8 cm, width 38.5 cm.

PLATE XXVII
Large Cycladic amphora. N.M. Stockholm 1. Main proportions. Analysis based on the author's measured drawing.



Painted decoration of the lekythoi. Motion in relation to the static repose of the vase.

The transition from the geometrical division and ornamentation of the surface to simple pictorial decoration offers a wealth of interesting material. From our point of view the most exciting point of the development occurs at the stage when the decorations *ceased to be purely static.* Our analyses of a number of lekythoi will explain this phrase better than words can. Lekythoi being almost cylindrical in shape, one may imagine the picture on the side being "rolled off" like that of a Babylonian cylinder seal, where the perimeter of the cylinder becomes the length of the impression. By analogy *the edge* between the body and the shoulder of a lekythos becomes the horizontal length of the side when the surface is shown flush into the picture plane. Normally the picture on a lekythos covers only part of the surface, frequently bounded by a border just below the edge. But we shall see that the border and the picture may be clearly displaced in relation to one another, and each of them may also be displaced in relation to the handle and to the decoration of the shoulder. In this way, the painted decoration is released from the static repose of the vase. *Fig. 56* is an example of this: it is a lekythos with black figures of a maenad with krotalai between two silens c. 500 – 490 BC, (Etn.M. Oslo 11074), by the Gela Painter.

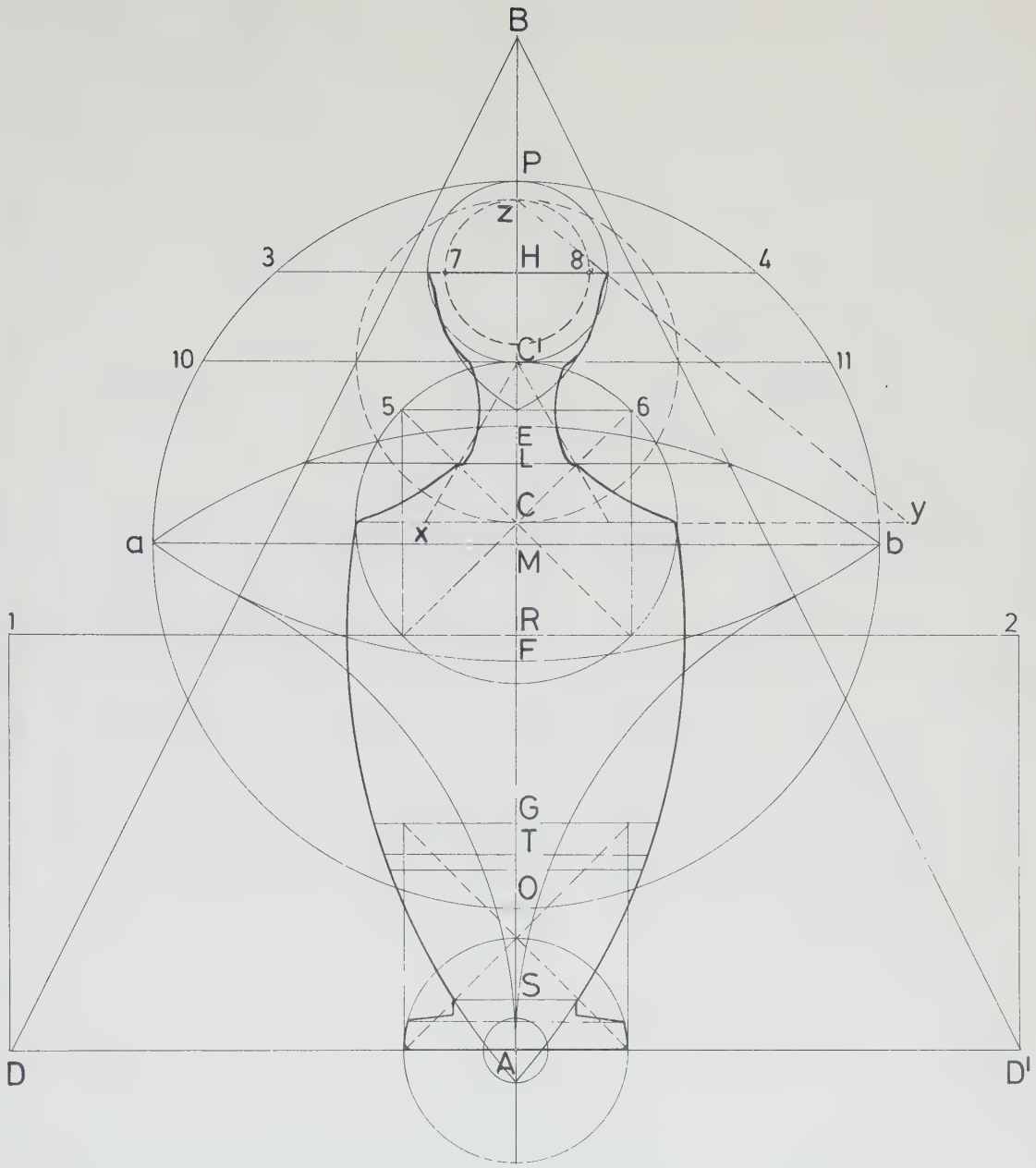
PLATE XXVIII A, B, C, measured by the author, the outline checked by a templet. Scale of analysis 1:1.

The geometry of the vase is very simple and clear: the perimeter at the edge between the body and the shoulder has been laid out and zy = AM = MB. Level C is determined, as zB is equal to the radius of the circle about C, and thus BC is equal to three times this radius. A square is inscribed in the circle about C, and it is repeated so that its lower horizontal side coincides with the A horizontal. The upper horizontal of the latter square determines the *base line* (G) of the painted scene. AB is divided in Φ at E and F, and points of intersection of the E arc and the Φ hypotenuses determine the L horizontal, the line of demarcation between the neck and the shoulder. The width at this level is determined by the points of intersection of the L horizontal and the diagonals of the square about C. The Φ arcs through E and F cut each other at a and b, and a circle centre M is described through these points. The vertex of this circle is P, and the height of the vase, H, is determined, as C′H = HP. The circle about H, diameter PC′, marks the horizontal projection of the greatest width of the mouth of the vase. The dotted circle about H through z marks the width of the opening at H. The lower part of the body of the vase is painted dark up to O, and AO is equal to the greatest diameter of the vase at H. The lower edge of the dark band (T) below the base line of the picture panel is determined, as TF is equal to TA. The body is widest at level R, and 1 and 2 are the centres of the arcs describing the outline of the body from level C down to the point where they meet the diagonals of the square bisected by AG. Here level S is determined. The two arcs about 1 and 2 intersect below A, at a distance from A equal to the height of the foot.

3 and 4, the points of intersection between the H horizontal and the large circle centre M, form the centres of the arcs describing the outline of the shoulder from level C to level L. The

Fig. 56
Lekythos, black figure. Left side. 'Maenad with Krotalai between two Silens'. By the Gela painter. Etn.M. Oslo 11074.

Fig. 57
Lekythos, black figure. Right side.



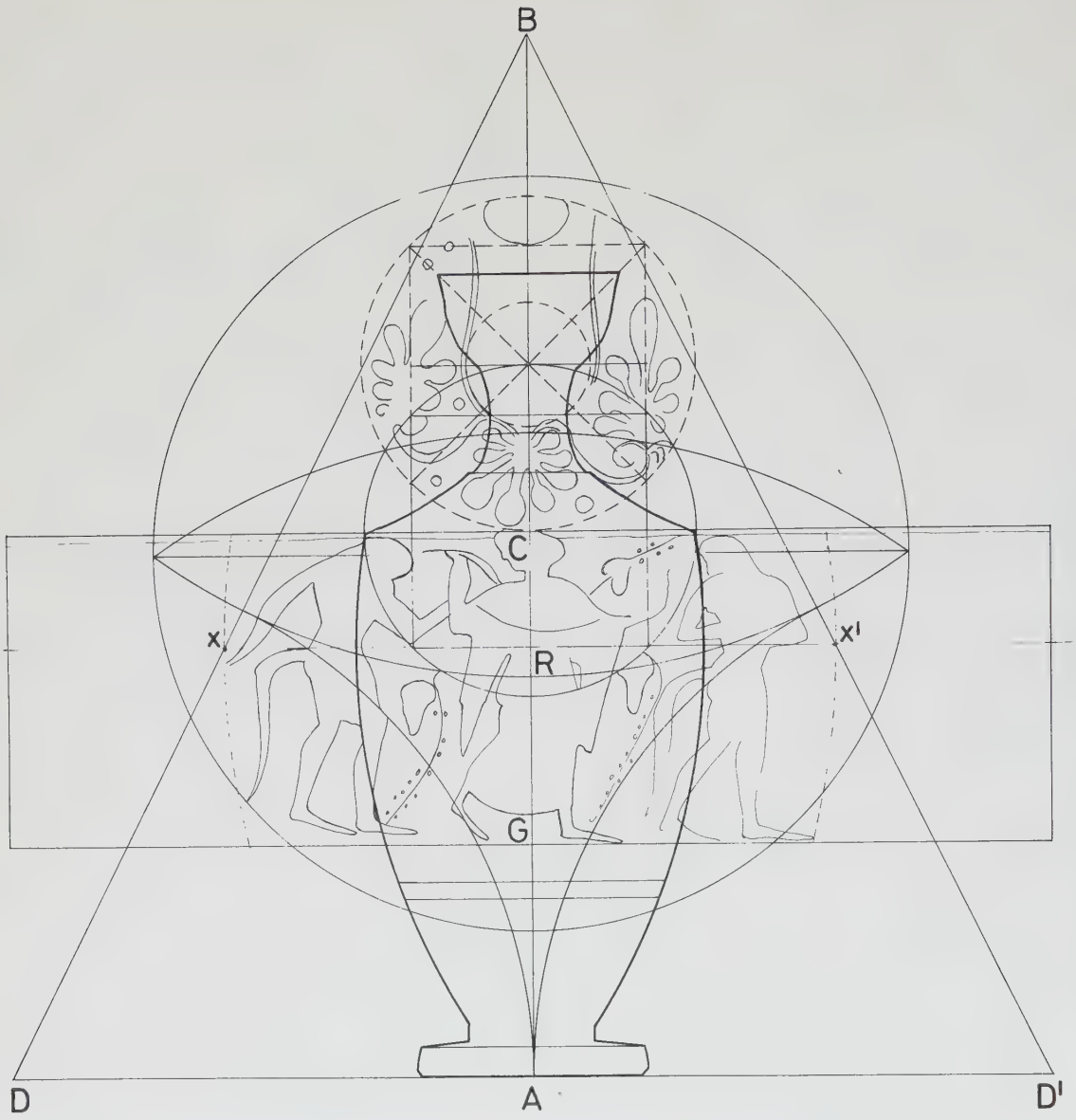
The following measures were taken:

Height AH 19.5 cm, diameter at H, external 4.6 cm, internal 3.6 cm
Height AL 14.8 cm, diameter at L 3.0 cm
Height AC 13.4 cm, diameter at C 8.1 cm
Height AR 10.5 cm, diameter at R 8.5 cm
Height AG 5.7 cm, diameter at G 7.3 cm
Height AT 4.8 cm, diameter at T 6.8 cm
Height AO 4.6 cm, diameter at O 6.6 cm
Height AS 1.2 cm, diameter at S 3.1 cm
Height AV 0.7 cm, diameter at V 5.4 cm
Diameter at A 5.7 cm

PLATE XXVIII A, B, C.
Maenad with Krotalai between two Silens. By the Gela painter. Mus.N.M. Oslo 11074.

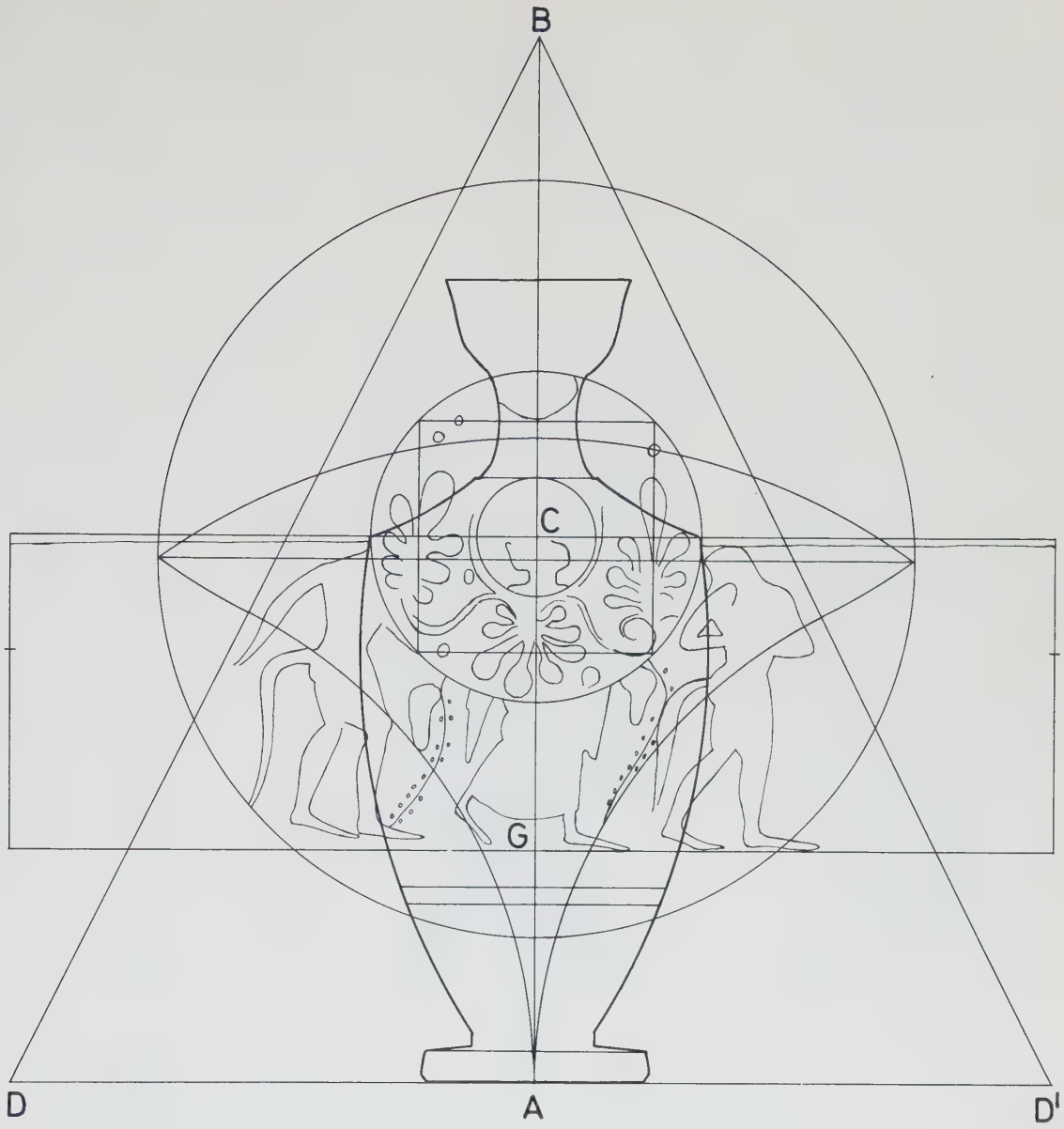
neck is then described by arcs about 5 and 6, up to level C′. Arcs about 7 and 8 on the H horizontal, through the mid-point of 5 — 6, describe the outline of the mouth almost to the top, where they are cut by arcs struck about 10 and 11 through the nearer of the two points denoting the diameter of the rim. This results in the slightly *concave* curve towards the top which is so characteristic. *Fig. 57.*

We see that the foliage on the shoulder is more or less *twisted* towards the left (PLATE B). The handle is central to the scene on the front, but the central leaf clearly points towards the left. It is much more difficult to draw or paint such distortion than to arrange the foliage symmetrically, and thus it seems likely that the artist had a special purpose in mind. On PLATE B the decoration of the shoulder is shown within the dotted circle about C′, while the picture on the body of the vase is shown on this surface placed flush. The drawings have been traced from



the vase, and due to its curvature they are only approximately correct, but they are nevertheless important for the purpose of this analysis. When holding the vase, one easily sees that the circle was transferred to the curving body of the vase, and that the lower horizontal (R) of the inscribed square of the circle (centre C) shown in the drawing was significant for the elbow of the left figure and the upper edge of the arm of the figure on the right. Moreover, the greatest width of the picture had clearly been calculated to coincide with level R, where the vase is widest. The verticals through x and x′ are here shown as arcs parallel to the outline of the vase. The figures are moving to the right while the central leaf of the shoulder points to the left. The right silen is looking back over his shoulder; on the photograph we can see that the original position of his head is marked by an incision.

Specialists frequently assert that distortions on a vase, particularly those connected with the position of the handle in relation to that of the scene or other decoration, are entirely due to coincidence, that the artist was unable to get it straight. In our view such a theory is untenable,



and the investigations for which the preceding chapter accounts surely indicate that such distortions and displacement form part of an intentional method. A good case in point is Dionysos on the Ocean, where the effect of the composition is due to its relation to the axis of the handles. We can also compare the vase to the rectangular block: both are three-dimensional, but while the block is changed, acquiring new vertical planes as a moving figure is hewn from it, the vase must always remain static. *Moving figures* can only be shown on *its outer surface.* Did the vase-painters perhaps employ the same means of showing movement as the sculptors — *displacement of the vertical central plane of the picture scene in relation to the vertical plane of the handle or handles*? If we define the back of a *lekythos* as the side to which the handle is attached, we shall frequently find *lekythoi* with some form of displacement between the handle and the picture on the front, possibly the border bounding it above or below, or even both.

Fig. 58 a, b, c
Lekythos, black figure. 'Apollo playing the Lyre between seated Women'. Etn.M. Oslo 36276. The vase, Fig. c, as seen from above, showing geometric marks.

The following measurements have been taken, the outline checked by a templet:
Height AL 12.2 cm, diameter at L 2.0 cm
Height AC 11.3 cm, diameter at G 5.3 cm
Height AG 5.8 cm, diameter at G 4.9 cm
Height AT 5.2 cm, diameter at T 4.7 cm
Height AR 4.6 cm, diameter at G 4.5 cm
Height AV 1.1 cm, diameter at V 1.9 cm
Diameter of foot at V 3.9 cm.
Diameter of foot at A 4.6 cm.
Height AS 0.5 cm.

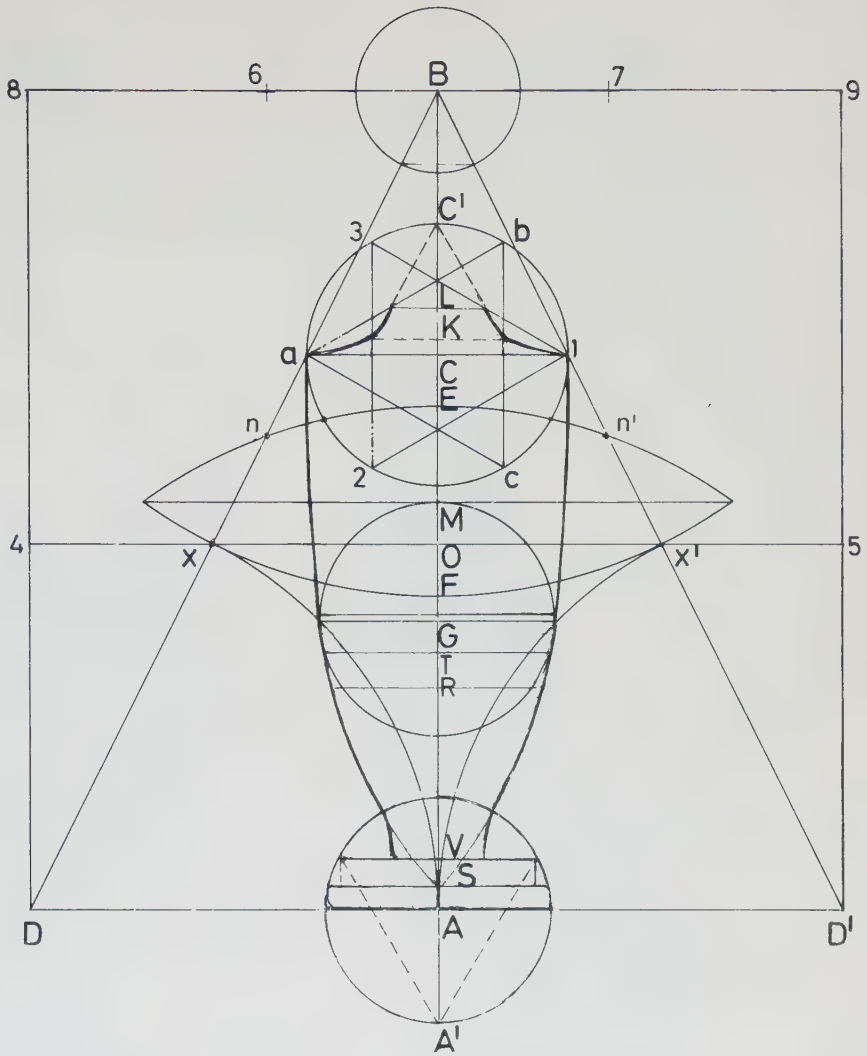
Then the decoration of the shoulder must provide a link between these two elements. Gisela M. Richter (*The Craft of Athenian Pottery*, Yale 1933) has shown that *lekythoi* were made in four parts, and it is obvious that these parts, which were to be joined, had to be very accurately calculated. In our opinion such calculation must have taken place on a plan drawing of the type described above, and the displacement must also have been shown on this drawing.

PLATE XXIX A, B, C. We shall now examine a *lekythos* made by an artist who clearly nursed a primitive, immense interest in geometry. A black figure, of Apollo playing the lyre, between seated women (Etn. M. Oslo 36276), here the displacement between the handle and the scene is so remarkable that it cannot possibly be the result of coincidence or inefficiency, *Fig. 58 a, b, c.*

The analysis of this unpretentious little lekythos reveals a simple way of geometrical proportioning. We note that *the total height of the vase, AB, is equal to the perimeter at the edge between the body and the shoulder. Further, the distance from this edge, C to the top B, is equal to the radius of the circle about C.*

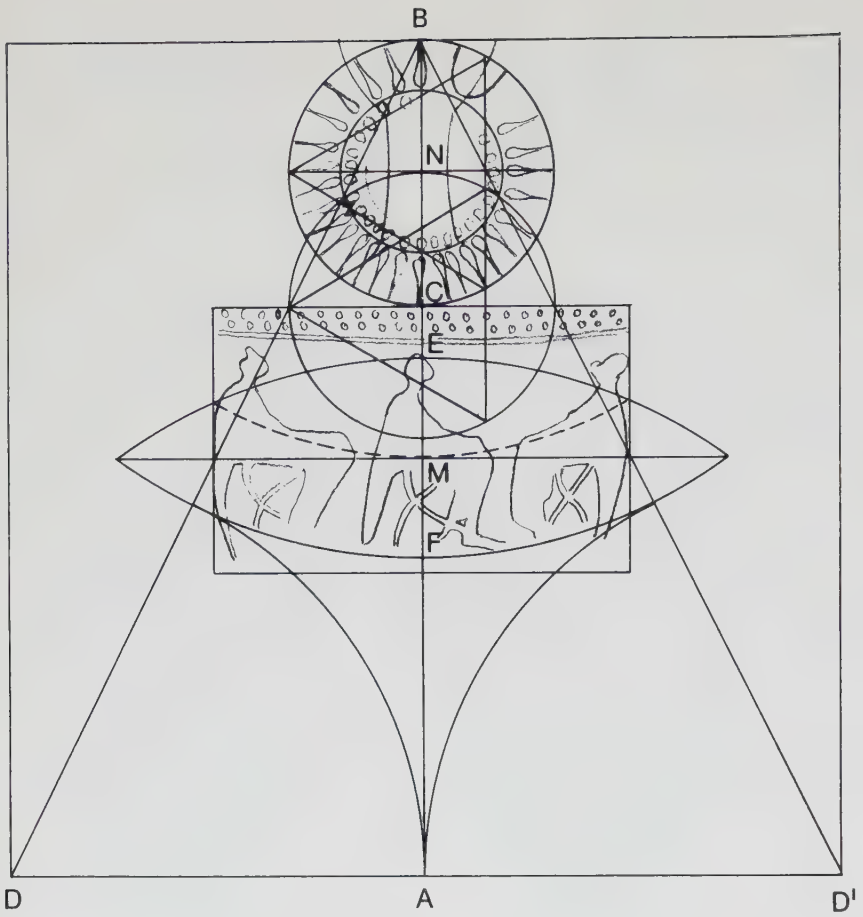
The picture panel is bounded by a simple border at the top; the distance from the left end of

PLATE XXIX A,B.
Lekythos, black figure. 'Apollo with the Lyre, between seated Women'. Underneath a grafitto. Etn.M. Oslo 36276. Revelstoke no. 72. From the Hermann Weber Collection. Analysis based on the author's measured drawing. Scale 1:1.



the border to the centre of the handle, measured along the edge, is 5.6 cm, while the corresponding distance on the right is 2.8 cm. The sum of these, 8.4 cm, equals half the perimeter measured along the edge. Thus the length of the border is equal to AM (shown on PLATE A). If AB is the central vertical axis of the border, the position of the attachment of the handle on the shoulder is determined by an isosceles triangle inscribed in the circle about C. This triangle has its left corner at a, to the left in the horizontal diameter; this point is marked on the edge of the shoulder by *a thin stroke pointing towards b.* A corresponding triangle, 1-2-3, has its right corner at 1, to the right on the horizontal diameter, and at point 2 we find a *thin stroke pointing to 3.* These marks do not belong to the decoration of the vase, and they must surely represent traces of measuring marks which should have disappeared. The first may be seen on *Fig. 58c.*

The circle centre C with its two inscribed triangles determines other dimensions of the vase and of its decoration. The upper edge of the dark zone at the bottom of the vase, R, lies at a distance above A which is equal to the sides of the isosceles triangles inscribed in the circle about C, which also determines the width at A. An isosceles triangle inscribed in the circle about A and pointing downwards determines the height of the foot as well as its width at the top. Arcs about the geometrical points trace the outline of the vase: arcs about 4 and 5 (level



x—x′) describe the outline from this level down to where they intersect with the circle of the base. These arcs cut each other at S, which determines the height of the lower step of the foot. The transition from the body of the vase to the foot is traced by arcs about D and D′. Arcs with radius 2AB are struck through a and 1 from points on the E horizontal, outside the drawing. They describe the outline of the vase between level C and level R. The outline of the shoulder up to level K is the result of arcs about 6 and 7 (which lie on the verticals through n and n′), and from level K to level L it follows arcs about 8 and 9. Here we shall stop, as the neck of the vase has been restored, and the mouth appears to be misshaped. All the main dimensions, however, are clearly intact.

The painted decoration of the vase expresses and confirms the geometrical idea on which its shape is based. We have already seen that the length of the picture panel is equal to ½AB. We have traced the scene, and on PLATE B it has been transferred to the constructional diagram. The error is only slight in this case, as the shape of the part of the vase in question is not far from being cylindrical. The scene depicts Apollo with his lyre, between two flanking figures. All are seated in profile, two of them facing right and one facing left. It appears that the lower

Fig. 59
Lekythos, black figure. Attic. Vth century. London B.M. 578.

sides of the triangles inscribed in the circle about C intersect within Apollo's head, and his lyre also lies within the circle about C. The two flanking figures both depend on an arc about B through M, its radius thus equal to half the perimeter. This arc forms part of the flush projection of the perimeter, for it passes through y (PLATE A). A dotted Φ hypotenuse from B meets this arc at the hand of the left figure. The same construction is repeated in the right half of the scene. Here we surely have another proof that the shape and the decoration of the vase were planned and designed on one and the same drawing: the shoulder has twenty-six leaves surrounding a ring of brush-strokes, probably intended to depict stalks. This ring is carelessly executed and broken under the handle, but like the leaves it continues round the entire front of the vase (not shown on the plate). The break in this inner ring is determined by the handle, as appears from the drawing. The ring of long leaves also shows a break under the handle and extending to the right. The leaf immediately to the left of the handle almost touches it, but there is a large open space between the handle and the first leaf on the right, whose point touches 1, a displacement which we find also in other *lekythoi.*

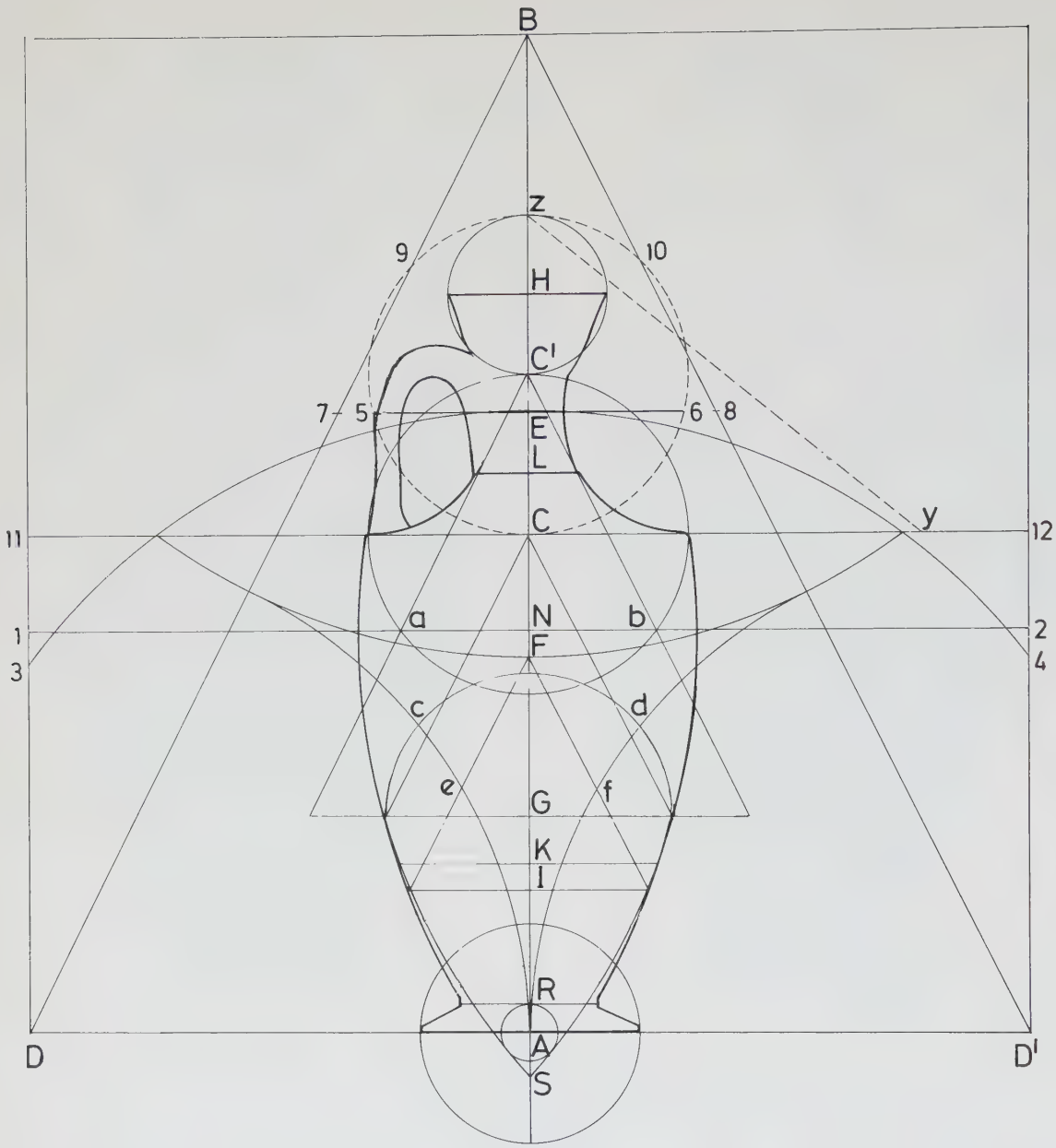


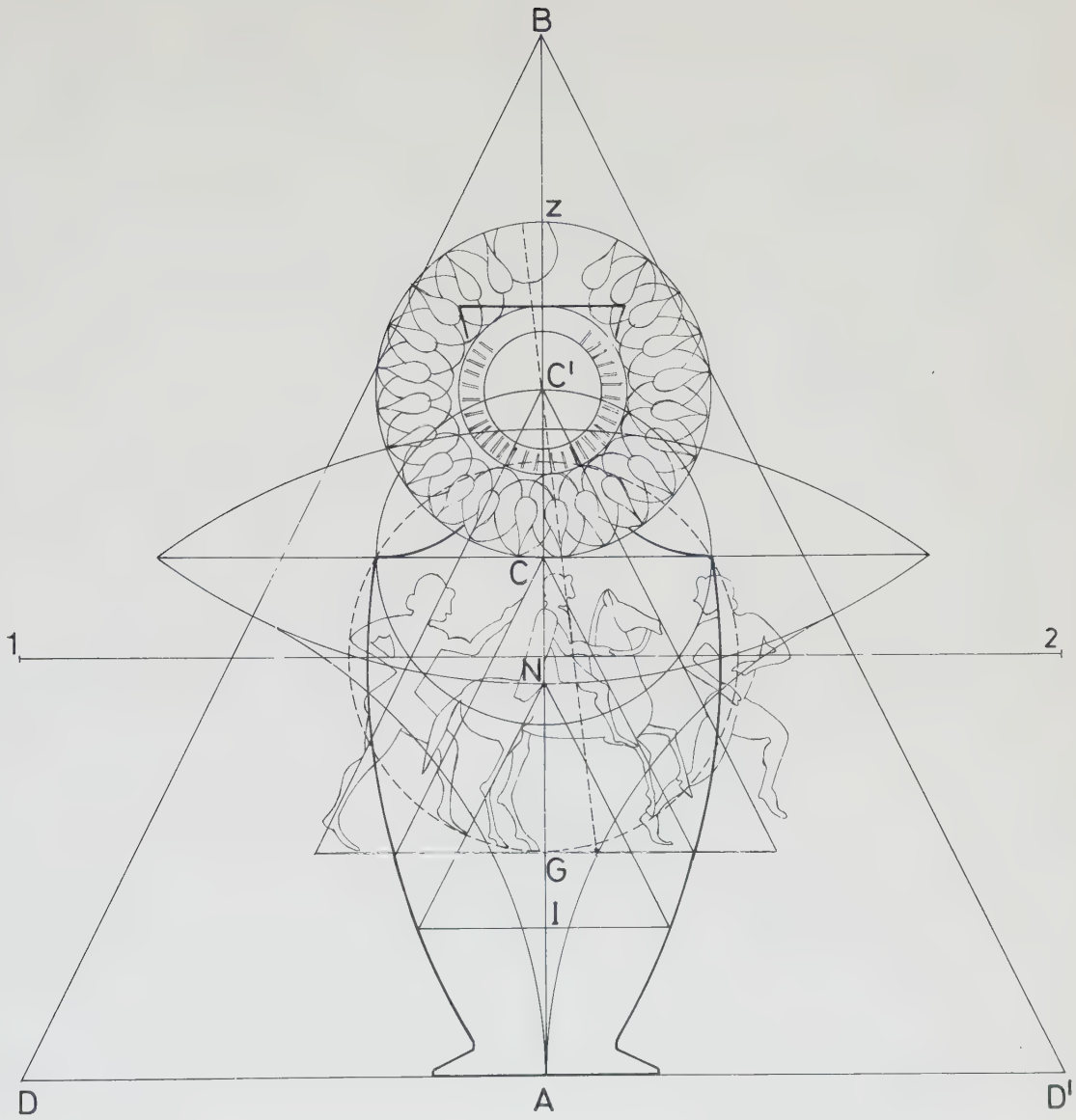
PLATE XXX A, B.
Lekythos, black figure.
London B.M. B 578. Att
Vth century B.C. Analysi
based on the author's mea
sured drawing. Scale 1:1.

The following measurements have been taken:

AH 19.0 cm, diameter A 5.7 c
diameter at H 4.1 cm
AL 14.5 cm, diameter L 2.8 cm
AC 12.8 cm, diameter C 8.2 c
AG 5.6 cm, diameter G 7.4 c
AR 0.7 cm, diameter R 3.5 c

PLATE XXX A, B. The painted decoration of the next vase studied may be more accomplished than that of the preceding one, but the basic principles underlying the connection between the shape of the vase and its decoration are the same. It is an Attic black figure *lekythos* of the fifth century BC. (British Museum, London. M.B.578).

The perimeter at the edge between the body and the shoulder of the vase is projected in the usual way, and it appears that the *height AC is equal to half the perimeter* (zy = AC = CB). The height of the vase is determined, for C'H = Hz. The points of intersection between the Φ hypotenuses from C', a and b, and the circle about C determine the N horizontal. This is the point at which the vase is widest. Points 1 and 2 lie on this horizontal, vertically above D and D'. Arcs about 1 and 2 describe the outline of the body of the vase between level C and G. The G horizontal, the base line of the picture panel, is determined by the points of intersection



between the outline and the Φ hypothenuses from C. From level G, the outline is traced downwards by arcs about 3 and 4 (vertically below 1 and 2), until the foot is reached. The height of the foot, AR, is determined by the R horizontal, which is tangential to the small circle about A, inscribed between the large arcs about 1 and 2. The diameter of the vase at A is equal to the distance between c and d, the points of intersection of the arcs about D and D′ through A with the circle about G, and e and f determine the diameter at level R. The lower edge (at I) of the light band below the G horizontal is determined by the outline of the vase and the Φ hypothenuses from F, its upper edge (K) lies half-way between A and the lowest point of the circle about C.

PLATE B shows a reconstruction of a sketch of the main features of the drawing on which the painted decoration of the vase must have been based. The shoulder of the vase, in horizontal projection, is shown flush with the surface of the body, in the same way as that we em-

ployed in the analyses of rectangular blocks. Our drawing has as its point of departure the vertical centre line of the *picture*, and it is quite clear that the handle is displaced towards the left in relation to this, *Figs. 65, 66.* In the same way as in the preceding analysis, the inner ring of brush strokes is broken under the handle, and the open space takes its direction from the axis of the handle. This passes through C′, pointing towards the point of the base line of the picture panel where this is cut by the arc about D′ through A. The leaves on the shoulder resemble those of the preceding vase, in that the left leaf lies close to the handle, while there is an open space between the handle and the leaf on its right. Is this merely coincidence?

Fig. 60 and *Fig. 61* show a roughly made wooden model to which the geometrical lines have been transferred. This was extremely simple and straightforward as measurements taken horizontally along the edge of the vase correspond to of AB and parts of AB. The points of intersection of the G horizontal with the Φ hypothenuses from C' were easily marked off on the edge of the model by laying a soft band of this length horizontally with C as mid-point. When the direction of the hypothenuses was determined, the Φ hypothenuses from C' and N could be drawn parallel to them.

The circles were also transferred to the model. The Greek vase-painters were entirely familiar with the fact that a circle changes shape on being transferred from a flat to a curved surface.

To give a simple example: the circle about C does not, on the model, reach the two points at the edge which mark the vertical central line of the vase seen from the side. On the right side, the circle marks the position of the head of the runner; measured along the edge this point lies about 2 cm from the vertical central line of the vase seen from the side. The runner also clearly depends on a circle (dotted) about N through the point of intersection between the circle about C and the C′ horizontal; the model shows that this circle, when transferred to the curved surface, is of decisive importance for this figure.

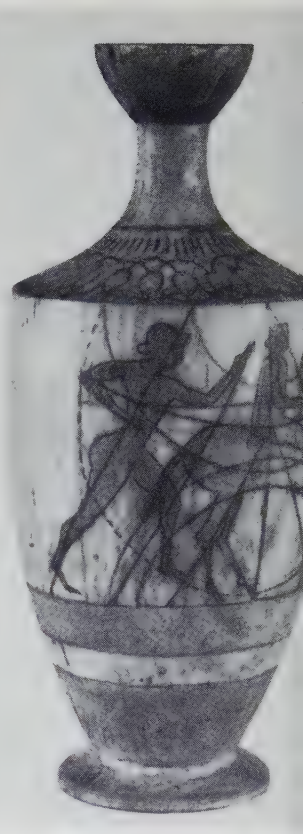
It should thus be obvious that the painter could not slavishly copy a drawing prepared beforehand. What actually happened may have been this: the vase-painter formed and placed his figures on the basis of the geometrical pattern when it had been transferred to the body and the shoulder of the vase. Thus his design is free and, at the same time, controlled. The reason why we have nevertheless shown the painted scene on PLATE B is that it will help the reader to find the geometrical lines on the model. It is, moreover, not impossible that the vase-painters could have worked according to some kind of drawn design, or that they could have marked off the main lines of figures on such a drawing.

The next vase, decorated with a scene of the flying *nike* with an apple — probably a fertility symbol — dates from *after* the transition from the black-figure style to one where the light figures stand out in contrast against the dark background.

PLATE XXXI A, B (Etn.M. Oslo 11076). Analysis based on the author's own measurements. Scale 1:1.

The outline of the vase was checked by a templet.

Again we find that the same methods were applied to the design of the vase as well as to its decoration. As usual, the basic dimension AB is determined by the greatest perimeter of the vase, and here the edge between the body and the shoulder of the vase, C, lies half-way between A and B, a proportioning we also noted in the previous analysis. The height of the vase, AH, is determined as CH = HB. AB is divided in Φ at E and F, and EF determines the diameter of the vase at the base line of the picture panel, level G. AG is determined, for the diagonals from A of the two squares on AC cut the circle about G through A at the end of the



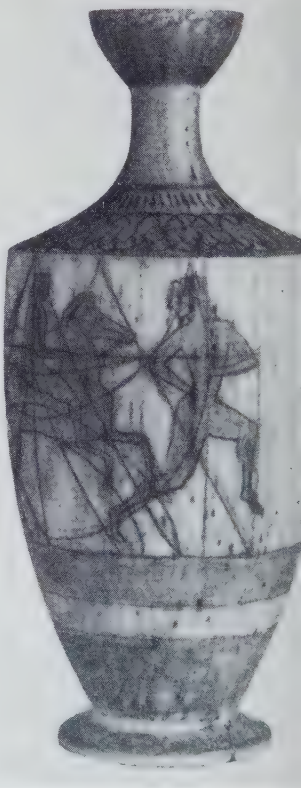
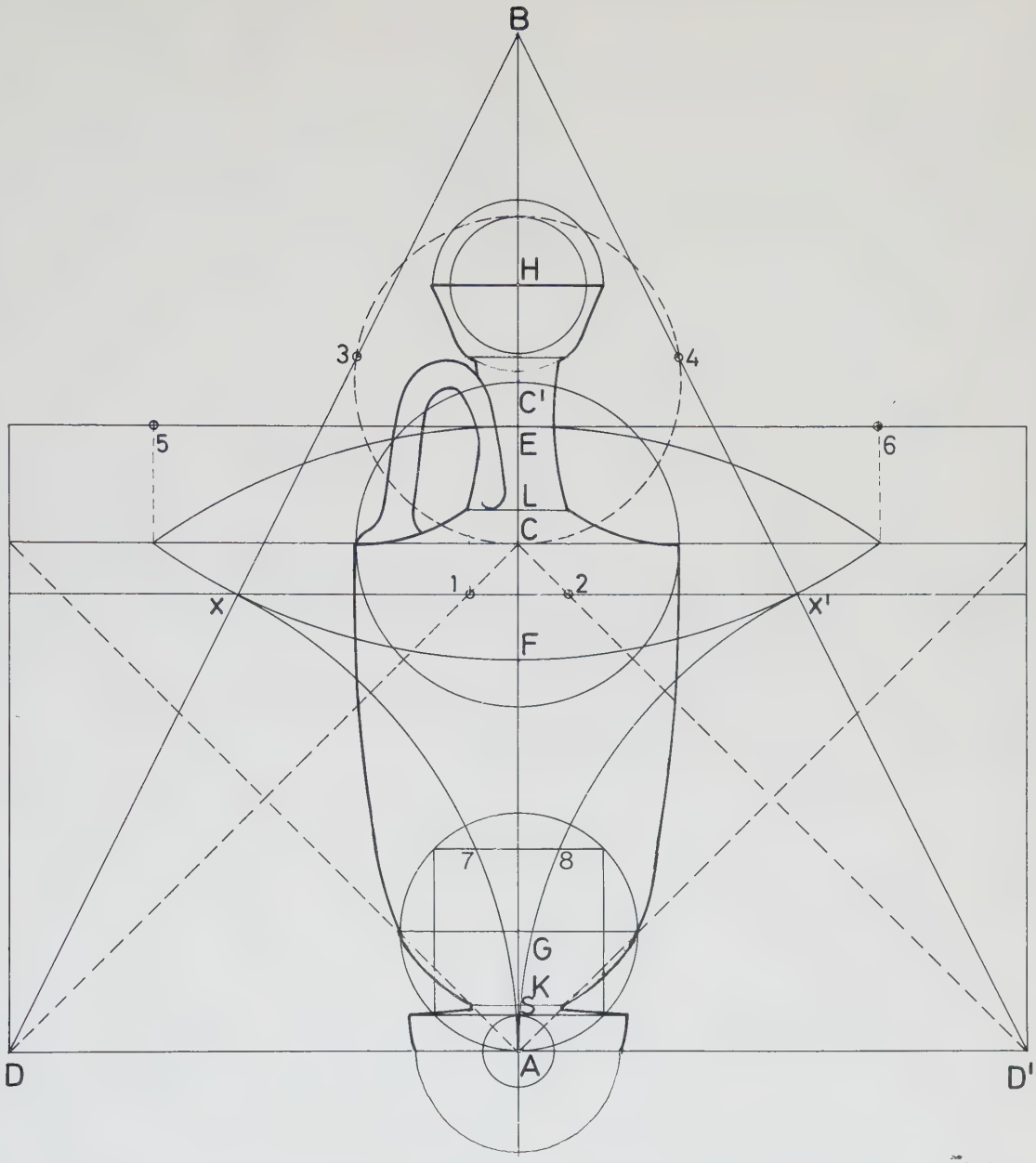


Fig. 60, 61
Wooden model of the vase shown on Fig. 59.

PLATE XXXI A, B.
Lekythos, red figure. 'Nike flying to the left, with Apple (?)'. Etn.M. Oslo 11076. 460–450 B.C. Analysis based on the author's measured drawing. Scale 1:1.



following measurements
been taken:
= 16.5 cm,
eter at H = 3.6 cm and 3 cm
= 11.5 cm, diameter at L = 2.2 cm
= 10.9 cm, diameter at C = 7.0 cm
= 2.6 cm, diameter at G = 5.2 cm
= 1.0 cm, diameter at K = 2.0 cm
0.7 cm, diameter at S = 4.5 cm
= 4.3 cm of depression
th the foot = 1.5 cm.

horizontal diameter. A square is inscribed in the circle about G, and the lower side of this square determines the height of the horizontal upper edge of the foot (S) (while the side of the square determines the greatest width at H). A circle about A through S is clearly marked by a depression underneath the foot. The greatest diameter of the foot, at A, is equal to the distance from A to the upper side of the inscribed square. 1 and 2 on xx′ determine the width at L, and the outline of the shoulder is drawn by arcs about points 3 and 4 up to level L, which is thus determined. The outline of the neck is traced by arcs about 5 and 6 until they meet the larger circle about H. This circle determines the outline of the lower part of the mouth; the top is concave, drawn by arcs about 5 and 6 through the nearest end of the diameter through H.

The outline between levels G and C is given by arcs about centres which lie far outside our diagram, on the xx′ horizontal produced, and at a distance from x, respectively x′, equal to AB. The outline below the G horizontal is the result of arcs about 7 and 8, which continue until

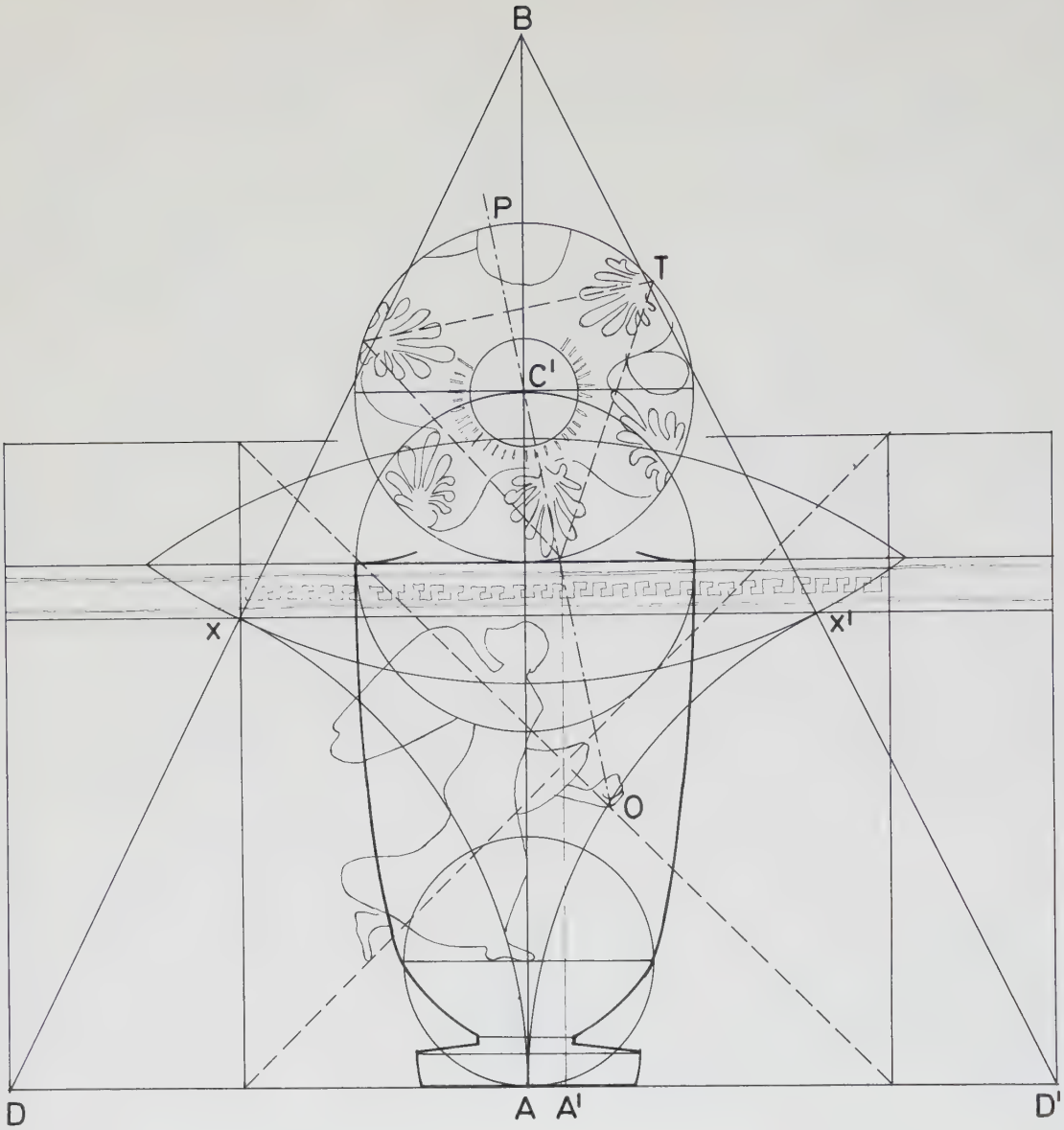


Fig. 62
Lekythos, red figure. 'Ni
Flying to the Right with
an Apple or Pomegranat
460–450 B.C. Etn.M.
Oslo 11076.

Fig. 63
Lekythos, red figure.
Showing a very clear dis
placement of the upper
and the lower border, ap
rently decided by the mo
ment of the figure. Stock
holm N.M. 1700.

they again meet the A diagonals. Now AK is determined, and also the width of the vase at level K. Arc about A through the end points of the horizontal diameter at A give the width at the top of the foot and its outline between level A and level S.

A border bounds the picture panel at the top (PLATE B), at level xx′. This is not centrally placed on the vase, if we let AB pass through the handle: there is considerable displacement to one side. As both ends of the border are clearly stopped, this displacement cannot be accidental, and the more we examine the decoration of the body and the shoulder, the more clearly we realize that this is not a matter of coincidence. We have traced the *nike* figure and the decoration of the shoulder. The error due to the curvature of the vase is slight enough to allow us to use these tracings as material for study, *Fig. 62*. We see that the *nike* is flying forward towards the vertical centre line of the *scene* (determined by the border), but she has already reached the centre line of the *vase* through A (determined by the handle). The shoulder also shows irregu-

larities, for the distance between the handle and the leaf on its right is shorter than that between the handle and the leaf on its left.

The length of the border measured along the edge between the body and the shoulder of the vase is equal to BE. It starts at x, the point of division of the left Φ hypotenuse. A square is drawn, its horizontal sides through A and E and its left side on the x vertical. The apple, the fertility symbol, which the *nike* holds, lies on the point of intersection (O) of the diagonal of this square and the arc about D′ through A. It now appears that a line through this point and C′ marks the displacement of the painted decoration of the shoulder. An isosceles triangle has been inscribed in the circle about C′, its upper side perpendicular to C′O. The foliage on the shoulder depends on this triangle, and we experience a very definite displacement, emphasized also by the ring around the neck, which suddenly stops. Only the central lobe of the central leaf turns slightly inwards, making for a more gradual transition to the figure below. Like the potter, the vase-painter was a craftsman of note.

We may sum up in the following way: *the painted figures move in relation to the static repose of the vase.* This applies both to the foliage on the shoulder and to the *nike*, flying forwards with her symbol of fertility. The decoration marks not only the vertical central plane of the vase, which was determined by the handle, but also another, new vertical central plane: seen from above this follows C′O, and seen from in front it is marked by the line passing through the point of intersection of C′O and the edge between the body and the shoulder of the vase. The plate shows that this new vertical passes centrally through the picture panel. Surely a striking analogy with the new central plane of the rectangular block, as shown on the cardboard box. On Fig. 63 (Stockholm N.M.1700) we see a related lekythos where the movement of the figure is very strongly marked by the displacement of the upper and lower border.

PLATE XXXII A, B, C, D, E. Our next subject is a *lekythos* on a white ground. (It is in private ownership in Oslo.) The analysis is based on the author's measured drawing. Scale 1:1. Every measurement checked during the work. Height 20.7 cm, diameter at C 7.4 cm.

The following measurements have been taken:

AH 20.7 cm, diameter at H 4.0 cm
AL 14.7 cm, diameter at L 2.5 cm and 2.8 cm
AC 13.9 cm, diameter at C 7.4 cm
AG 5.9 cm, diameter at G 6.7 cm
AP 5.0 cm, diameter at P 6.5 cm
AI 1.1 cm, diameter at I 2.5 cm
AK 0.8 cm, diameter at K 5.0 cm, diameter at A 5.2 cm.

Underneath the foot lies a depression bounded by a circle 2.2 cm in diameter. The outline of the vase was checked by a templet.

Before leaving the subject of *lekythoi*, we shall attempt an analysis of a more complicated and involved scene, painted on the side of a white *lekythos*. We are here dealing with a highly dramatic scene of combat between Heracles and four Amazones, painted by the Diosphos Painter (Plate 20 in *Corpus Vasorum Antiquorum, Norway 1,* Oslo 1964). *Figs. 64 – 67.* The scene covers a large part of the side of the vase, and thus it is impossible to take it all in at one glance; we must turn the vase (or move around it) in order to see all the figures and to study their movements, thus understanding the action depicted. This complicated drawing looks like a deliberately designed and drawn composition, but one cannot help wondering how the artist could paint so vividly on such a long stretch of double curvature while retaining complete control of the situation. But before we examine the decoration of the vase, we must study its proportions.

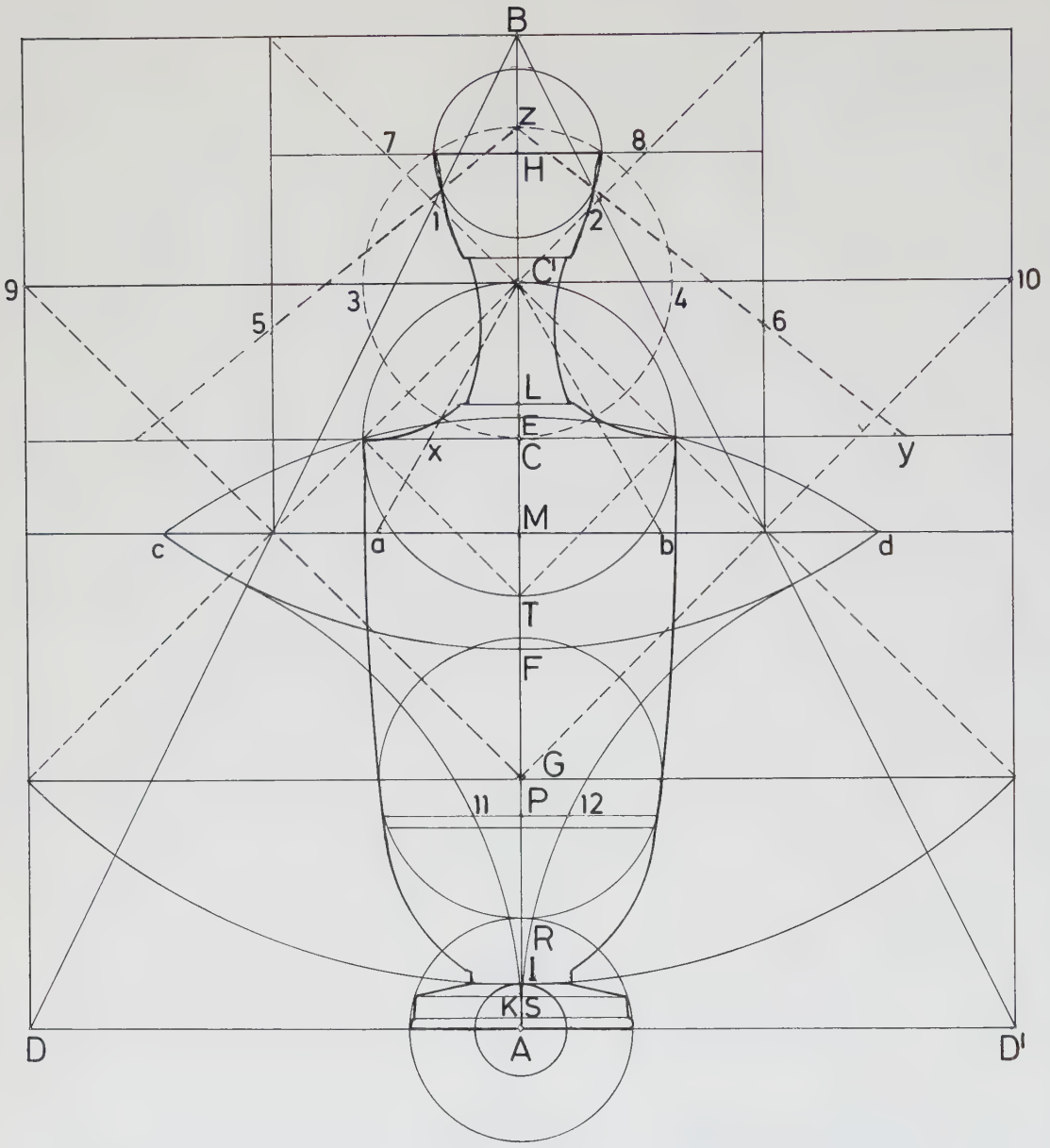


PLATE A. In this vase C′, the vertex of the circle about C, lies at the centre of MB, and thus *C is geometrically dectermined.* The base line of the picture lies at level G, and the width at G equals ab = aC'. Three squares are shown on PLATE A — all with sides equal to ½AB, the upper square about MB, and the two lower squares with one side on GC′. 1 and 2 are the upper points of intersection of the Φ hypotenuses from B and the diagonals of the upper squares. The radius at H is equal to the distance to points 1 and 2, and AH is determined by the diameter through H, HL = AG.

The outline of the vase depends entirely on the geometrical pattern. Arcs struck about 3 and 4 trace the outline of the shoulder from the body of the vase up to level L, while arcs about 5 and 6, their radii equal to the perpendicular distance to the M horizontal, describe the outline of the neck. The outline of the lower part of the head (the same radius), is produced by arcs

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ear.

about 7 and 8 through the lower points of intersection of the Φ hypothenuses and the circle about H. An arc about C′, its radius equal to the diagonal of the square on GC′, determines I, the level at which the body of the vase meets the foot. The diameter here is equal to that of the neck at L. The diameter of the foot at A is determined by the circle about A tangential to that about G (radius ab), while a circle about A through I is marked beneath the foot, where a sharp edge divides an outer from an inner depression. The lowest step of the foot at level K is determined by arcs about the ends of the horizontal diameter of the circle about G, their radii equal to that diameter (not shown on the diagram). From level P (P = midpoint of AT) down, the outline is traced by arcs about 11 and 12, which meet at S at the upper edge of the foot (not drawn). The diameter at this level is determined by the large circle about A.

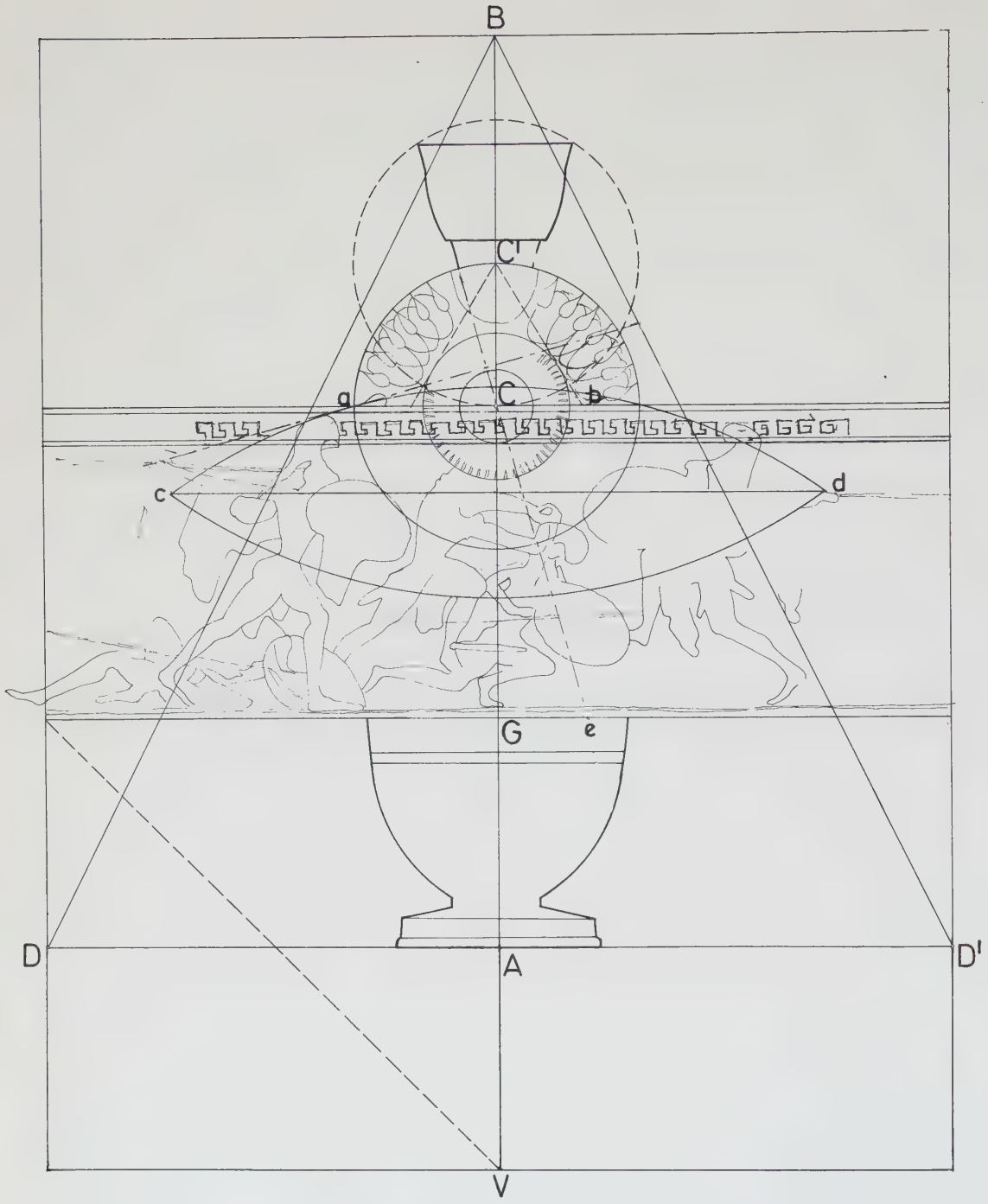
The outline of the vase is vertical from the edge between the body and the shoulder down to level M, from where it curves very slightly down to level G – this part of the outline may have been drawn from a point on the M horizontal, far beyond the diagram.

The body of this *lekythos* is so very nearly cylindrical that a tracing of the scene shows only very slight errors. The motive is a highly dramatic combat between Heracles and Amazones (PLATE B). Heracles charges forwards from the left, and behind him we see an Amazon attacking him with her lance. The hero dispatches one Amazon, between two others attacking; a fourth Amazon has fallen. The composition is drawn on the white ground with supreme ease and sureness of hand, but we must project it on to a plane surface in order to appreciate its tremendous force and outstanding homogeniety to the full. If we suppose that the handle indicates the back of the vase, the decoration shows two clear cases of displacement in relation to the vertical central axis of the handle, a fact which requires investigation and explanation: the meander border just below the edge between the body and the shoulder of the vase does not extend all around the vase – it stops short at both ends, *Fig. 65.* This border is 16.8 cm long, measured along the edge, and this length equals the distance between the two points of intersection of the Φ arcs through E and F (c and d).

PLATE B shows the displacement of the border to the right as seen in relation to the vertical central axis of the handle, *Fig. 67.* Another striking displacement is that of the row of small strokes at the top of the shoulder. We note the same phenomenon as in previous analyses: these brush strokes form a ring, which suddenly stops short at both sides of the handle. The horizontal projection about C of this ring on its left side touches Heracles' head. The foliage runs around the shoulder without a break, but it is not shown below the C horizontal on our drawing. We have picked out the displacement of the shoulder, denoted by the ends of the ring of brush strokes, by an oblique line running up through a. PLATE C will show how this line is geometrically determined.

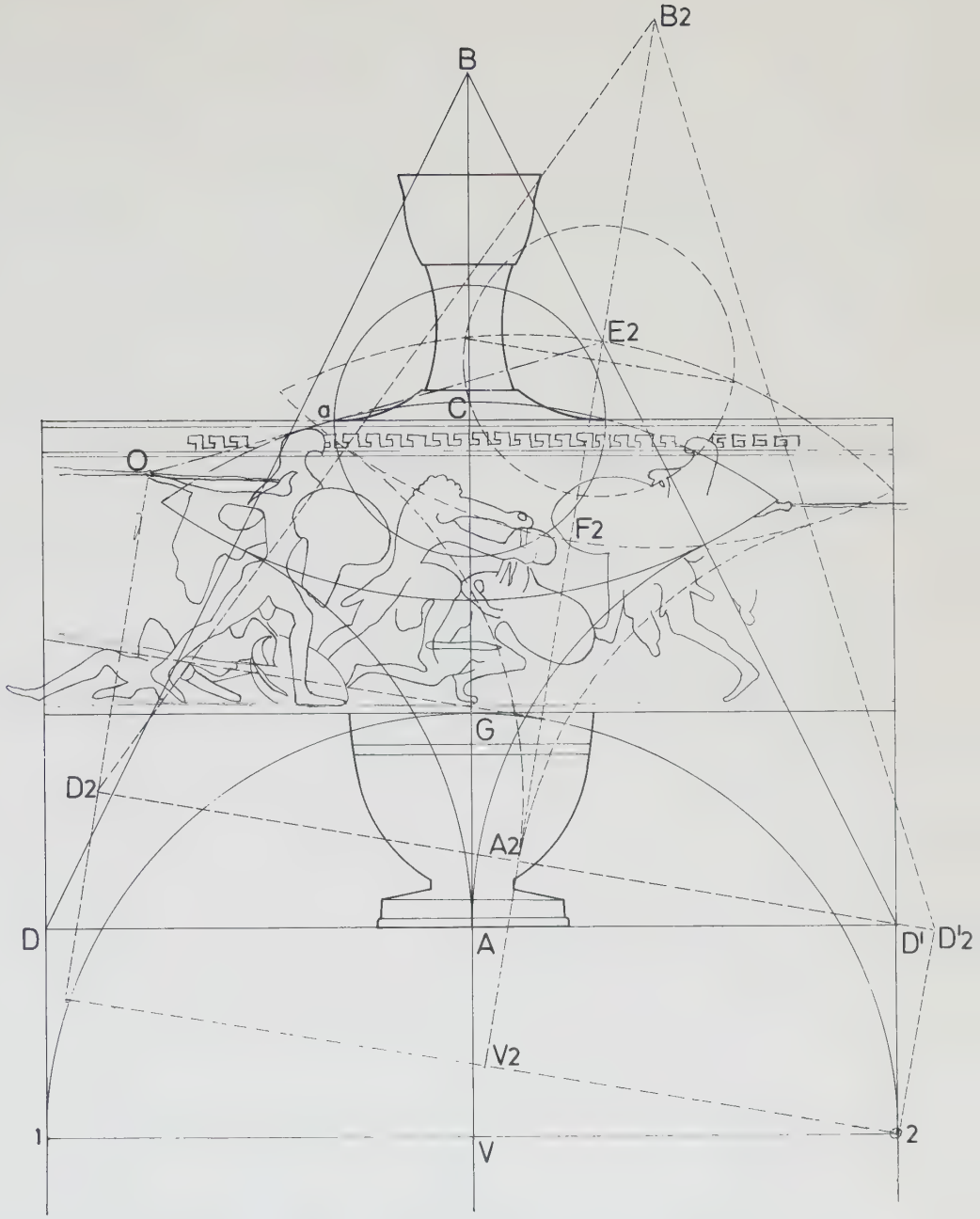
Our analysis of the decoration of the body of the vase presupposes that this is a *flat picture* (PLATE C). We construct a square whose upper side is the G horizontal and *V is its centre point;* the lower part is not shown. The circle about V through G represents the horizontal projection of the space at the disposal of the combat scene. The evidence of previous analyses suggests that the progress of the action depicted is described by means of pivoting movements of the geometrical pattern, and the displacement we have noted must be of decisive importance to this process. As usual, the first pivoting movement takes place in the horizontal projection; the pivot is point 2, the right end of the horizontal diameter of the circle about V.

The geometrical lines pivot to the right until E2 lies on BD′. It now appears that the line joining E2 and the left end, a, of the "horizontal" diameter of the circle about C is *identical with that which denotes the displacement on the shoulder on PLATE B. Thus the displacement of the*



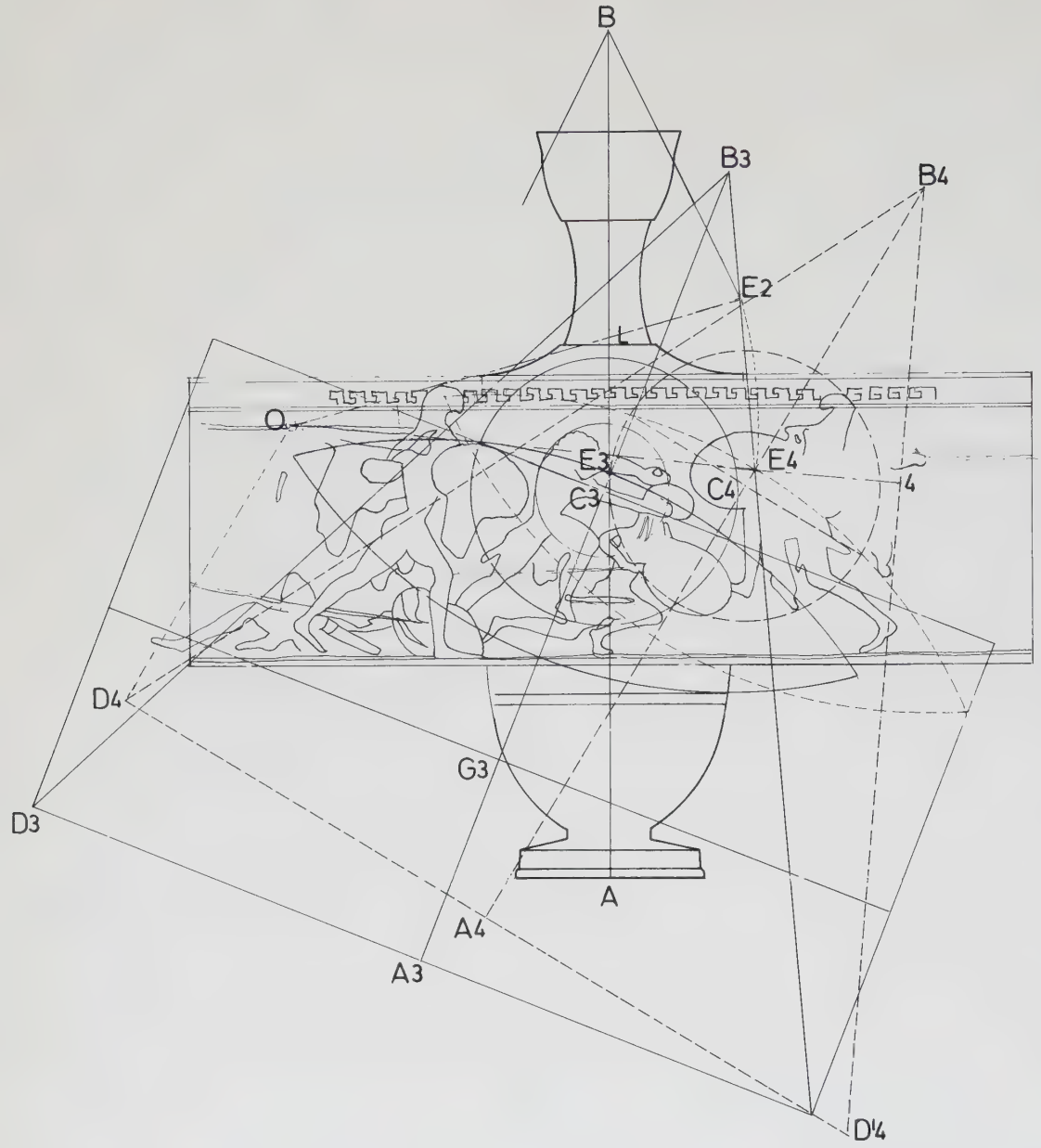
shoulder is geometrically determined, and it is closely related to both these positions of the geometrical lines. We note that the new position, here shown by a dotted line, has a decisive effect on the action depicted: the Φ arc from B2 through F2 runs along the lower edge of the right shield, passing the lower edge of Heracles' right arm and hand gripping the sword, until it touches the top of the left Amazon's helmet. Her hand gripping the lance lies on point O, the point of intersection between E2—a produced and the left side of the dotted picture panel. This side is marked by an *oblique brush stroke, about 1 cm long*, a little below the hand.

Fig. 65
Lekythos, white ground. Left side. 'Heracles and Amazons'. By the Diosphos Painter. In private ownership, Oslo. Plate 20, Corpus Vasorum Antiquorum I, Norway.



This mark is entirely without any bearing on the story of the scene — it must be a *geometrical* mark, see *Fig. 65*. In its dotted position, the G horizontal is marked by the lance of the fallen Amazon, bottom left, while D2B2 affects the forward movement of the standing Amazon.

While the lance of the figure on the extreme right is aligned with the M horizontal (PLATE A), the left Amazon has raised her lance hand to point O in the F2 horizontal. Now the two connected geometrical positions pivot together about this point, moving downwards until E3 lies on AB. On PLATE D we see how the circle about C3, which marks the ring of brush strokes at the top of the shoulder (PLATE B), *encloses Heracles' neck and back, as well as his hand gripping the sword, while the angle of his left leg is determined by A3B3*. The foot of the fallen Amazon, bottom left, lies beyond the vertical on the back of the vase (determined by the



handle). Here we see that this foot is determined by the dotted side, the back vertical of position 4. The weight of the composition is displaced towards the right, a displacement marked by the "verticals" through C3 and C4 and the "horizontals" perpendicular to them; at the bottom left, the long Φ hypotenuses from B3 and B4 are important. The C3 "horizontal" runs right across the picture from the upper edge of the large shield at the left, along Heracles' right arm and along the right Amazon's calf. The figure on the right is in other respects closely connected with the position shown in broken line: her helmet and the drapery between her legs lie within the circle about C4, and the C4 horizontal traces the upper edge of her scabbard. The lower part of the drapery on the extreme right is determined by the E4 arc and the Φ hypotenuse from B4. We may say that Heracles and the Amazon on the right are both at the centre of the scene in this position of the geometrical lines; the dying Amazon collapses between them.

Fig. 66
Lekythos, white ground. Front. In private ownership, Oslo. Plate 20, Corpus Vasorum Antiquorum I, Norway.

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ythos, white ground.
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pus Vasorum Antiquorum I,
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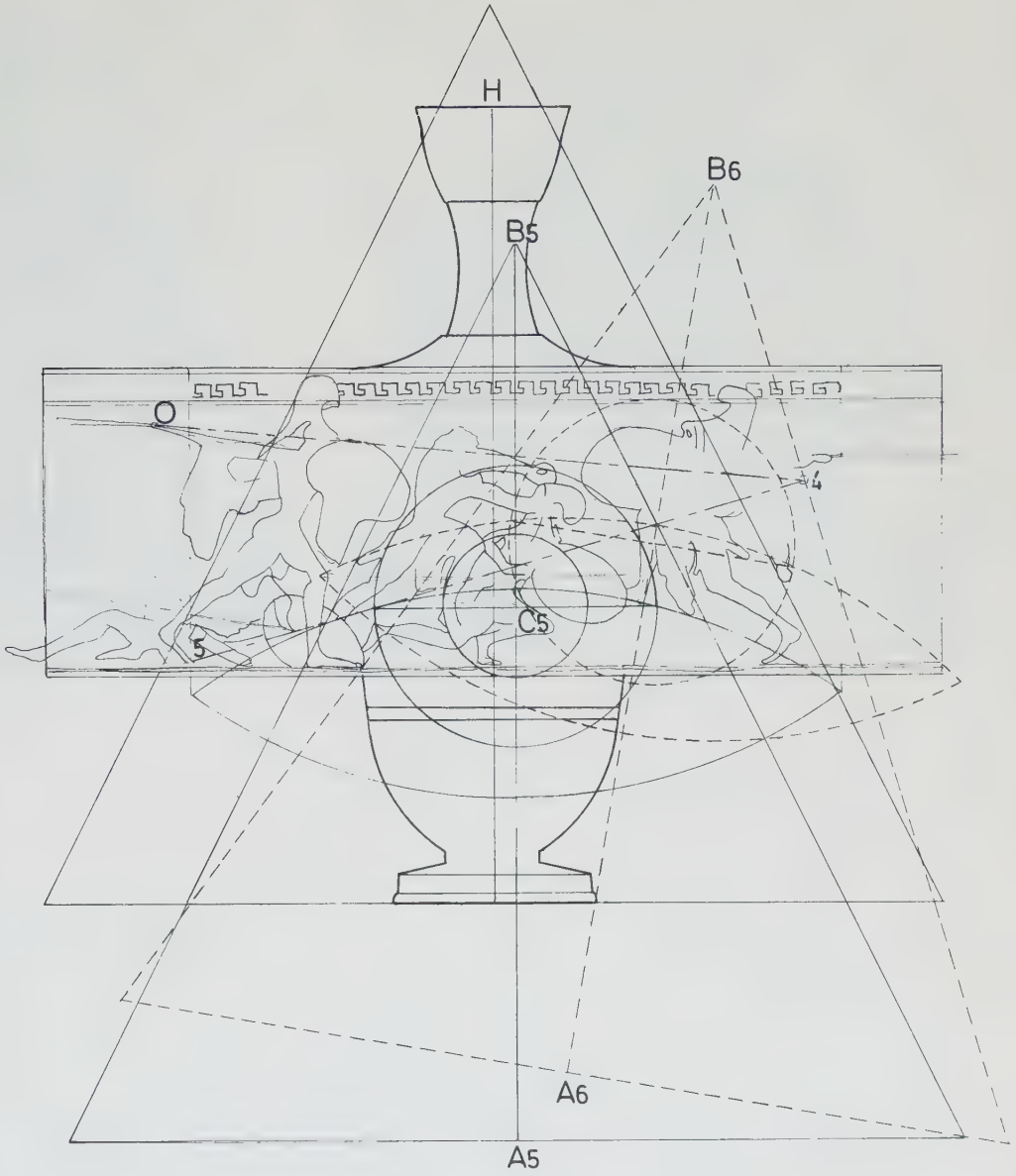


A final pivoting movement downwards, about point 4, just below the lance hand of the right Amazon, straightens up the geometrical pattern (PLATE E). The pivot is determined, as 4 is the point of intersection of B4D′4 and the line from O (of Plate A) to E4 produced; the extent of the pivoting movement is determined by A5B5 on PLATE E being vertical. The combat has reached its final phase: the vertical central axis of the scene, *A5B5 lies at the centre of the meander border.* The fight has come to an end. The artist's interpretation of it is based on the same methods as those accounted for in the preceding chapter, and thus his work must of necessity have required a constructional drawing. Even though new features have made their appearance – such as the foreshortening of the shields – the composition still expresses the way of thinking which we have demonstrated in Greek art from the Archaic period onwards.

Different vase forms, their decoration based on the geometry of the vase

Fig. 68, 69
Corinthian aryballos from Boeotia. Front view and left side. Etn.M. Oslo 6922

The vases which form our next subject show the problems from a somewhat different aspect, thus giving us some idea of the wealth of possibilities and variation which is so characteristic of Greek pottery. The strict and logical method of designing seems to have served as an inspiration, making for imaginative freedom rather than for restraint.

PLATE XXXIII A, B, C. Our first vase in this section, a Corinthian *aryballos* from Boeotia (Etn.M.Oslo 6922), decorated with five padded dancers, is approximately spherical, *Fig. 68*, *Fig. 69*. The men do not dance out of their positions, they remain in the space allotted to them. There is no displacement between the handle and the decoration, which is naturaly enough in a vase with a static scene. The analysis is based on the author's measured drawing. Scale 1:1.

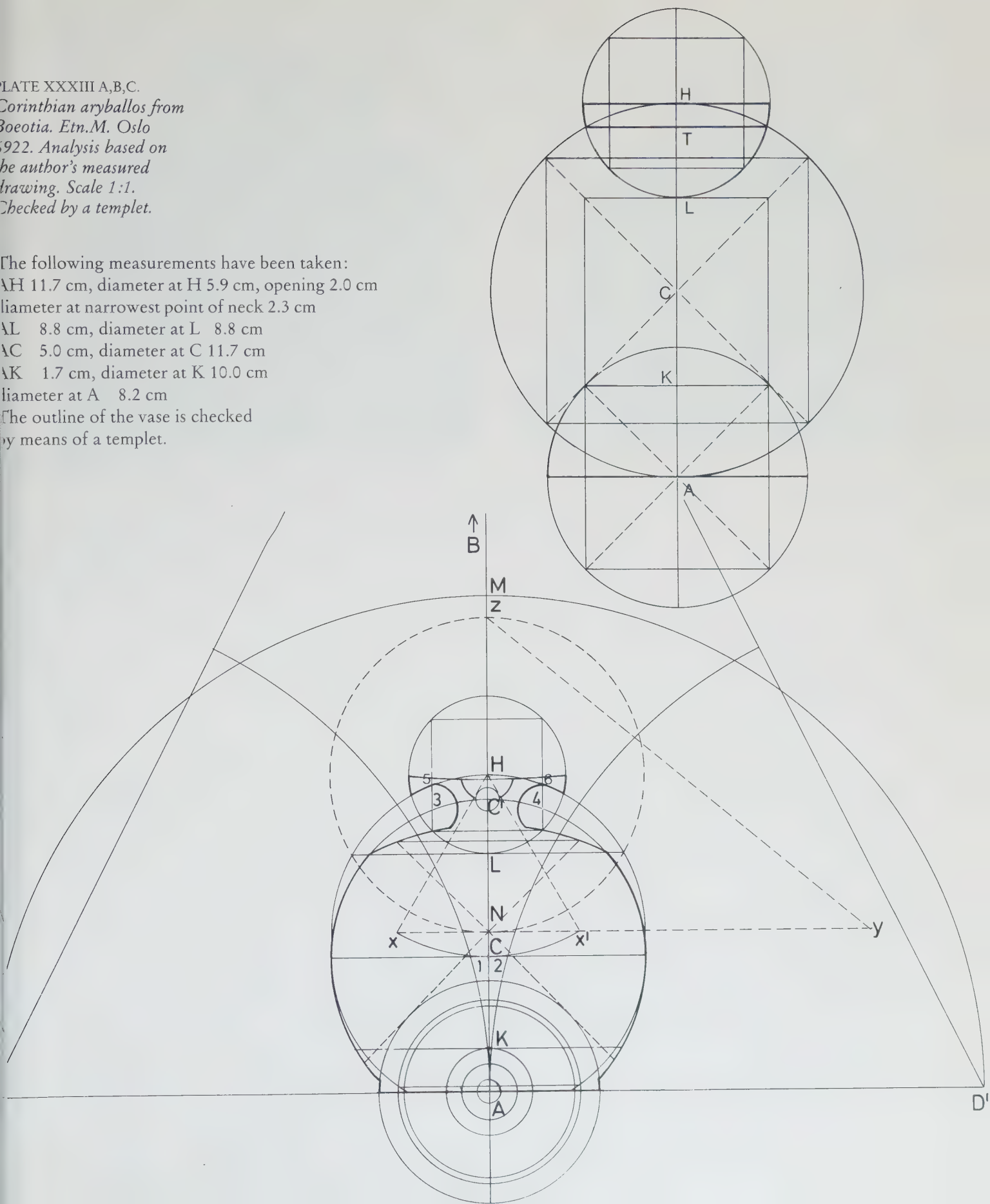
PLATE A shows a geometrical diagram explaining the basic dimensions of the vase. The circle about C marks the greatest width of the vase, seen from above, and the height AH is equal to the diameter of this circle. The side of the inscribed square of the circle determines the diameter of the foot-ring. The side of the inscribed square of the foot-ring gives the greatest diameter of the mouth-disc (at H). A further square has been inscribed in the circle of the mouth-disc.

PLATE B shows that the greatest width of the vase (C) lies below N; an investigation shows that level C has been calculated in connection with the flush projection *of the greatest perimeter of the vase.*

This construction is shown on PLATE B, in broken line, and zy is determined as being equal to half the perimeter, cfr. *Fig. 52.* AM is set off equal to zy = AD = AD′. *An arc about H through x determines C,* and the circle denoting the greatest width of the vase can now be

PLATE XXXIII A,B,C.
Corinthian aryballos from Boeotia. Etn.M. Oslo 5922. Analysis based on the author's measured drawing. Scale 1:1. Checked by a templet.

The following measurements have been taken:
AH 11.7 cm, diameter at H 5.9 cm, opening 2.0 cm
diameter at narrowest point of neck 2.3 cm
AL 8.8 cm, diameter at L 8.8 cm
AC 5.0 cm, diameter at C 11.7 cm
AK 1.7 cm, diameter at K 10.0 cm
diameter at A 8.2 cm
The outline of the vase is checked by means of a templet.



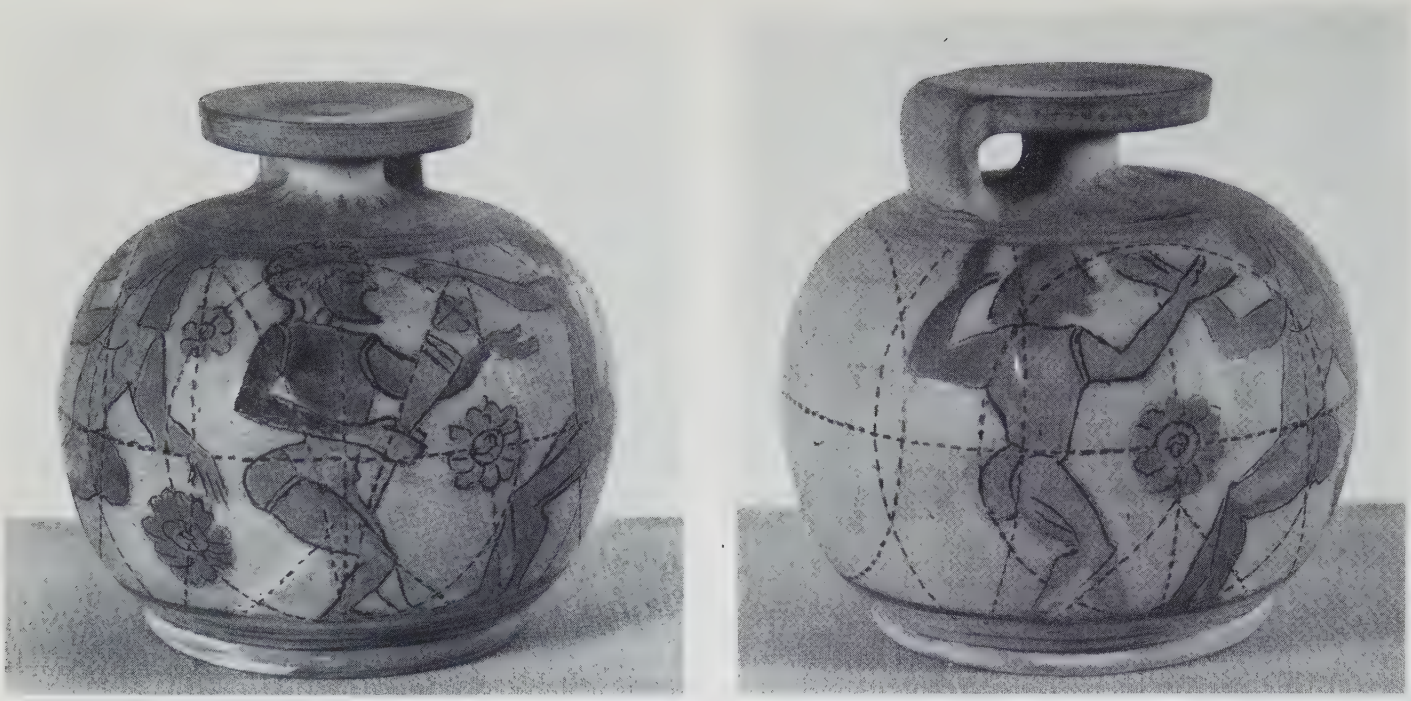


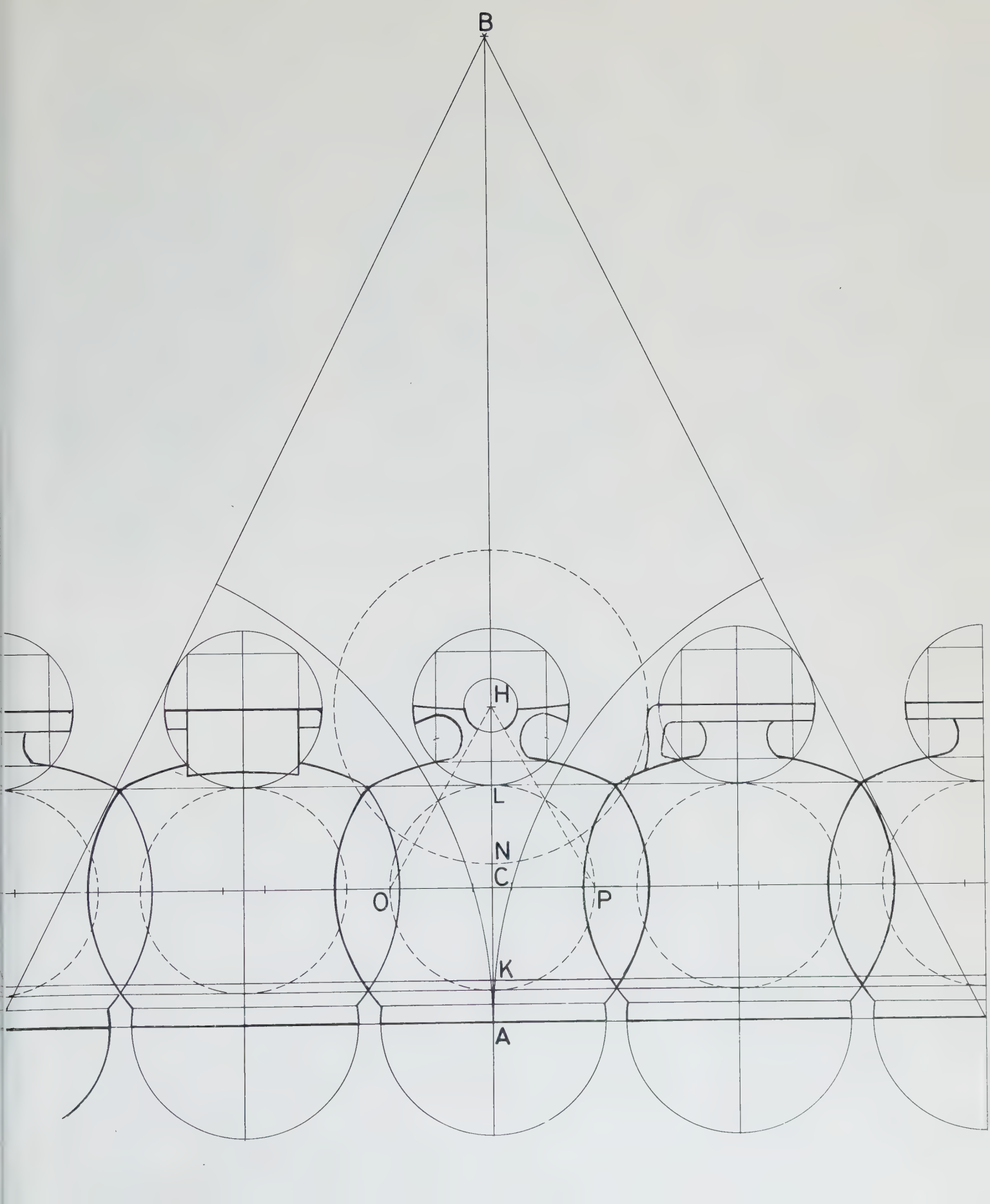
Fig. 70, 71
Pottery copy of the vase shown on Fig. 68, with the geometrical lines of PLATE XXXIII transferre to the double curved side of the vase.

drawn at its correct height (about C). The outline of the vase from level C to level L follows the circle about C. Points 1 and 2 on the C horizontal (determined by arcs about D and D′ through A) are the centres of the arcs describing the outline of the vase from level C to the foot-ring, the arc about 1 passing through the right end of the diameter, that about 2 through the left one. An arc about A describes the outline from level L up to the neck, while arcs about 3 and 4 through 5 and 6 extend the outline up to these points. The opening of the mouth is determined by a circle about H through C′, and the inner surface of the mouth follows this circle until it meets a circle about C′ which is tangential to Hx and Hx′. It is interesting to note that this small inner opening is repeated in a painted version *under* the foot-ring, where other dimensions from the mouth also occur. The diameter of the inner edge of the smaller painted ring is equal to half of the side of the square centre H while that of its outer edge passes through K (determined on PLATE A).

Seen from below, the dividing line between the foot-ring and the bottom of the body of the vase a few mm above it is determined by a circle about A, its diameter equal to xx′. There is a painted ring about A at this point also, and its inner edge is determined by the points of intersection of the A horizontal and the circle about C. The painted horizontal rings on the side of the vase are also entirely dependent on the geometry of the vase, their level being determined by points of intersection which are marked on the plate. Thus the painted figures are dancing on the K horizontal.

PLATE C shows the vase from the front, from both sides and from the back (shown in two parts). All the main features of PLATE B are repeated about AB. The mouth-disc is slightly concave, and its section may possibly have been determined by an arc about B through the ends of the diameter of the disc, level H. The handle reaches up to level H. Its width is equal to the side of the inscribed square of the circle about H (see left drawing). The right drawing shows the handle from the side, where it is tangential to the dotted circle about H.

Our clay model of this vase facilitates the study of the importance of the geometrical pattern when transferred to the double curvature of the surface, *Fig. 70* and *Fig. 71*. The model was


B
H
L
N
C
O
P
K
A

thrown on the wheel from a templet based on the constructional drawing, and then it was fired. This was very easy, and there would seem to be no technical objection to pottery production as accurate and precise as our analyses presuppose. The geometrical pattern was transferred to the fired model. The two vertical central planes — seen from the front and from the side — were marked, as was also the horizontal at level C. We now had four points of intersection, and the two concentric circles about C were struck about these (dotted). The vase has five padded dancers, the one in the middle holding a drinking-horn. The geometrical pattern changes on being transferred to the model; the figures can now be seen to follow the pattern, to express it, as it were. The dancer on the extreme left, for instance, depends on the circle about the centre of the rosette, the point of intersection of the C horizontal and the vertical centre plane of the side. While the small circle encloses the dancer's buttocks, his left elbow is supported on the large circle whose centre lies centrally on the back of the vase, on the C horizontal. The relation between the corresponding circle and the corresponding figure on the back of the vase is exactly the same — but apart from this, these two dancers do not resemble each other. There are four rosettes on the front of the vase. The right one lies on the point of intersection of the C horizontal and the large circle described about the centre of the right side of the vase, while all the others have their centres on the small circle of the front: the two to the left, where this circle intersects with the large circle on the left side of the vase. The man with the drinking-horn marks the vertical centre line of the front — his weight rests on this line. An arc about D′ through A traces his horn and the rosette right above, while the corresponding arc on the left describes the curve of his back. (The method by which these arcs were transferred is explained in connection with the next analysis.)

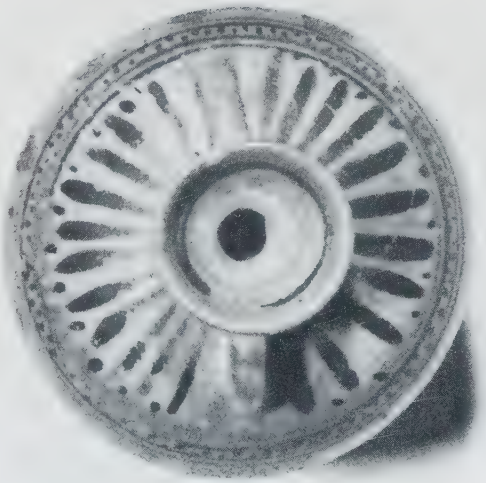
To sum up: both these circles were transferred to the vase four times, and the direction of the lines as well as their points of intersection are decisive for the design of the scene. As the pattern undergoes considerable change on being transferred from the flat drawing to the vase surface, *the decoration must have been designed on the vase*; this applies to a far greater extent to spherical vases than to lekythoi. The painter has conceived a scene incorporating five padded dancers and filling rosettes — he designed the figures and the compositon on the vase.

PLATE XXXIV A, B, C. The small Corinthian *amphoriskos* is also very nearly spherical at the point where the principal motive occurs. Here we have a *motive in movement.* A procession is moving slowly around the vase, an unbroken meander border above it also runs around the whole vase, *Fig. 72.* We quote from Corpus Vasorum, Norway (I): "In the frieze, sacrificial procession to the left: three girls with trays, bull, ribbons of consecration tied to his horns, a male(?) figure in chiton and cloak, woman flutist, two women." It is of the sixth century (Etn. Oslo M. 6809).

Analysis based on the author's own measurements. Scale 1:1.
The outline of the vase is checked by means of a templet.

On PLATE A the greatest perimeter of the vase has been projected from A to B, and from D to D′. *The height of the vase, AH, is equal to ½AB.* C, which marks the level of the greatest width of the vase, lies half-way between A and H, *Fig. 73* and *Fig. 74.* A ring of eleven lobes runs from the foot up to the lowest horizontal band on the body of the vase, and on both sides of the shoulder there are eleven leaves between the handles. We have inscribed an eleven-pointed star in the circle about C: Its perimeter was divided into eleven parts, and chords were drawn from the points of division across 3/11 of the perimeter. The resulting figure, an eleven-pointed star, is of great practical importance. — The scene on the vase is one of decided movement, as expressed by the chords when we follow them around the vase — this is here

Fig. 72 – 75
Corinthian amphoriskos.
Etn.M. Oslo 6809.



The following measurements have been taken:

AH 12.0 cm, diameter at H 3.0 cm
AK 9.4 cm, diameter at K 1.4 cm
AL 8.6 cm, diameter at L 6.2 cm
AP 7.9 cm, diameter at P 7.0 cm
AC 6.0 cm, diameter at C 7.7 cm
AG 3.5 cm, diameter at G 5.9 cm
AR 2.9 cm, diameter at R 5.2 cm
AV 0.7 cm, diameter at V 2.0 cm
diameter at A 3.0 cm

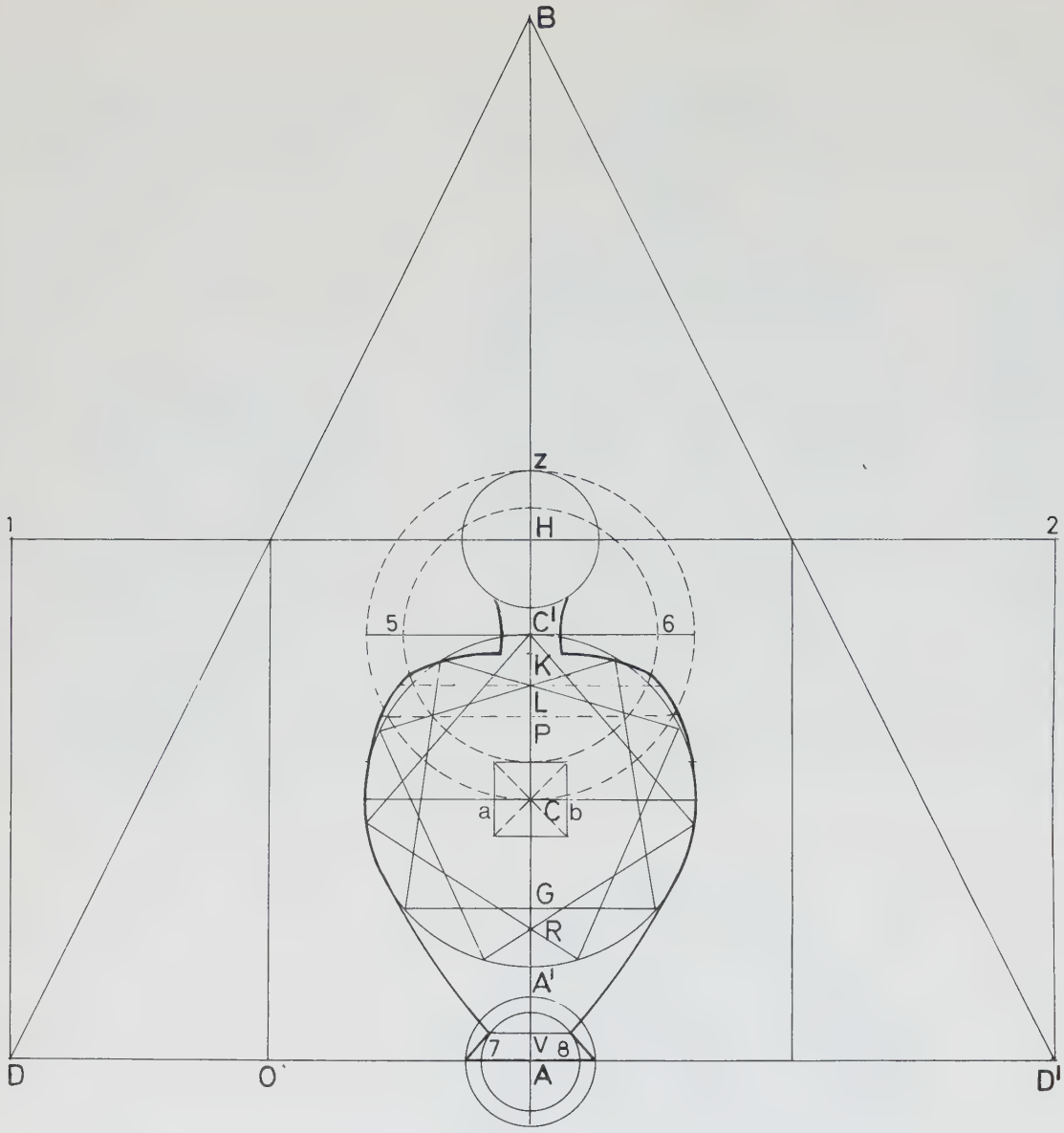
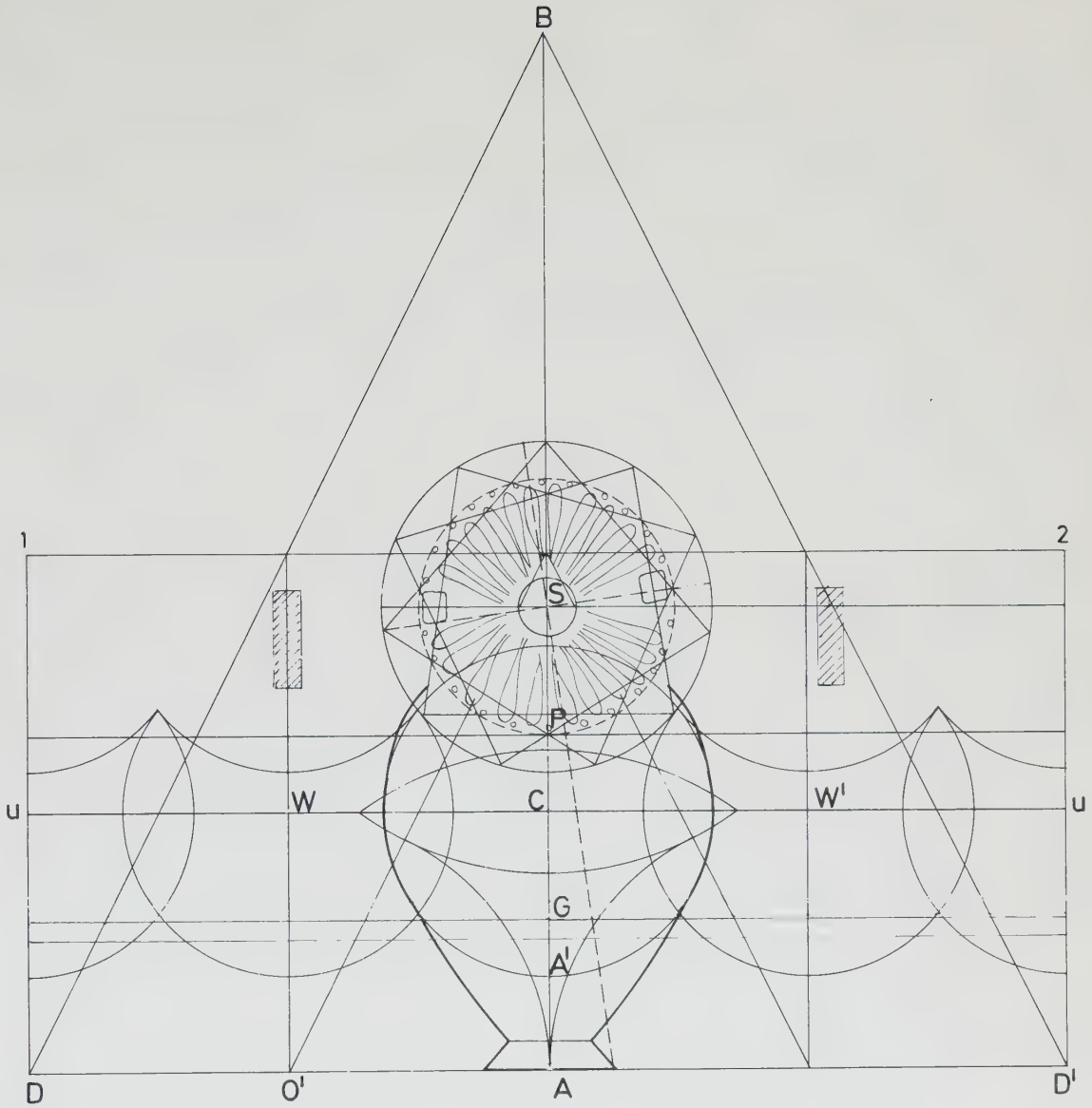


PLATE XXXIV A,B,C.
Corinthian amphoriscos. Etn.M. Oslo 6909. Analysis based on the author's measured drawing. Scale 1:1. Checked by a templet.

shown in horizontal projection about C. It now appears that the length of the bull is equal to one chord, and thus the bull never stops.

The circle about C is repeated about its vertex, C′ (dotted). A smaller circle has also been drawn about the same centre, its diameter equal to the length of the chords. The greatest width of the foot at A and of the mouth at H are alike. Both are determined, for the circle about H, which lies within the large circle about C', touches the latter at its vertex (z). The side of the small square constructed about C is equal to half the distance from C to C'. The lower chord of the eleven-pointed star determined G, which lies on the base line of the picture.

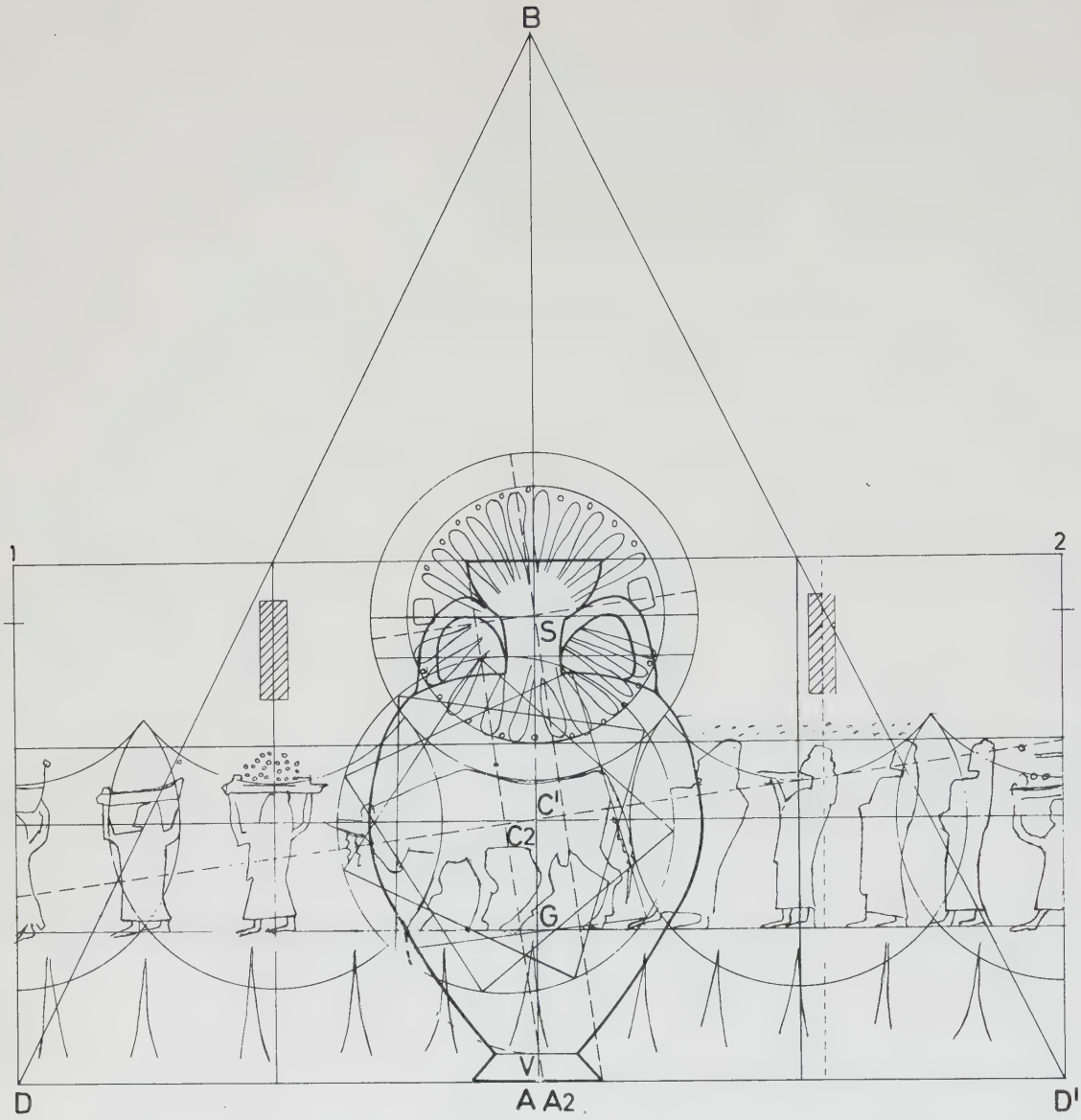
The outline of the vase is determined by the geometrical pattern in the usual way. It follows the circle about C from level P down to level G, and further down it follows arcs struck from 1 and 2 on the H horizontal vertically above D and D′. The upper part of the outline is traced



by arcs about a and b on the C horizontal (the right outline about a and the left about b) until they meet the smaller circle about C′. Arcs from the lower corners of the small square describe the shoulders until they meet the outlines of the neck traced by arcs from 5 and 6, tangential to the two chords of the star which meet at C′. The outlines of the head will be determined on PLATE B. The diameter of the foot at A equals Hz, while the inner circle about A has a diameter equal to the distance between two points of the star. The intersection points 7 and 8 thus determine both the height and the least width of the foot.

On PLATE B the greatest length of the picture panel has been projected equal to DD′ and we have repeated the circle about C centrally on both sides (about W and W′), and on the back (about u). Higher up, the circle is also repeated about S, on AB, in such a way that it touches the circles about W and W′.

It will be noted that the S horizontal does not pass through any of the points of the star nor through any of the points of intersection of its chords; if, however, the horizontal central axis



is tilted slightly down to the left about S (against the sun), it will be seen to pass through one handle and two points of intersection. *Fig. 75.* Here we have the explanation of the position of the handles which are apparently awry: *while the left handle is determined by the horizontal diameter, the right is aligned with this tilted diameter.* In fact, movement away from the original position is indicated even by the position of the handles (seen from above).

Seen from the side (on PLATE B) the left handle is determined by the vertical through W, while the right handle is slightly displaced towards the right of the corresponding vertical on that side. As PLATE C shows, the vertical through W is clearly marked by the basket that the woman immediately in front of the bull carries on her head; the central figure on the right side is in contact with the displacement of the right handle. The circles of the shoulder extend into the painted scene on the left and also on the back of the vase.

The bull takes up a great deal of room, and he tends to dominate the scene. He has already moved a fair way beyond the vertical centre line of the vase, AB, and we may assume that his

position is the result of a pivoting movement of the geometrical lines, according to the way of thinking with which we have grown familiar. This means that V (on PLATE C) represents the horizontal projection of AB, for this point lies at the centre of an (imaginary) square, whose upper horizontal side coincides with the diameter of the vase at G. The geometrical lines now pivot about V, until AB in its new position is perpendicular to the dotted diameter of the shoulder through S, which was determined on PLATE A. PLATE C shows the circle about C with the inscribed eleven-pointed star in its new position, about C2. The circle about C2 clearly limits the space at the disposal of the bull. On the right, it traces the front of the high priest, whose determined position lies between the bull and the flute-player; at the front, it touches the points of the bull's horns. Not only is the movement of the bull towards the left clearly marked in this way, but important lines from the original position are also of great significance in this new position of the lines. The figures are only sketchily indicated on the plate, their purpose is that of making the text, and the way of thinking on which this is based, more comprehensible.

A comparison of the constructional drawing with the actual vase shows more and more clearly that the geometrical lines were transferred to the surface of the vase, with its double curvature. The pattern is altered, and heavily marked lines on the figures indicate this alteration. Circles were transferred from the drawing, and employed on the vase. This appears especially clearly in the row of figures whose back is obviously the result of arcs changed on being transferred to the vase. This corroborates our theory that the eleven-pointed star was "rolled" around the vase from its position in the circle about C2 showing the solemn procession advancing.

PLATE XXXV A, B, C, D, E, F. The final investigation of this chapter concerns a *kylix, Hermes in Flight over the Sea* (Br. M. E 58) by Makron, about 490 BC. Diameter without handles = 28.7 cm. Full scale analysis based on the author's measured drawing. We shall study the shape of the vase as well as the circular, slightly convex picture inside it. This is an eye cup whose curved outside surface is also decorated, but we shall here concentrate on the picture of Hermes which may be regarded as a flat picture, a typical movement motif. Even though it was painted as late as 490 BC, it reflects none of the classic Greek painters' interest in the naturalistic representation of the human figure. Hermes is shown flying right across the picture surface, from left to right, and he holds his *caduceus* aloft in his right hand. Only the feathers on his right leg touch the crest of one of the waves, and his flowing draperies trail behind him on the left. *Fig. 76.*

On PLATE A we shall consider the shape of the vase. As usual, the basic dimension is the greatest perimeter of the vase. We set off this basic dimension from A to B and from D to D′ (outside the plate) and divide in Φ in the normal way. AB is divided in Φ at F below, and *this is the only point of this construction which is included on the drawing.* AF is divided in Φ at I above and at K below. AN = NF. The arc about A through I cuts the Φ hypotenuses from F at 1 and 2. The distance between these two points determines the external diameter of the border bounding the circular picture inside the vase. The points of intersection between the Φ hypotenuses from F and the arcs (centres outside the plate) about D and D′ through A (3 and 4), determine the internal diameter of this same border. Verticals through 3 and 4 intersect with the horizontal through N, at 5 and 6. From 5 and 6, dotted diagonals run downwards at an angle of 45°, intersecting with one another at S, which marks the height of the vase when *seen in profile*, and the centre of the picture when *seen from above.* The inner circle about S determines G below, and an arc about F through G determines the curve of the vase between the

Fig. 76
Kylix with round picture.
'Hermes in flight'.
By Makron. About 490 B.

Φ hypotenuses, when seen in profile. From that level the curve is drawn from x and y, Sx = Sy = SI. An arc (dotted) from A through 7 and 8 determines the distance between the extreme ends of the handles.

The diagonals from A intersect with those from S at 11 and 12. T is the centre of the lower horizontal side of the inscribed square of the inner circle. Diagonals from T meet the diagonals from A at 13 and 14, and we now construct circles about these two points through 15 and 16. These circles give the outline of the foot from the points of intersection with the diagonals from A and upwards. From the A diagonals the outline continues downwards drawn by circles from 11 and 12 until they meet the diagonals from G, and coincides with these down to the A horizontal, and thus the lower width of the foot is determined. The lower arcs of the handles are drawn about 17 to 18 through 19 and 20, all on the large circle about S.

PLATE B shows the horizontal projection of the *kylix*. We assume that the height of the figure is equal to the outer diameter of the border, OL, and its width is equal to the side of the inscribed square of the inner circle. The lower side of the square forms the "surface of the

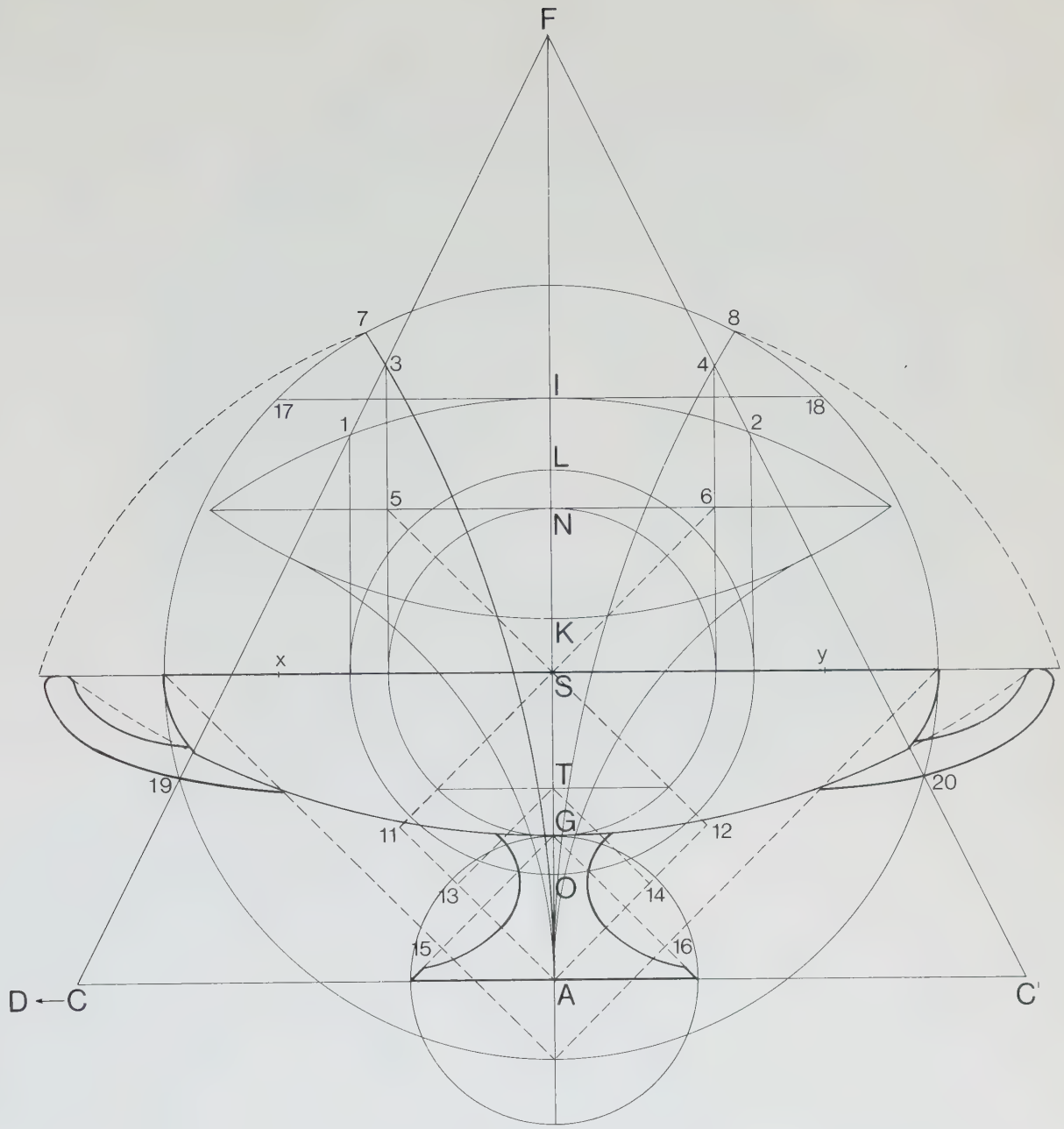


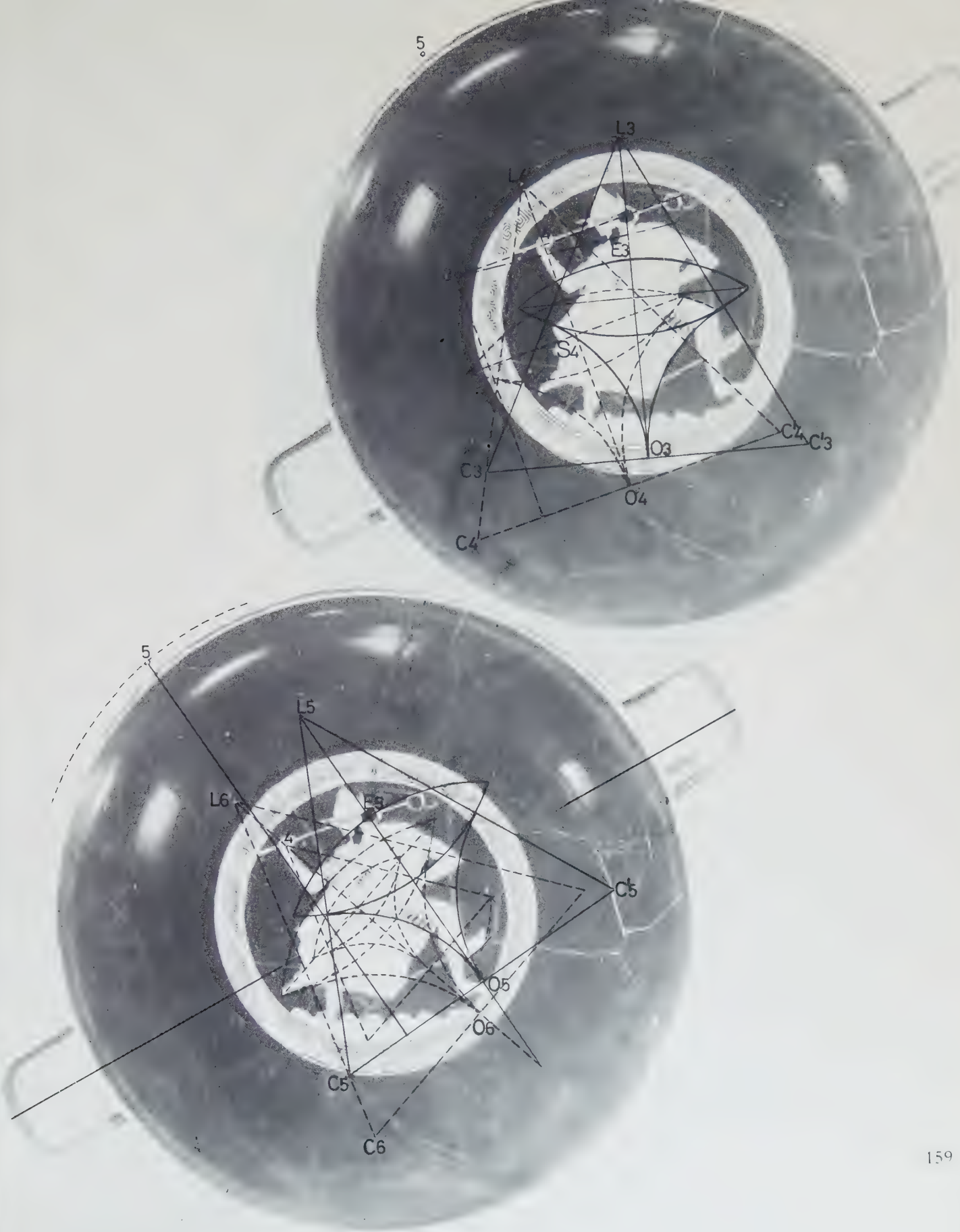
PLATE XXXV A,B,C,D,E,F. *Kylix with round picture. 'Hermes in Flight'. London B.M. E 58. By Makron. About 490 B.C. Analysis based on the author's measured drawing. Scale 1:1 (Pl.A), and on the museum's photograph (slightly over full size).*

sea", which determines the direction of the original position. OL is divided in Φ in the normal way. The O horizontal is produced towards the left and the L horizontal towards the right, and they meet the edge of the vase at P and P′, the two points which determine the axis of the handles. Circles are drawn about P and P′ with diametres equal to the horizontal marking the sea. The width of the figure is projected from O to A, and a pivoting movement of the geometrical lines is determined in the horizontal projection about 2. The geometrical lines pivot downwards into the dotted position, which is determined when the left, "vertical" side of the square centre V is tangential to the outer circumference of the border. The dotted "vertical" centre line marks Hermes' winged *foot on the crest of a wave*, and also his hand holding the *caduceus*. The interested reader will be able to find other points marked in this position.

On PLATE C we shall trace Hermes' first step in the flight from his position determined by the "horizontal of the sea" until he finally reaches the "axis of the handles"; the method employed should be familiar by now. The geometrical pattern pivots upwards about 3 in the upper, left-hand corner (dotted position) of the inscribed square, until the direction of the *caduceus* is determined by the dotted "horizontal" through O4 becoming tangential to the outer perimeter of the border. The vertical centre line is strongly marked in the dotted position through his right hand, as is also the arc about C'3 through O3, which is most striking.

Next, on PLATE D, the geometrical lines pivot about 4 (in Hermes' upstretched hand) until the corner C3 as C5 lies on the perimeter marking the outer edge of the border. The whole geometric pattern in its new position can be studied on the plate. The importance of the foot (to the left) is most striking as is the front line of his other leg. Here we observe a tremendous force in action.

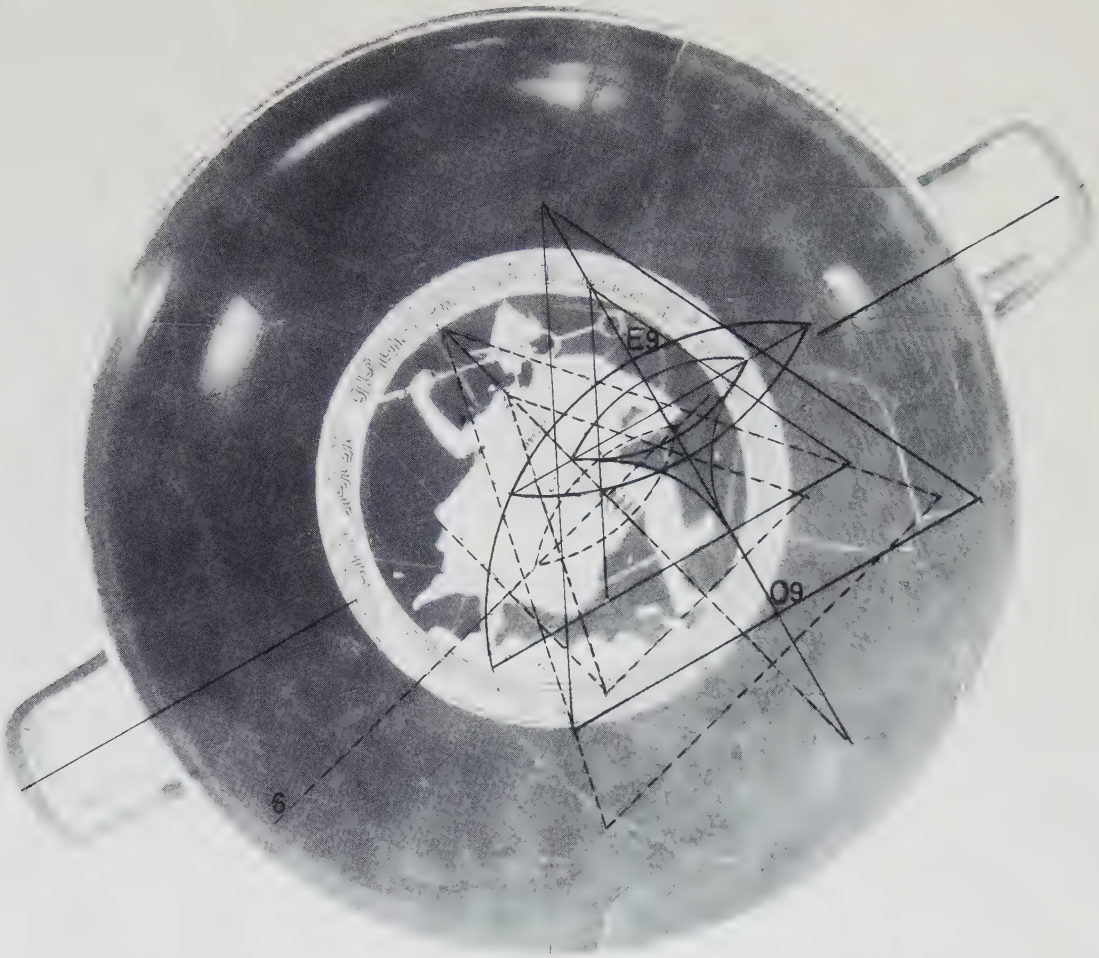
The geometrical lines continue their pivoting movement this time anti-clockwise; they pivot about 5, the point of intersection between the extension of the left side of the square in position 5 (Pl. D) and the perimeter of the vase in the same position, until O6 lies on the inner perimeter of the border. On PLATE E the arc about C7 through O7 gives Hermes' left thigh, while the dotted " vertical" centre line determines the axis of the lyre. We should also note the signific-


5
L3
L6
E3
S4
O3
C'4
C'3
C3
O4
C4
5
L5
L6
E5
C'5
O5
O6
C5
C6

ance of the dotted position as regards the course of the folds flowing towards the left. The dotted "horizontal" centre line meets the perimeter of the vase in the dotted position at 6 on the left.

On PLATE F the geometrical pattern then moves back to some extent, pivoting about 6 in such a way that *the main position* is orientated along the axis of the handles. The "vertical" centre line of the main position runs down through the middle of the ring of the *caduceus*, and the wing on his left foot rests on the projected line of the inscribed square — the side which marked the sea in the original position. The large arc about O9 through E9 encompasses the wing on the surface of the sea. Here, too, the upper edge of the right thigh is traced by an arc, this one forming part of the Φ division of the centre line of the inscribed square. Now we get the impression that Hermes with a *caduceus* in his upstretched hand has moved a long distance forward in the picture, and that he is still moving on at a good speed, trailing his heavy garments behind him. Here we have a masterpiece from the great period of Greece, conceived and executed entirely according to Archaic methods. The motive is the abstract concept of *speed* or *flight*; the artist mastered the methods to perfection.

Some of the finest Greek vases are signed, some bearing one signature, others two. Sometimes one artist was responsible for the design and execution of the vase and for its painted



decoration, sometimes these are the work of specialists. The vase painters were highly esteemed during their life-time. In this connection it is worth noting that E. Loewy based his research for his work on the great painter Polygnotos (Vienna 1929) on the evidence of vase-paintings. Polygnotos' monumental paintings are lost, they are known to us only through literature. But the vase-paintings of his day give us some idea of what his free paintings looked like. The methods used by the vase-painters as shown in this book may give us an idea of how Greek monumental paintings were conceived.

Analyses of Sebastiano Serlio's vase drawings. Venice 1559.

We have found support for our theory of the flush construction of rectangular blocks in Ceasariano's drawings illustrating his Vitruvius edition of 1521. We found this method of proportioning represented in his two drawings of the Cathedral in Milan. Similar support is provided by Sebastiano Serlio's vase drawings in "*Libro Primo d'Architettura", which appeared in Venice in 1559. On these drawings we find a number of geometrical lines marked. From our point of view the indication of a horizontal at the widest point of the vase is particularly interesting, as is also the fact that the horizontal projection of the vase is shown on the same drawing as the projection of the outline (the vertical projection), Fig. 77.* This in itself provides evidence in support of the theories we have submitted with regard to Greek vases.

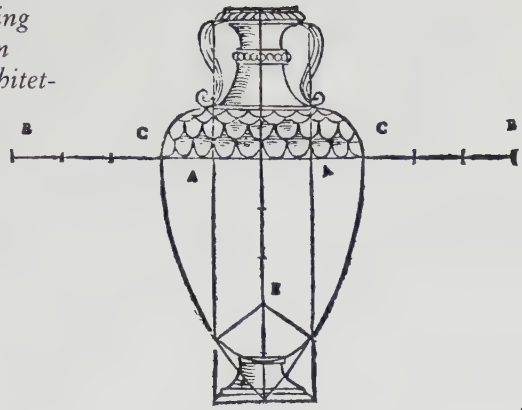
A question immediately arising is this: is the horizontal at the widest point of the vase, which is shown produced beyond the vase, in any way connected with the greatest perimeter, which must occur at this level? A close study of the drawings shows that this horizontal line forms part of the flush projection of the greatest perimeter, the construction which we have above demonstrated in our analyses of Greek vases. This construction has been superimposed on Serlio's drawing, and zy has been determined as equal to half the perimeter at level C in the usual way.

Plate XXXVI. By employing the methods by which we analysed Greek vases, we can determine Serlio's drawings further, on a purely geometrical basis. The length of the perimeter is set off from A to B, and from D to D′, and AB is divided in Φ in the usual way. The plate shows that AC is equal to the minor of AB, while the height of the vase is equal to the major, AE. The two verticals marking the diameter of the foot at level A are determined by the dimensions of the neck, a feature we frequently encountered in our analyses of Greek vases. A circle has been struck about C′ through E. The circumscribed square of the circle is also shown; the vertical sides of the latter, produced through the body of the vase, are identical with Serlio's two verticals which determine the greatest diameter of the foot. Arcs about Y and Y′ trace the outline of the vase from level C until they meet the closer of the two verticals. Lines are now drawn from these points of intersection towards a and b on the C horizontal. These oblique lines cut AB at the centre of the outline of the body down to the foot. The outline above level C is the result of arcs about 1 and 2 through the lower corner of the square about C′. From here, arcs about c and d on the Φ hypotenuses follow the outline of the neck.

Plate XXXVII. The next vase drawing, *Fig. 78,* has many points in common with the one above. Here, too, the greatest diameter occurs at the lower point of the Φ division of AB. The outline of the body of the vase from this level downwards is constructed in the same manner, and the centres are y and y′. In this vase the outline up towards the neck follows the circle about C. On our drawing we have produced upwards the large arcs which trace the outline of the body of the vase, and also lines xC′ and x′C′. The points of intersection between these arcs and lines determine the level at which the lid rests on the vase, as well as the diameter at this level. The

Fig. 77 – 79
Vase drawings by Sebastiano Serlio in his Libro Primo d'Architettura. Venice, 1559.

PLATE XXXVI
Analysis of vase drawing by Sebastiano Serlio in his Libro Primo d'Architettura. Venice 1559.



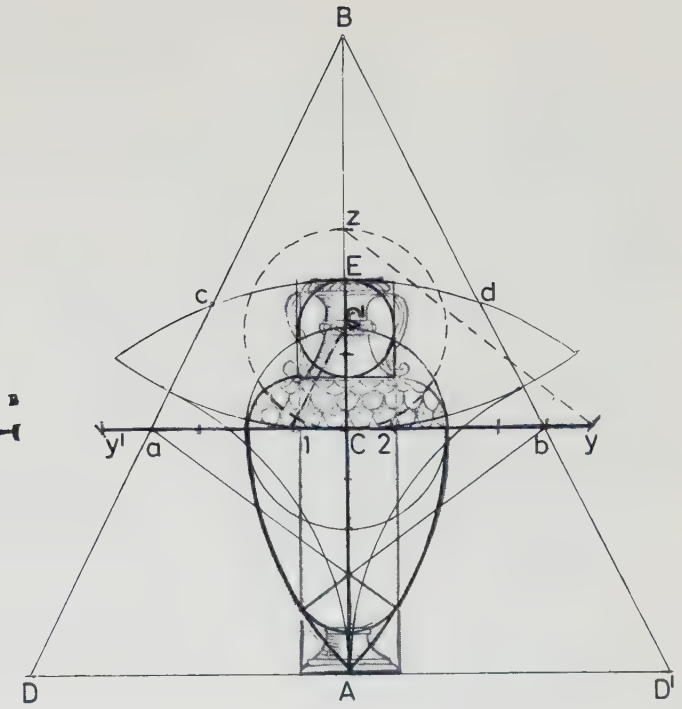
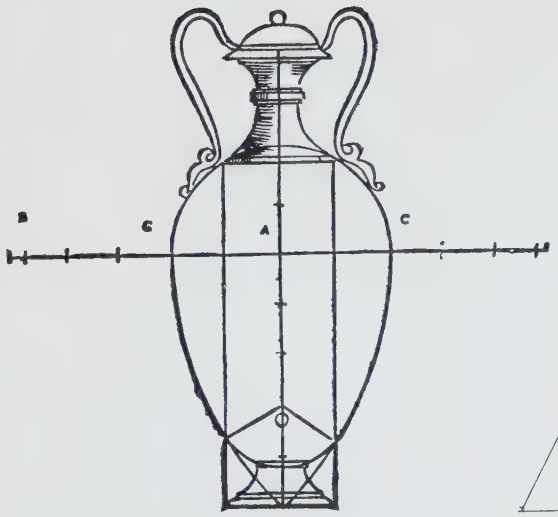
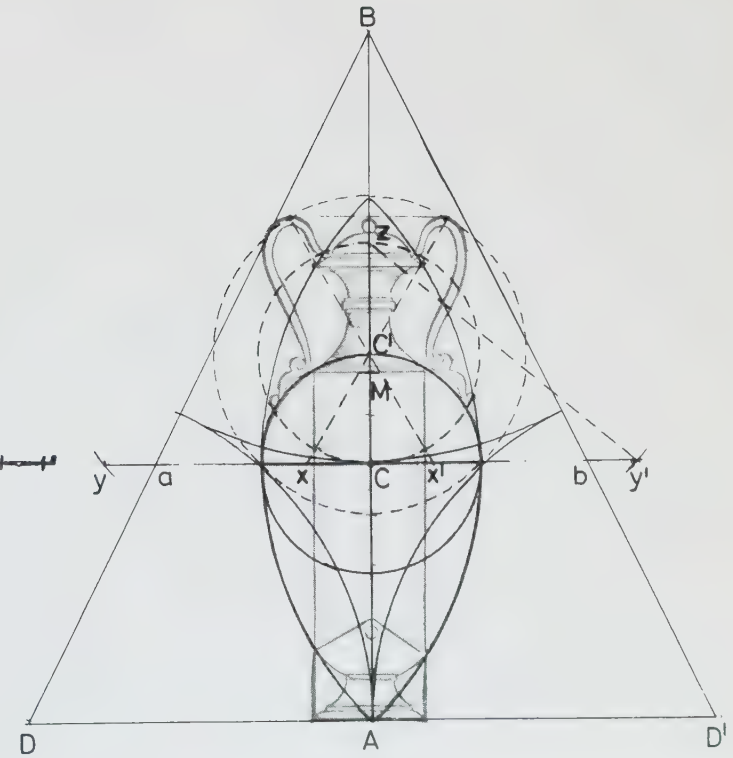


PLATE XXXVII
Analysis of vase drawing by Sebastiano Serlio in his Libro Primo d'Architettura. Venice 1559.





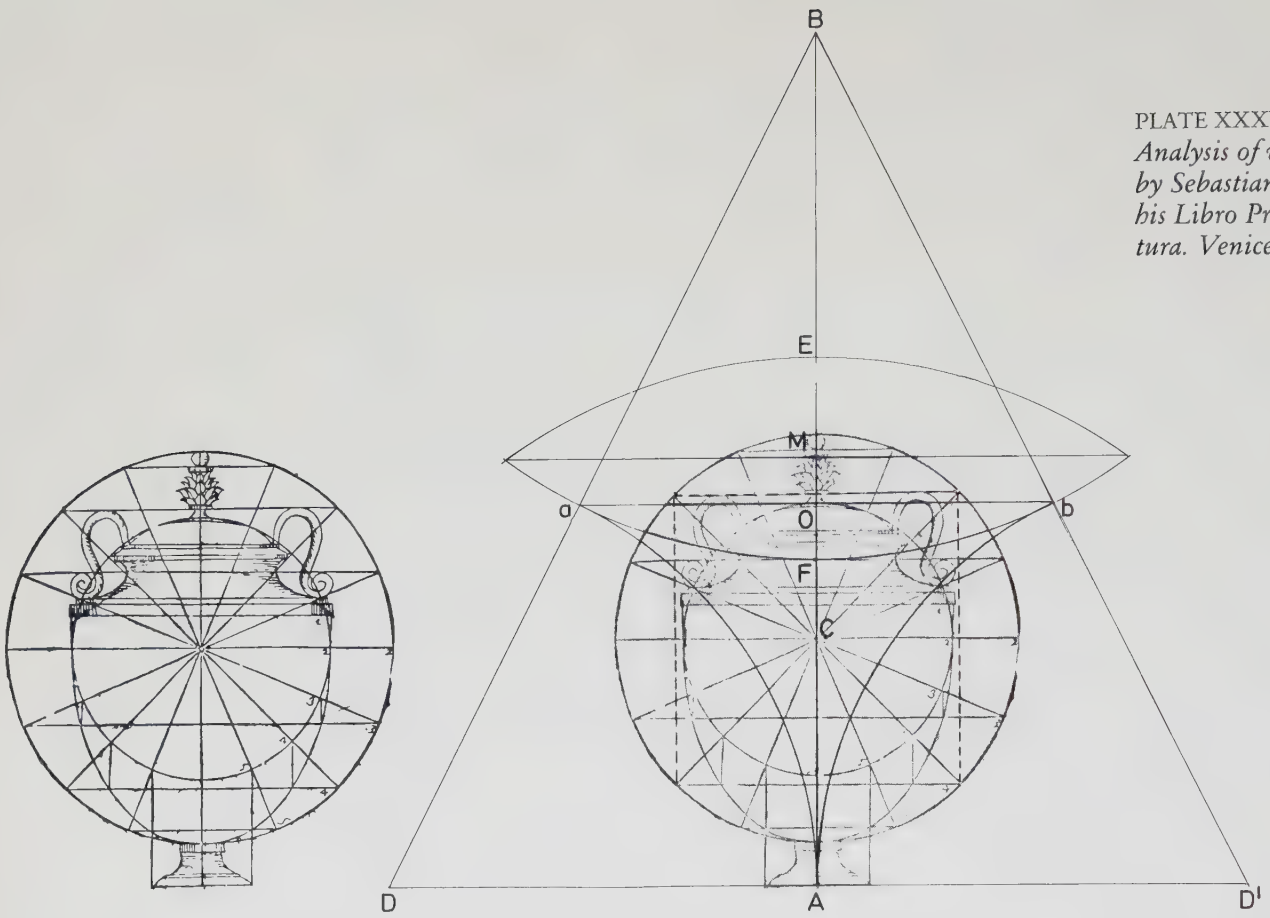


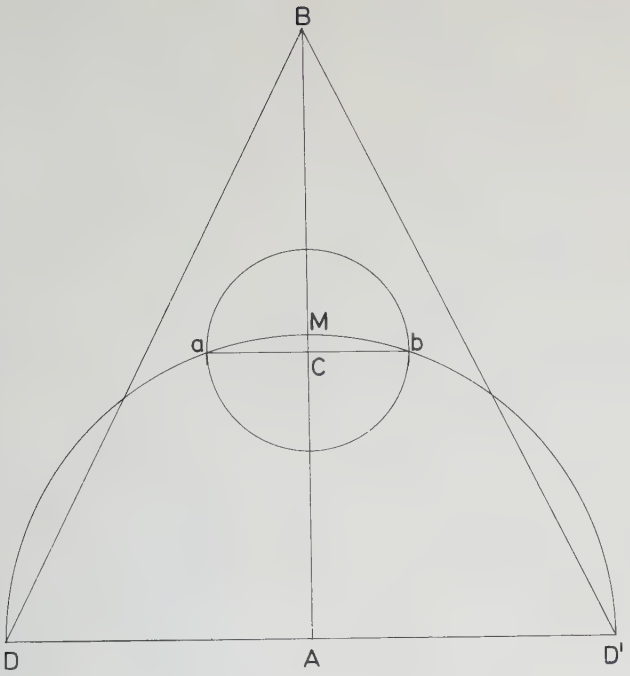
PLATE XXXVIII
Analysis of vase drawing by Sebastiano Serlio in his Libro Primo d'Architettura. Venice 1559.

largest dotted circle about C′ passes through the points of intersection of the C horizontal and the circle about C, and touches the top of the handles. The points of intersection of the produced oblique lines and this circle determine the total height of the vase with lid. The dividing line between the body and the neck of the vase lies at level M.

Plate XXXVIII. Our next analysis concerns a vase drawing of an entirely different type. The most immediate point of interest here, *Fig. 79,* is that the circle about C, which marks the greatest width of the vase as seen from above is shown on the drawing of the vase seen from the front. Its vertex passes through O (determined by a and b, the points of intersection on the Φ hypotenuses) – it depends from O. The Φ division of AB also determines other dimensions, such as the diameter of the foot, level A (= FM). The outline of the corpus is drawn by a circle with radius equal to the diameter at C from the end of this, passing from level C down to the vertical tangents to the foot.

The large circle about C (radius = EF) encompasses the upper part of the lid at the top, and the body of the vase at the bottom. We have inscribed a square (dotted lines) in the circle, bisected to AB – the diagonals of this square appear on Serlio's drawing (diagonal 4). The upper side is tangential to the outside of the top of the lid. Diameter 3 is determined by the F horizontal, where it intersects the large circle, and 5 is dependent on the width of the foot.

The above analyses show that Sebastiano Serlio's drawings of "Greek" vases are based on an entirely Greek way of thinking. In shape, his vases may be Hellenistic or even Roman, but the method is Greek in all its main features. The use of a "governing" circle may be a late phenomenon, or its use in Greek art may simply have escaped our attention. In any case, it appears to be in strictly logical keeping with the geometrical way of thinking.



Comments on Cretan and Mycenaean art

PLATE XXXIX
Storage jar from Knossos, 'Kamares' style. Herakleion Museum Galleries. Analysis based on the author's measurements. No number.

Our investigations of Greek art started with the Archaic period, with works created some three to five hundred years after Crete and Mycenae had fallen. We chose this period for our early analyses because of the striking relationship between a number of Archaic sculptures and Egyptian statues. Their point of contact, we found, lies principally in a method which offered so many hitherto undiscovered possibilities that, in the hand of the Greeks who developed it further, it was after some centuries to lead to what is commonly known as the "Greek Triumph". But the contact between Egypt and Crete/Mycenae was at least as close and at least as lively when these realms were at the peak of their power as any later Greek/Egyptian contacts. This historical fact prompts the question: can we trace this same geometrical way of thinking also in the art of Crete and Mycenae? Its presence there would not be in the least surprising.

We shall not deal with this question at length, hoping that some analyses will provide a clue to the answer. Test pieces are very readily available in the museum at Heraklion on Crete: on the floor, in the corners of galleries I and II, which include exhibits from, for instance, Knossos, there are some large storage jars. (These vases were without numbers.). They are simple in shape, and some of them are decorated with spiral ornaments. The catalogue describes them as *pithoi* of the polychrome "Kamares" style. We shall not study these jars in detail, merely examine the principal proportions of one of them, in order to find out whether these depend on the greatest perimeter of the vase in the way which we established in the chapter on Greek vases.

PLATE XXXIX. The vase has a height of 0.81 m, a width of 0.54 m. It is widest at the top. *Fig. 80.* The longest perimeter runs from A to B and from D to D'. The height of the centre C is determined by a and b, the intersection-points between the arc through M, and the perimeter about C.

Fig. 80
Minoan jar. Heraklion Museum.

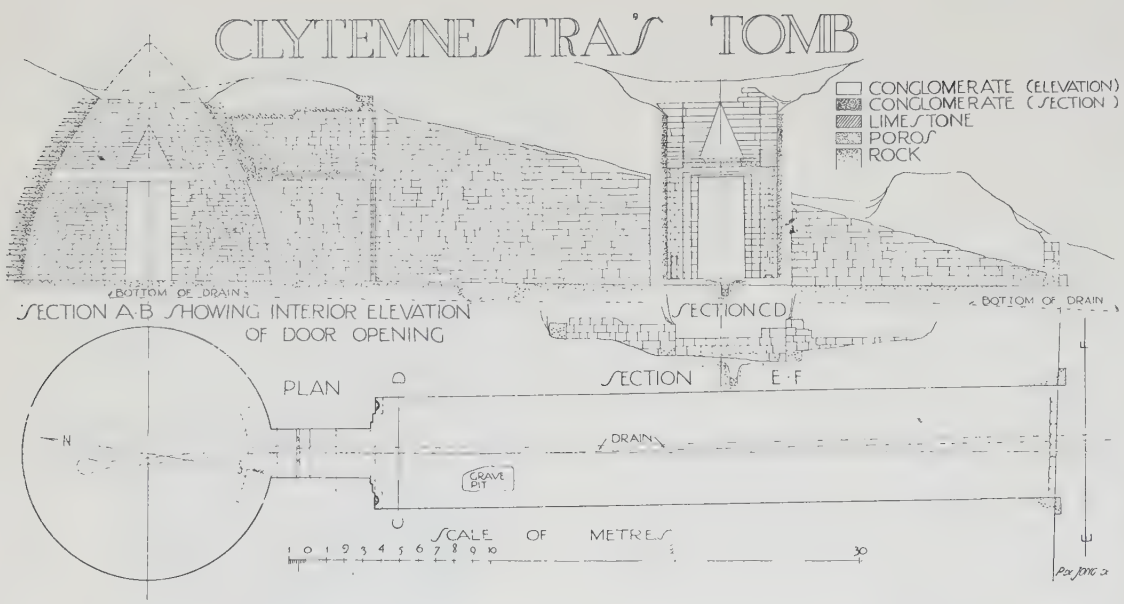


Fig. 81.
Clytemnestra's Tomb.
Mycenae. About 1300 B.C

Fig. 82
Egyptian Ostracon. Cairo
M. 25184.

Again an example of the way of thinking with which we are familiar. The further development of Cretan pottery during its most bizarre period cannot be examined except in a separate study, but it seems unlikely that the basic principles, once adopted, should later have been abandoned.

Mycenae contributes to our study large buildings, so that both plan and height can still be measured today. Nine *tholos* tombs are preserved, all dating from the period 1560 – 1300 BC. Although Mycenae's Acropolis was destroyed in 468 BC, after the battle of Thermopylae, these tombs have been visibly in evidence ever since they were erected. A building such as Atreus' Treasury, also known as Agamemnon's tomb, is bound to impress the visitor with its quality of great architecture. Alan J. B. Wace, who published a study of these buildings in 1949, expressed himself thus: "That unknown master of the Bronze Age who conceived and created it was as bold a designer and as able an engineer as any great architect of historic times. His artistic instinct planned the perfect proportions. His engineering skill calculated thrusts and stresses and how to counteract them and with his knowledge of materials brought the great design to triumphant completion. Archaeology here leads us to a fuller appreciation of a true artist."

All the *tholos* tombs have a long, narrow passage, the *dromos,* which leads, via a short, narrow doorway, into the beehive tomb proper. Unfortunately none of the published sets of measurements are today considered reliable. In spite of this, we shall briefly study Wace's drawing of Clytemnestra's tomb, *Fig. 81.* Cutting up a photostat copy of his measured drawing, we lay the plan and the greatest height flush with each other in the normal way. The resulting similarity with the Egyptian drawing of a temple on an ostracon – although of much later date – is striking. On the Egyptian drawing, the doors are shown flush with the picture surface, so that the horizontal plane and the vertical plane are shown as a connected whole on one drawing *Fig. 82.*

PLATE XL. We have every reason to believe that the *tholos* tombs were designed on the

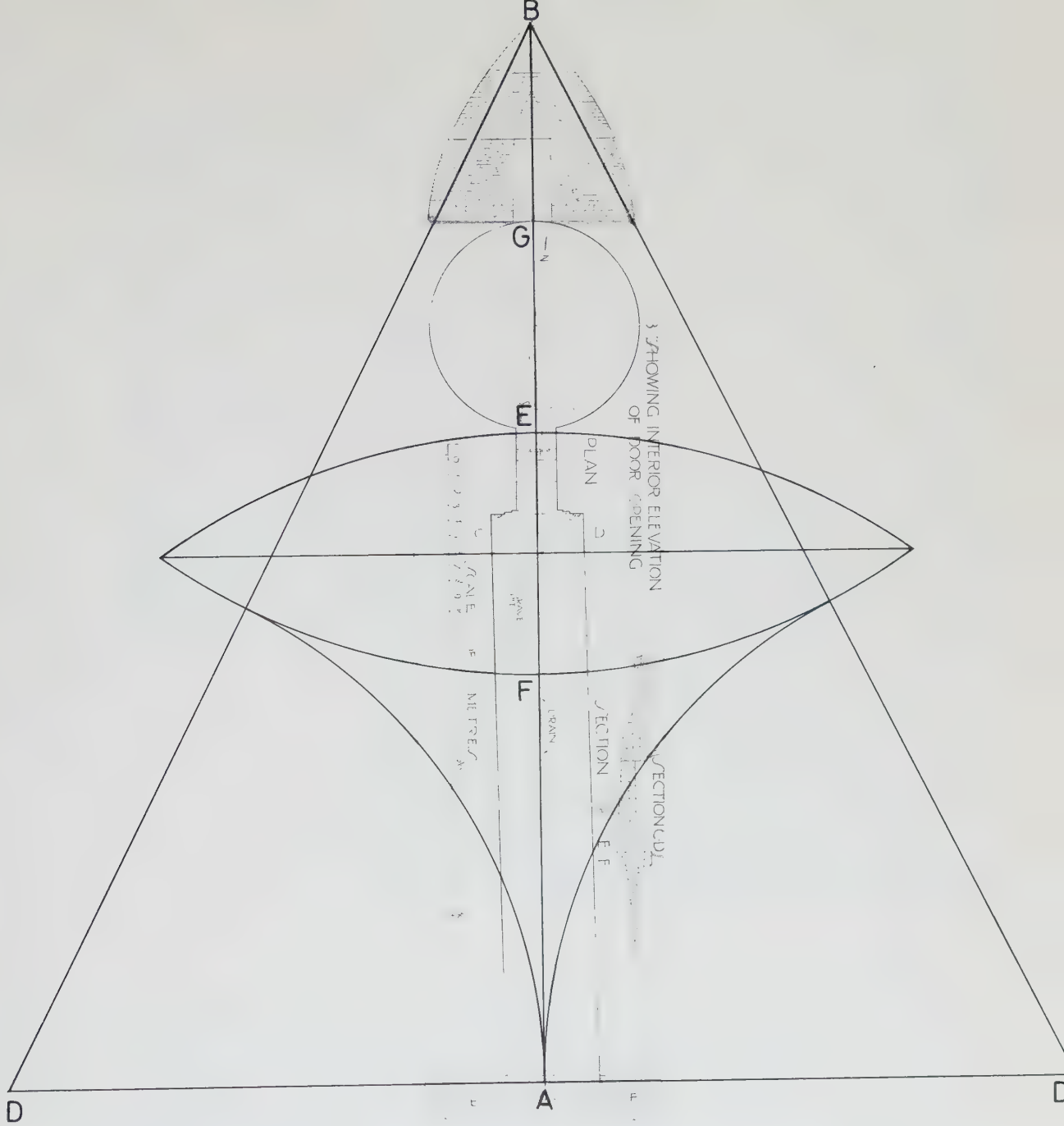


PLATE XL
Clytemnestra's Tomb. Mycenae. About 1300 B.C. Analysis based on Alan J.B. Wace's measured drawings in 'Mycenae', Princeto, 1949.

basis of the sum of the length of the entire tomb, and the height of its cupola. Wace states that the *tholos* of Clytemnestra's tomb measures 13.4 m in diameter, and he estimates the height, which is uncertain, to be approximately equal to the diameter. The doorway is 5.4 m deep, while the *dromos* measures 37 m on the drawing. The following must be noted: doorway + dromos = 42.4 m, on Wace's drawing. If we divide this dimension by 3.14 (π), we arrive at the figure 13.5 m, in other words, the diameter of the *tholos*. Thus *the perimeter of the tholos determines the length of the dromos + the doorway*. On our plate, this dimension should equal the major of the basic dimension AB. We calculated according to the Φ division:

Fig. 83
Relief. Mycenaean. Athens N.M. (No number).

AB equals the major FB × 1.618 42.2 m × 1.618 = 68.6 m
— AG (*dromos* + doorway + diameter *tholos*) 42.4 m + 13.5 m = 55.9 m
= Height of *tholos* 12.7 m
The top of the *tholos* is lost. While Wace states that the height is "approximately the same as the diameter", Mylonas gives it as 12.9 m in George E. Mylonas, *Ancient Mycenae*, London 1957. The whole analysis can be proved or disproved only by a new survey.

In the realm of two-dimensional art we can also find material which enables us to demonstrate that the work of the artists of Mycenae was based on the same way of thinking as that with which we are familiar from Egypt. A relief on one of a number of Mycenaean *steles* is of particular interest in this connection, for it depicts a chariot scene. In *The Figured Mycenean Stelai* (American Journal of Archeology 1951), G. E. Mylonas surveys all the known Mycenaean *steles*. Seventeen were found in the grave circle of Mycenae, and eleven of these had pictorial decoration; of these, three — all from tomb V — bore chariot scenes. Mylonas discusses a number of such scenes from Cretan and Mycenaean art, all of them with one or two men standing in the chariot, one of them leaning sharply forwards. He interprets these scenes as chariot races and not, as was the earlier interpretation, as battle or hunting scenes. He does

Fig. 84
Relief of Amenophis II shooting at a target. Karnak.

not, therefore, adduce the Egyptian parallels from the days of Ramses III and Tut-Ank-Amon, which are of particular interest for our purposes. These kings are also shown in their chariots and, regardless of whether they are shooting at a target, hunting wild beasts, or engaged in battle – the god is with them. He may stand *behind* the Pharao on the moving chariot, and he may be symbolised by a bird hovering above his head and aiding him, *Fig. 84.* The Mycenaean artist did not work in so naturalistic a manner, his symbols seem to be simpler, more geometrical. The method, however, is the same, as the analysis clearly reveals. These *steles* were carved when the "New Kingdom" of Egypt was at the height of its power. In *Geometry in Egyptian Art* we have accounted for the method in which the Egyptian chariot scenes were designed, and we shall now investigate how the same way of thinking underlies the Mycenaean relief, which can be studied in the National Museum of Athens (without number).

PLATE XLI A, B, C, D. Wace gives the height of the *stele* as 1.33 m, the width as 1.06 m, but he does not give the measurements of the relief. We shall here attempt to analyse the relief from the photograph, *Fig. 83, aided by control meansurements taken at the museum. On* PLATE A, scale 1:9, AB is equal to the sum of the height and width of the relief. M is the mid-point of AB, which is divided in Φ at E (above) and F (below). It appears that a and b, the points of intersection between the arc about A through M and the arc about B through F determine the inner width of the relief. This is equal to AG, and thus the height of the relief, GB, is also determined. N is the mid-point of GB. An arc is struck about G through N, and a vertical is constructed through x, its point of intersection with the left Φ hypotenuse. This vertical is tangential to the front of the wheel, and its diameter is equal to the distance between the vertical and the edge of the relief. V, the centre of the square bisected by AG, must be regarded as the horizontal projection of the vertical centre axis of the horse, (cf. *Geometry in Egyptian Art*), while W is the horizontal projection of the vertical centre axis of the wheel. A vertical is constructed through y on the V horizontal where this line is cut by the arc from D′ through A.

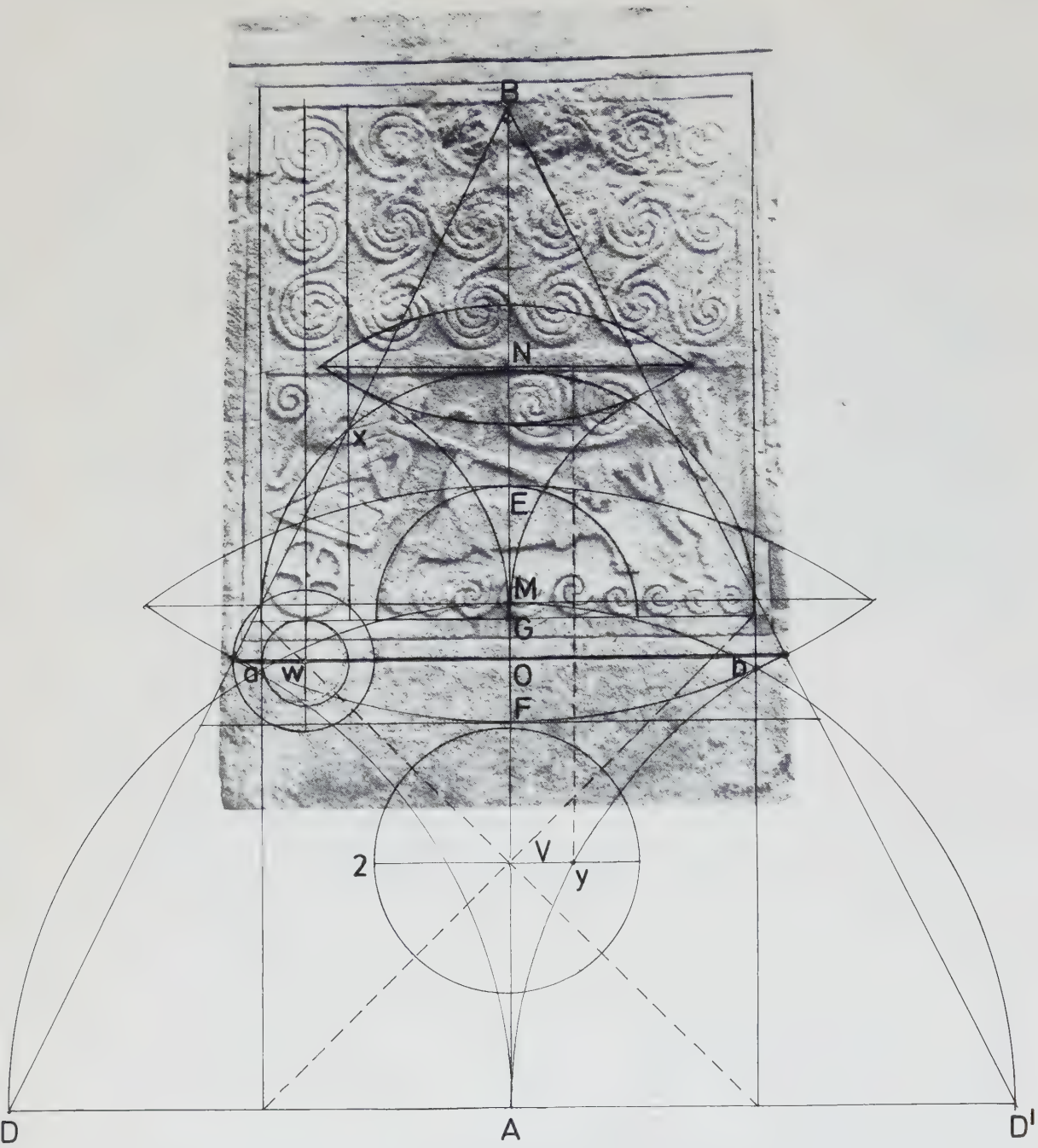
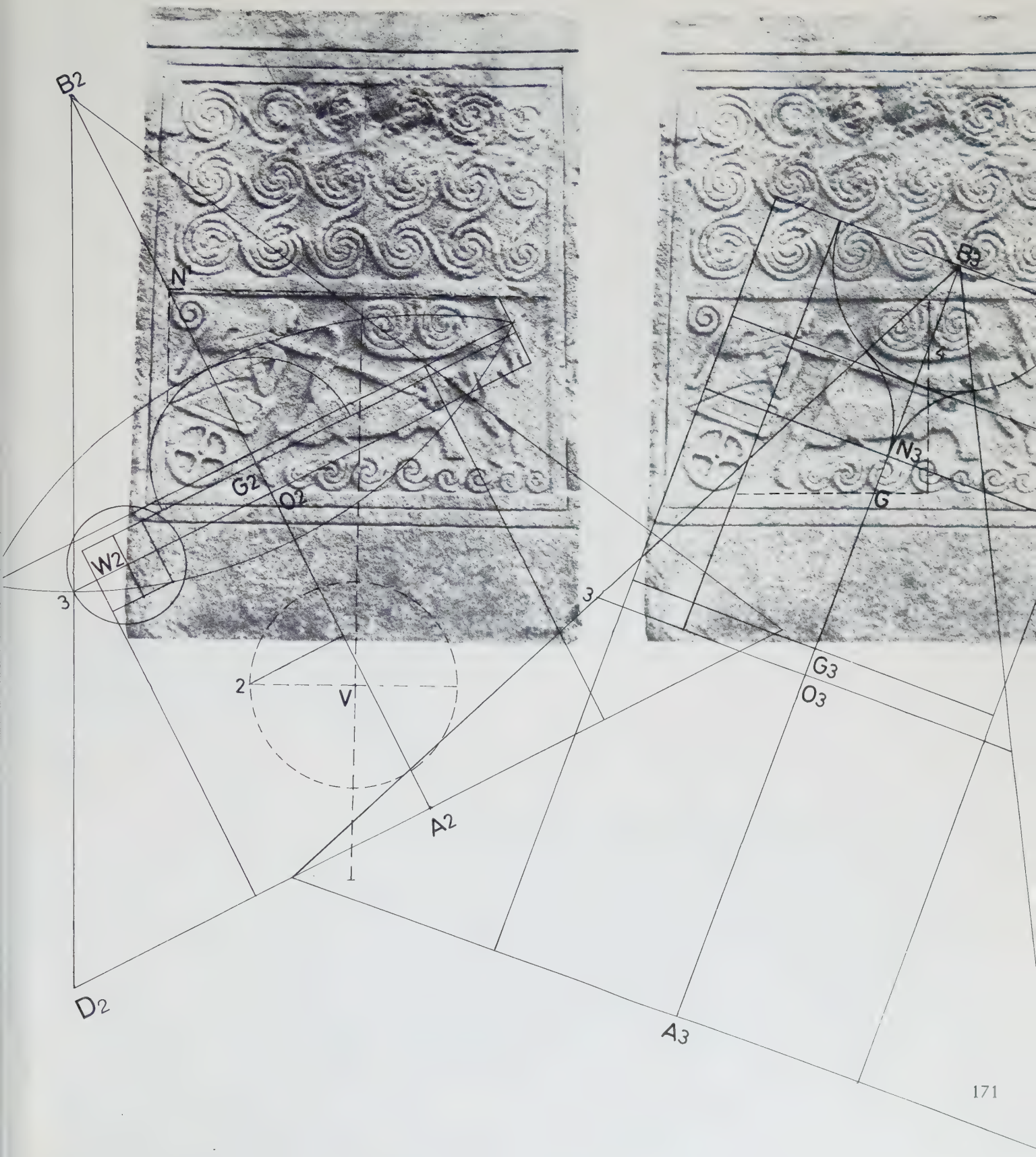


PLATE XLII A,B,C,D.
Relief from Mycenaen Tholos. 'Chariot Race'. Athens N.M. (no number). Analysis based on the Museum's photograph. Measurements checked by the author. Height of stele 1.33 m, width 1.06 m (after Wace.)

The dotted line is the mid-line of the double spiral. The front and the hind leg of the horse both depend on an arc about G through E. The radius of this arc is repeated about V, and the resulting circle about V marks the space at the disposal of the horse, seen from above. Point 2 marks the hind hooves of the horse; about this point the geometrical lines pivot backwards to the left, with the result that the horse rears on his hind legs. The geometrical pattern pivots about 2 until N as N2 lies on the vertical through a. We notice the right intersection point of the two Φ arcs from A2 and B2, which is most remarkable. Both underarms are strongly dependent on this position of the geometric pattern.


B2
N2
G2
O2
W2
3
2
V
A2
D2
B3
N3
4
G
3
G3
O3
A3

The arc about A2 is also decisive for the man in the chariot, for it encloses the sword in his left hand.

The geometrical lines pivot towards the right about 3, and the movement is geometrically determined by A3B3 passing through G. In this position, PLATE C, both the long lance and the bottom of the chariot are "horizontal", while the axis of the wheel is "vertical". A3B3 cuts the vertical through y (PLATE A) in the double spiral, and point 4 is determined.

The geometrical lines pivot about 2 until A4B4, on PLATE D, covers the dotted y vertical of PLATE A. In other words: the geometrical pattern has moved forwards, and at the same time the horizontal through N4 is tangential to the front hoof; the horse is running on the mid-line of the relief. G4B4 is here clearly marked by the symmetrical ornament immediately above the horse's head, both sides of which reproduce the dimensions of the wheel. It is tempting to ask whether the double spiral also symbolizes higher powers, who have decreed the result of the race? The double axe of Crete also suggests itself, another markedly symmetrical symbol.

Fig. 85
The large sarcophagus from Hagia Triada. Museum of Heraklion. Gal. XIV, case 171.

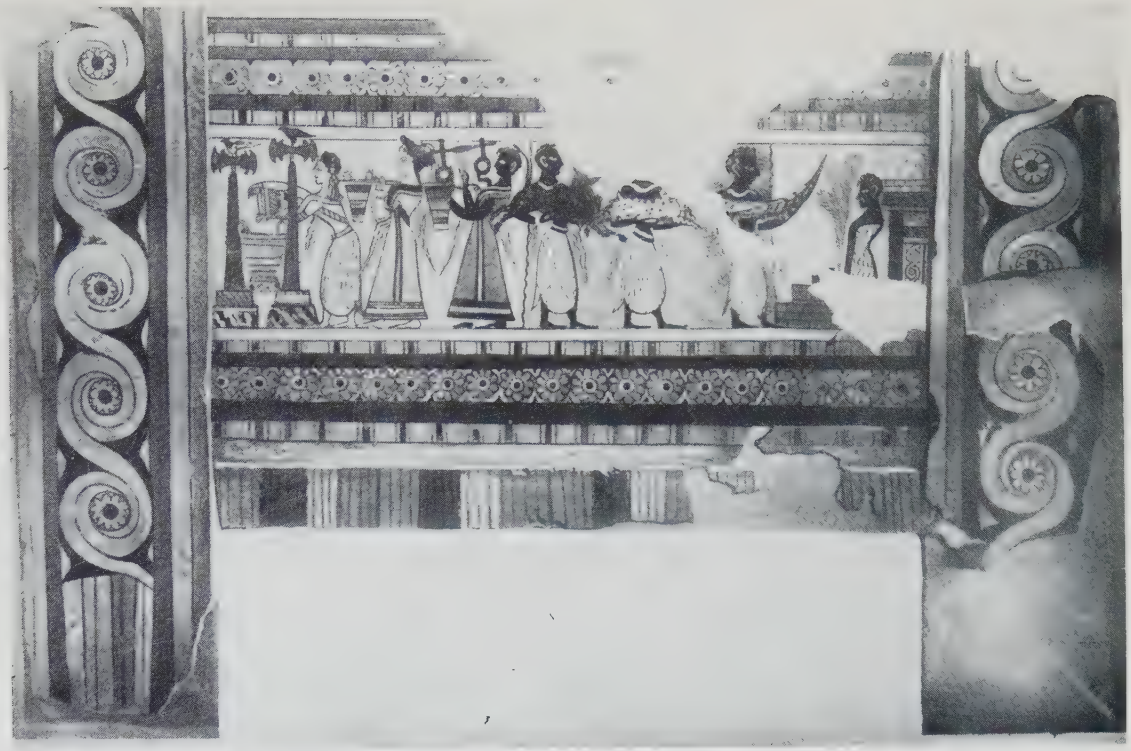


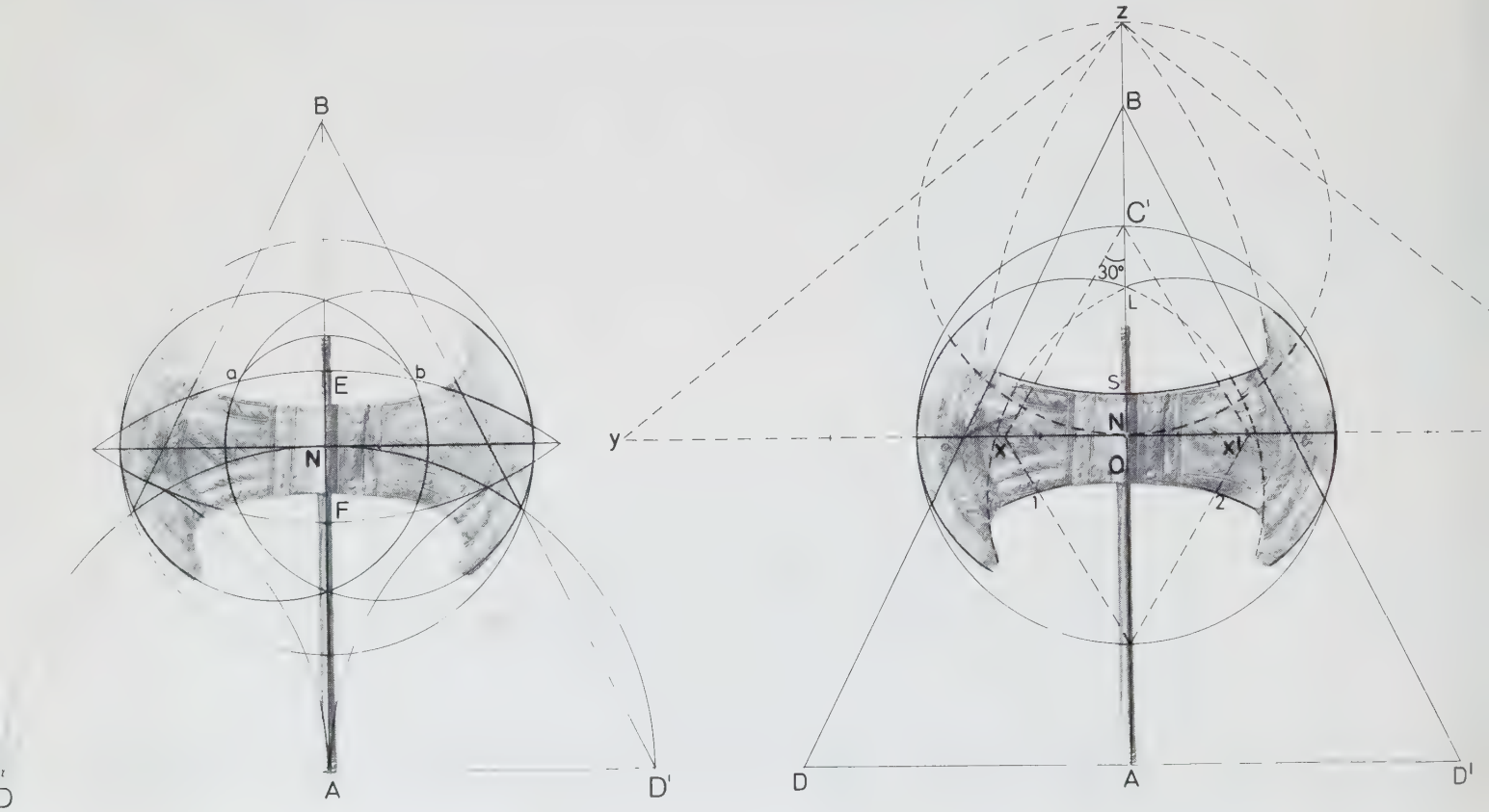
Fig. 86
Double axe of gold. From Marinatos I, Kreta und das mykenische Hellas, Munich 1959.

The famous, large sarcophagus from Hagia Triada, *Fig. 85*, is unfortunately practically inaccessible for our purposes, as it stands in a glass show-case. It was impossible to obtain the necessary measurements. With its al fresco decoration almost intact, this sarcophagus is one of the most important of Cretan finds. On one of its sides, two votive scenes are shown: on the right, three men are carrying a ship and two calves towards a figure, assumed by Wace to represent the deceased; on the left, three women in procession, carrying libations. And the object of their sacrifices? No image of any deity, but two double axes surmounted by a sitting bird. We are reminded of the Egyptian chariot scenes, where the bird hovering above the Pharao, the deified king, directs his arrows. On the Cretan sarcophagus we have a bird above each double axe, the recipients of the votive gifts.

PLATE XLII A, B. We have analysed a double axe of gold, using for the purpose a photograph from Marinatos' "*Kreta und das Mykenische Hellas*", München 1959, slightly larger than life-size, *Fig. 86*. From cutting edge to cutting edge, this axe measures 8.6 cm. It appears that the basic dimension AB is equal to half the perimeter of a circle coinciding with both the cutting edges, whose centre N lies on the mid-point of AB. AB is divided in Φ at E and F, and EF determines the length of the radii of the two circles describing the two cutting edges of the axe. a and b are the inner points of intersection between the E arc and these two circles. The lowest point of the handle is determined by A, while its uppermost point is cut by an arc about N through a and b. On PLATE B, the flush construction of half the perimeter of the large circle centre N is shown in dotted line on both sides, so that y,y′,x and x′ are determined. The isosceles triangle xx′C′ is repeated upside down, AS = Sz, and NO = NS. Arcs about y and y′ through z describe the inner edge of the upper parts of the axe, while the slack curve describing the upper outline between the Φ hypotenuses forms part of a circle about Z through S. The arc through O has the same radius. The circles describing the two cutting edges intersect at L above, and arcs about 1 and 2 through L trace the inner edge of the lower parts of the axe.

It appears quite clear that the geometrical flush construction of half the perimeter was not merely known and used, not only did it fulfil the function assigned to it, that of calculating a straight line on the basis of a circle — above all, it is an integral part of the very form of the double axe. How tempting, then, to see a connection between the cultic significance of the double axe, and the geometrically austere grandeur demonstrated by our analysis.

PLATE XLII A, B.
Double axe of gold. Analysis based on a photograph in Marinatos, Kreta und das Mykenische Hellas. Munich 1959. Slightly larger than life-size. From cutting edge to cutting edge 8.6 cm.



Summary

When attempting to summarize the results of our investigations we clearly see that the main lines of the Egyptian way of thinking dominated Greek art throughout all its phases of development. The idea of order, the heritage of the ancients from the Eastern Mediterranean, was to become the basis also of Greek art, as of Greek philosophy. Geometry opened for the Greeks a world of order, a world whose very essence was absolute.

The Greek development of the geometrical methods does not in any way depart from this basic view. There are logical innovations which, based on the old way of thinking, made possible a more naturalistic and free representation of the human figure. We like to say that the Greeks tried to express themselves with ever-growing freedom, but when doing this, freedom was always seen in relation to law. This law, according to Plato, "is of such a nature that it rules over the willing and not over the oppressed", and our analyses may give us some idea of what is implied in this phrase.

Our examples from Cretan and Mycenaean art showed that the Egyptian methods were known in the Greek Bronze Age realms, and that the latter belonged to the same cultural region as Egypt. It is thus possible that there should have been a continual knowledge and use of these methods in Greece covering the dark ages before the Archaic Period. Ventris' reading of Linear B would also seem to indicate this. But the finds from this period are few, and we have in fact no archaeological holds except those provided by pottery. But the excellent Geometric pottery, surely not made by beginners, may show that the tradition remained in use continually, just like the tradition of literature (mythology). The methods employed in pottery were so simple that plain pots may have kept them alive throughout the long period of decline, until Greece at last experienced a new dawn.

The analyses show how simple the basic principles of the artistic methods were, and it is not surprising that they should have been known and practised throughout large regions inhabited by different peoples, who endowed their art with their own characteristics. Their national features may easily be discerned: art became national, but the method remained international. The ancient methods were in use for a long period of time, and they seem to have formed an important part of the basis on which European art was developed.

Detailed investigations of mediaeval Norwegian material and pre-Christian Norse material show that these methods of Egypt and Greece spread all over Europe already in the Bronze Age and even earlier, thus making one cultural region of all Europe at a very early stage. Analyses of Mesopotamian material indicate that similar investigations towards the Far East would be of the greatest importance and might reveal the same dualism of law and freedom as shown in this book. (*Stave Churches and Viking Ships,* Dreyer 1981.)

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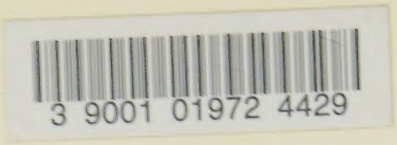
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